# Supplementary Materials for "Relaxing the No Liars Assumption in List Experiment Analyses"

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## A. Appendix

### A.1 Analytical Solution to Linear System (2)

In this section, I detail the steps to obtain the crude bound from linear system (2):

$$
\begin{aligned}
&\max / \min_{(p_{kL}, p_{kN}, p_{kT})_{k=0}^{J}} \textstyle\sum_{k=0}^{J} p_{kL} + p_{kT} \\
&\text{s.t. } c_0 = p_{0N} + p_{0L} + p_{0T},\ \ldots,\ c_J = p_{JN} + p_{JL} + p_{JT} \\
&\quad t_0 = p_{0N} + p_{0L},\ \ldots,\ t_J = p_{JN} + p_{JL} + p_{J-1,T},\ t_{J+1} = p_{JT} \\
&\quad p_{kL}/(p_{kL} + p_{kT}) \leq (p_{JN} + p_{JL})/(p_{JN} + p_{JL} + p_{JT}),\ \forall k = 0, \ldots, J-1.
\end{aligned}
$$

*Step 1*: Identify the sum of non-supporters and lying supporters $p_{kN} + p_{kL}$ and truth-telling supporters $p_{kT}$, $k = 1, ..., J$, from the distribution of responses under control $\{c_k\}_{k=0}^{J}$ and treated $\{t_k\}_{k=0}^{J+1}$.

In particular, I obtain

$$p_{kT} = \sum_{i=k+1}^{J+1} t_i - \sum_{j=k+1}^{J} c_j, \quad p_{kN} + p_{kL} = \sum_{j=k}^{J} c_j - \sum_{i=k+1}^{J+1} t_i. \tag{1}$$

| | |
|---|---|
| $c_0 = p_{0N} + p_{0L} + p_{0T}$ | $t_0 = p_{0N} + p_{0L}$ |
| $c_1 = p_{1N} + p_{1L} + p_{1T}$ | $t_1 = p_{1N} + p_{1L} + p_{0T}$ |
| $c_2 = p_{2N} + p_{2L} + p_{2T}$ | $t_2 = p_{2N} + p_{2L} + p_{1T}$ |
| $c_3 = p_{3N} + p_{3L} + p_{3T}$ | $t_3 = p_{3N} + p_{3L} + p_{2T}$ |
| $c_4 = p_{4N} + p_{4L} + p_{4T}$ | $t_4 = p_{4N} + p_{4L} + p_{3T}$ |
| | $t_5 = p_{4T}$ |

Table A.1: Relationship between latent attitudes and observable responses $(J = 4)$

*Step 2*: Calculate the maximal liar ratio used to construct the crude bound under the relaxed liars assumption:

$$\lambda \equiv \frac{p_{JN} + p_{JL}}{p_{JN} + p_{JL} + p_{JT}} = \frac{c_J - t_{J+1}}{c_J}. \tag{2}$$

*Step 3*: Calculate the maximal proportion of liars for respondents answering affirmatively to fewer than $J$ control items:[1]

$$p_{kL}/(p_{kL} + p_{kT}) \leq \lambda, \tag{3}$$

$$\Rightarrow p_{kL} \leq \frac{\lambda}{1-\lambda} p_{kT}. \tag{4}$$

[1]For list experiments with a modest sample size, $\hat{p}_{kT}$ may be negative even if $p_{kT}$ is positive in the true data generating process. For cases where $\hat{p}_{kT} < 0$, I set $\hat{p}_{kL} = 0$.

*Step 4*: Lower bound and upper bound are given by:

$$\sum_{k=0}^{J} p_{kT} = \sum_{k=0}^{J}(\sum_{i=k+1}^{J+1} t_i - \sum_{j=k+1}^{J} c_j) = \sum_{i=0}^{J+1} it_i - \sum_{j=0}^{J} jc_j, \tag{5}$$

$$\sum_{k=0}^{J} p_{kT} + p_{kL} \leq \sum_{k=0}^{J} p_{kT} + \sum_{k=0}^{J} \min\{\frac{\lambda}{1-\lambda} p_{kT}, p_{kN} + p_{kL}\} \tag{6}$$

$$= (\sum_{i=0}^{J+1} it_i - \sum_{j=0}^{J} jc_j) + \sum_{k=0}^{J} \min\{\frac{\lambda}{1-\lambda}(\sum_{i=k+1}^{J+1} t_i - \sum_{j=k+1}^{J} c_j), \sum_{j=k}^{J} c_j - \sum_{i=k+1}^{J+1} t_i\}, \tag{7}$$

where the lower bound is the standard difference in means estimate, and the upper bound is weakly smaller than $1/(1-\lambda)$ multiplied by the lower bound.

## A.2 Tables for Illustrative Example

Table A.2: Organizations included in the list experiment

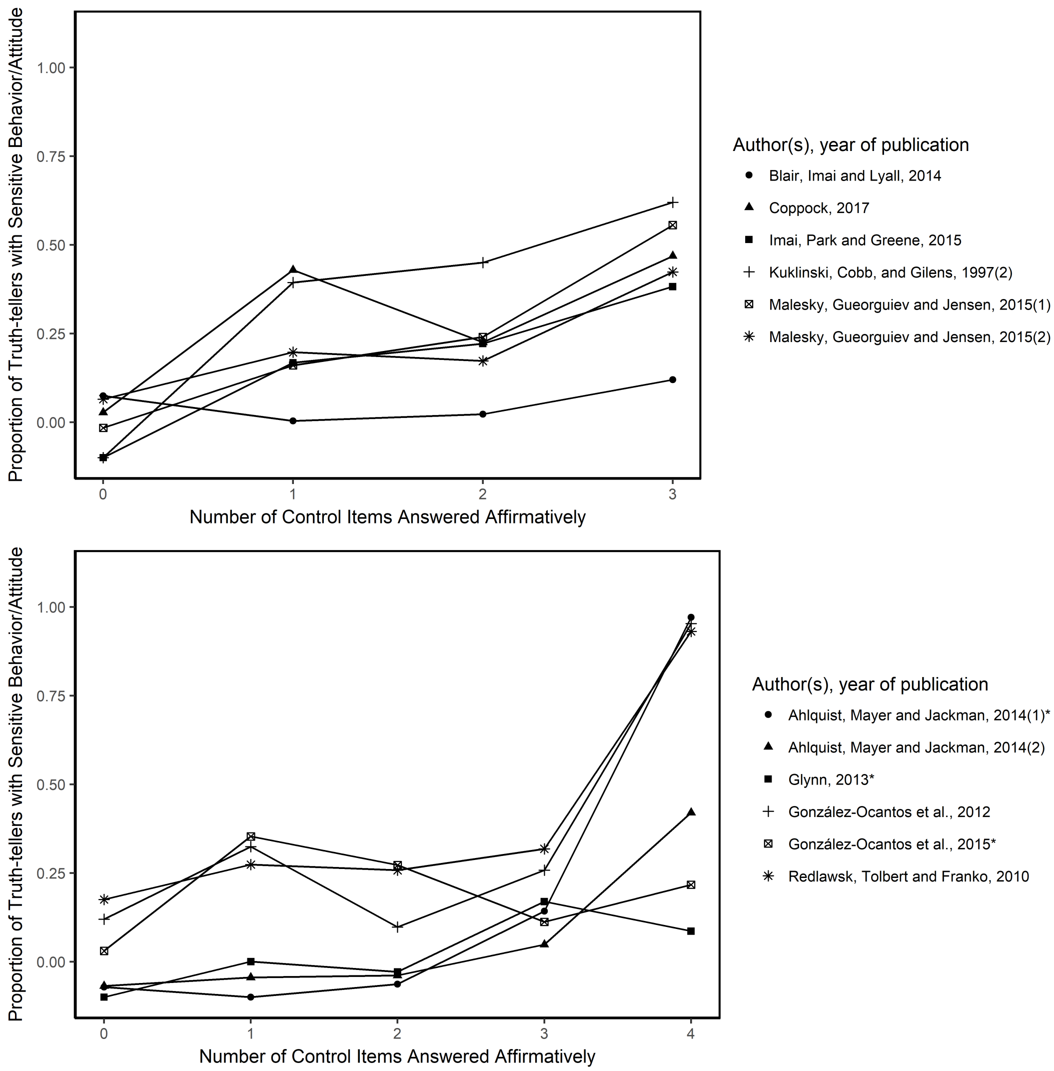
| Item Type | Name (Short Description) |
|---|---|
| Control | *Californians for Disability Rights* (organization advocating for people with disabilities) |
| | *California National Organization for Women* (organization advocating for women's equality and empowerment) |
| | *American Family Association* (organization advocating for pro-family values) |
| | *American Red Cross* (humanitarian organization) |
| X-treatment | *Organization X* (organization advocating for immigration reduction and measures against undocumented immigration) |
| Y-treatment | *Organization Y* (citizen border patrol group combating undocumented immigration) |

Table A.3: Distribution of responses under control, X-treatment and Y-treatment

| | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Control | .13 | .16 | .22 | .25 | .24 | |
| X-treatment | .12 | .11 | .21 | .24 | .15 | .17 |
| Y-treatment | .12 | .15 | .20 | .27 | .12 | .14 |

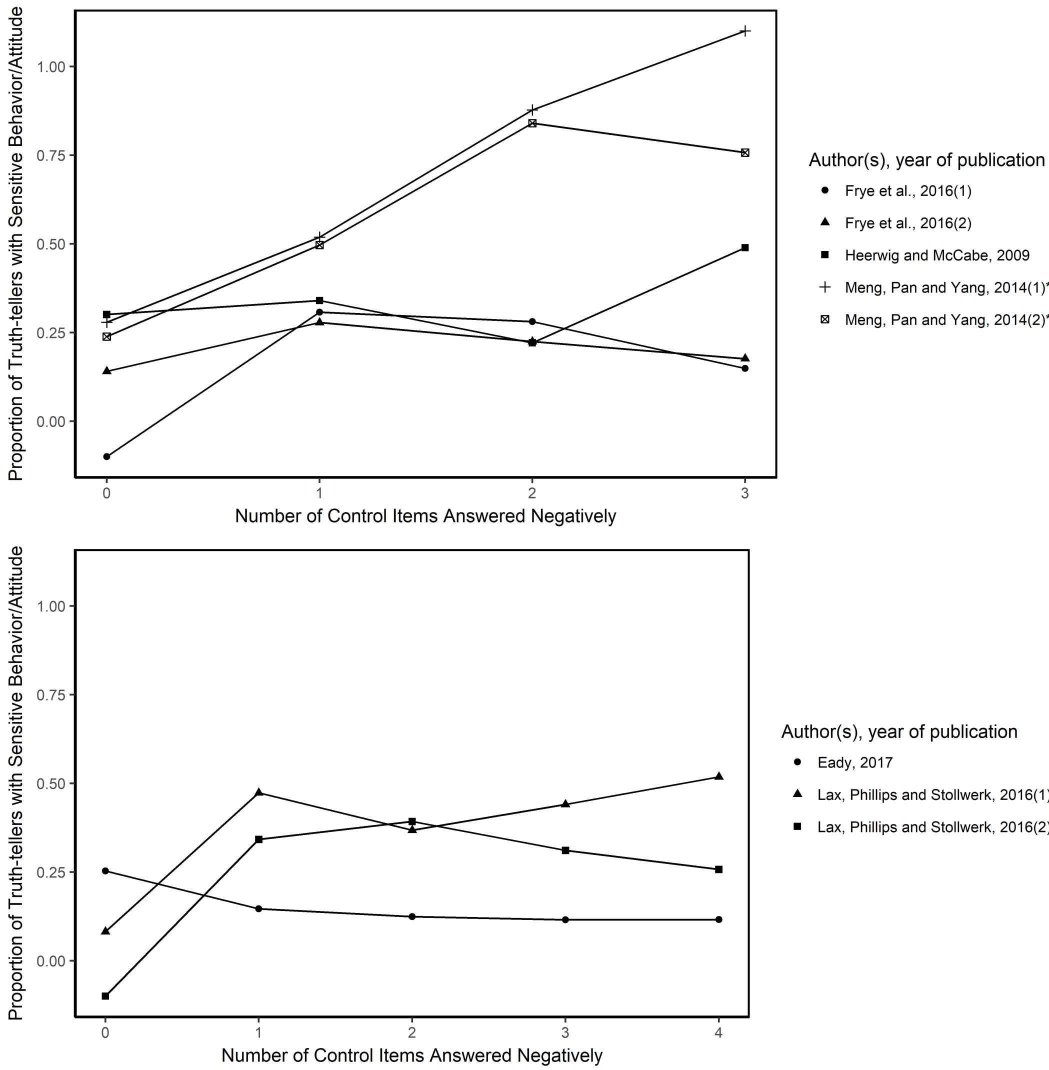
## A.3 Figures for List Experiments in Published Studies

Figure A.1: Proportion of respondents with sensitive behavior/attitude (the sensitive response is affirmative) conditional on the number of control items answered affirmatively




Note: The figure presents the proportion of truth-tellers with sensitive behavior/attitude conditional on the number of control items answered affirmatively for list experiments with affirmative sensitive response (top panel: $J = 3$, bottom panel: $J = 4$). List experiments with fewer than 50 respondents choosing the maximal number of items are marked with stars. Proportions smaller than $-0.1$ are trimmed for graphical presentation.

Figure A.2: Proportion of respondents with sensitive behavior/attitude (the sensitive response is negative) conditional on the number of control items answered negatively




Note: The figure presents the proportion of truth-tellers with sensitive behavior/attitude conditional on the number of control items answered negatively for list experiments with negative sensitive response (top panel: $J = 3$, bottom panel: $J = 4$). List experiments with fewer than 50 respondents choosing the minimal number of items are marked with stars. Proportions smaller than $-0.1$ or larger than $1.1$ are trimmed for graphical presentation.

## A.4 Additional Simulations

### A.4.1 High vs. Low Correlation between Sensitive Item and Control Items

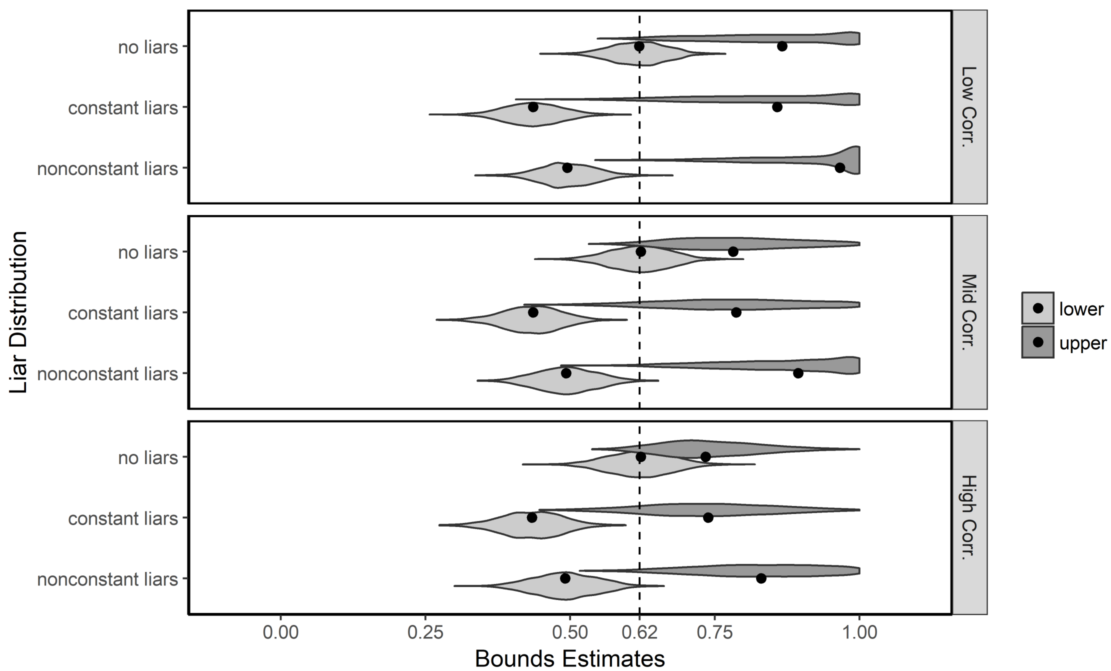



Figure A.3: Bound Estimates for Simulated List Experiments, II

Note: The figure presents the densities of lower bound and upper bound estimates for 2000 simulated datasets for each of the 9 types of list experiments. The high, medium, and low correlation lists are otherwise identical to the correlated lists except that the added pairwise correlation between items are 0.15, 0.1 and 0.05, respectively (instead of 0.1). Lower bound densities are shown in lighter grey and upper bound densities are shown in darker grey. Black dots are the median lower bound and upper bound estimates.

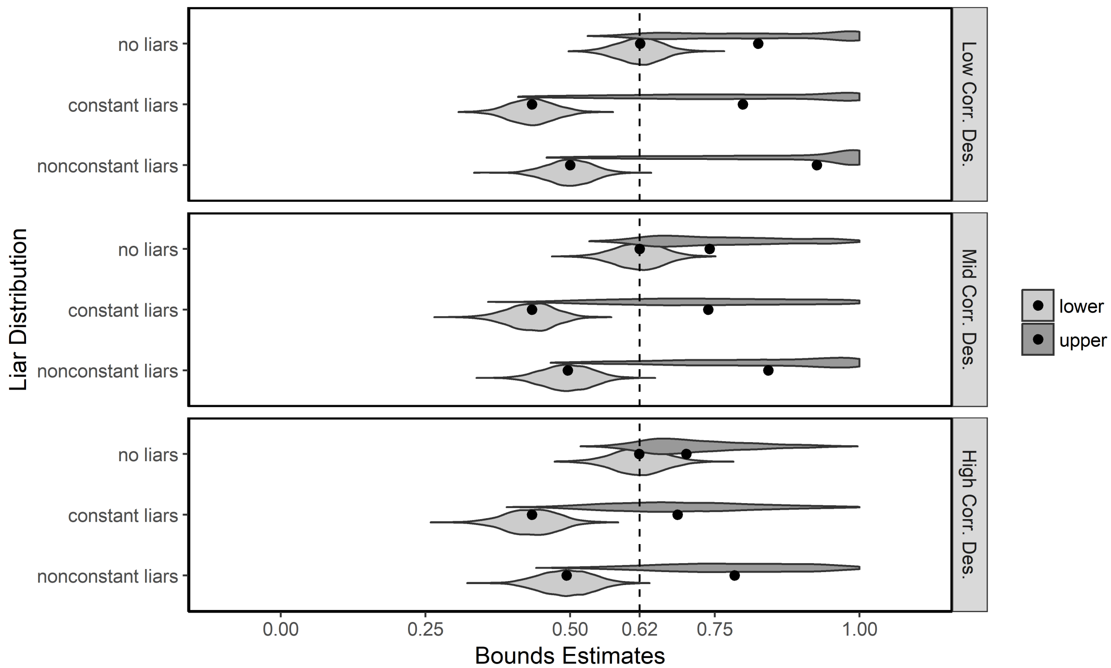



Figure A.4: Bound Estimates for Simulated List Experiments, III

Note: The figure presents the densities of lower bound and upper bound estimates for 2000 simulated datasets for each of the 9 types of list experiments. The high, medium, and low correlation lists are otherwise identical to the correlated design lists except that the added pairwise correlation between items are 0.15, 0.1 and 0.05, respectively. Lower bound densities are shown in lighter grey and upper bound densities are shown in darker grey. Black dots are the median lower bound and upper bound estimates.

### A.4.2 High vs. Low Prevalence of the Sensitive Item

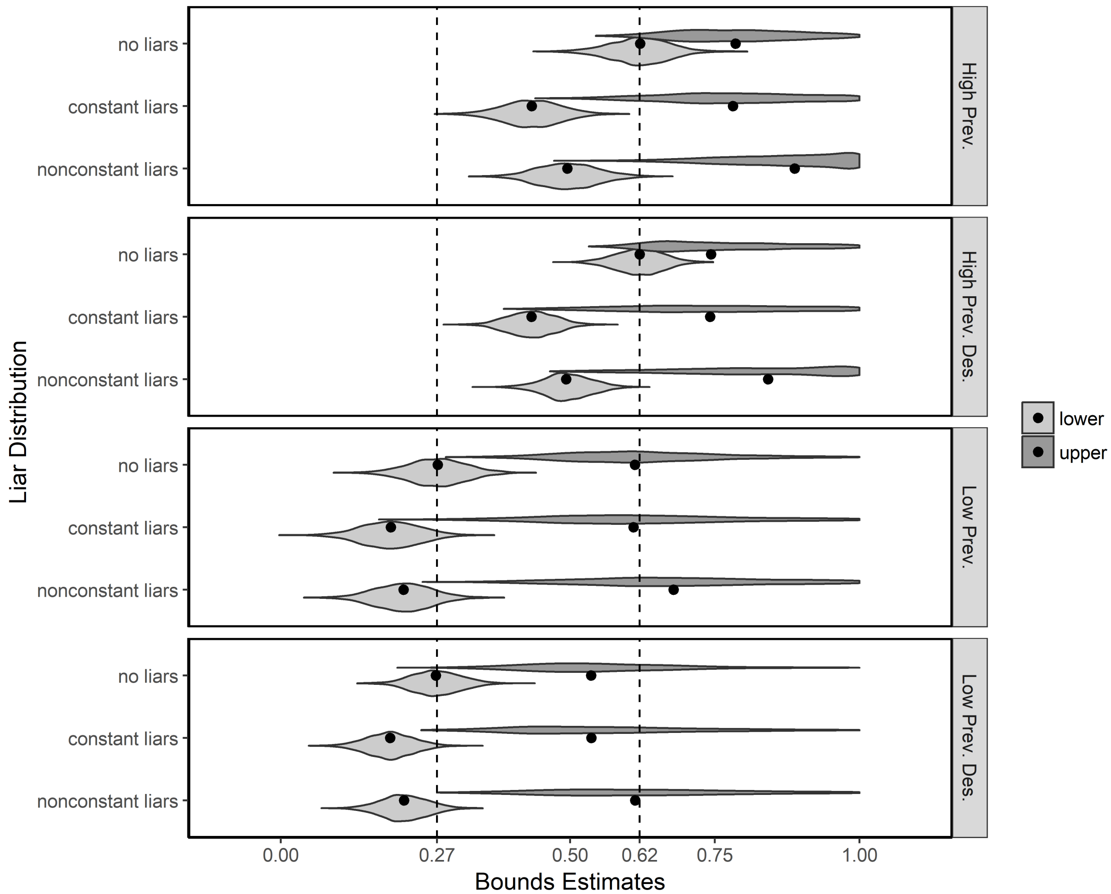



Figure A.5: Bound Estimates for Simulated List Experiments, IV

Note: The figure presents the densities of lower bound and upper bound estimates for 2000 simulated datasets for each of the 12 types of list experiments. The high and low prevalence correlated lists/correlated design lists are otherwise identical to the correlated lists/correlated design lists except that the prevalence of the sensitive item is 0.62 and 0.27, respectively. Lower bound densities are shown in lighter grey and upper bound densities are shown in darker grey. Black dots are the median lower bound and upper bound estimates.

### A.4.3 Affirmative vs. Negative Sensitive Responses

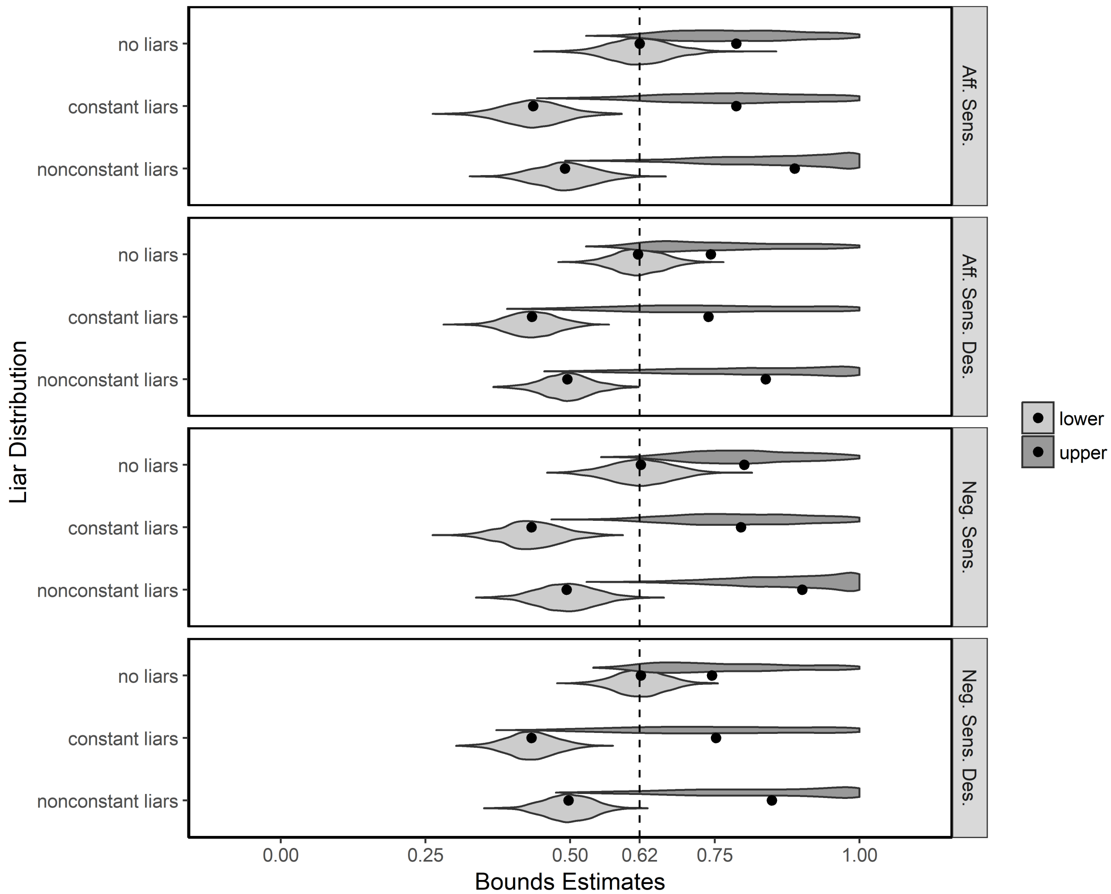



Figure A.6: Bound Estimates for Simulated List Experiments, V

Note: The figure presents the densities of lower bound and upper bound estimates for 2000 simulated datasets for each of the 12 types of list experiments. The affirmative and negative sensitive correlated lists/correlated design lists are otherwise identical to the correlated lists/correlated design lists except that the sensitive response is affirmative and negative, respectively. Lower bound densities are shown in lighter grey and upper bound densities are shown in darker grey. Black dots are the median lower bound and upper bound estimates.

## A.5 List Experiments with Negative Sensitive Responses

I can readily adapt my method to list experiments where a negative answer to the sensitive item is considered sensitive. As in the text, I denote the population fraction of respondents who answer affirmatively to $k$ control items and (1) without the sensitive behavior or attitude by $p_{kN}$; (2) with the sensitive behavior or attitude, but would *not* give the truthful answer for it by $p_{kL}$; (3) with the sensitive behavior or attitude, and give the truthful answer for it by $p_{kT}$. The difference from the case of affirmative sensitive responses is that the truthful answer is negative.

Similar to the case of affirmative sensitive responses, I can establish the relationship between the proportions of different types of respondents and the distribution of answers under control and treatment, shown in table A.4. It follows from a close inspection of the equations that I can identify $p_{kN} + p_{kL}$ and $p_{kT}$ for each $k = 1, ..., J$.

| | |
|---|---|
| $c_0 = p_{0N} + p_{0L} + p_{0T}$ | $t_0 = p_{0T}$ |
| $c_1 = p_{1N} + p_{1L} + p_{1T}$ | $t_1 = p_{0N} + p_{0L} + p_{1T}$ |
| $c_2 = p_{2N} + p_{2L} + p_{2T}$ | $t_2 = p_{1N} + p_{1L} + p_{2T}$ |
| $c_3 = p_{3N} + p_{3L} + p_{3T}$ | $t_3 = p_{2N} + p_{2L} + p_{3T}$ |
| $c_4 = p_{4N} + p_{4L} + p_{4T}$ | $t_4 = p_{3N} + p_{3L} + p_{4T}$ |
| | $t_5 = p_{4N} + p_{4L}$ |

Table A.4: Relationship between latent attitudes and observable responses $(J = 4)$ for list experiments with negative sensitive responses

For list experiments with negative sensitive responses, truth-telling is fully revealing about the sensitive item for respondents whose answer is negative to all control items (floor effects). I consider the relaxed liars assumption that states among all respondents with the sensitive behavior or attitude, the ones who respond negatively to all control items have the strongest incentive to lie:

$$\frac{p_{kL}}{p_{kL}+p_{kT}} \leq \frac{p_{0L}}{p_{0L}+p_{0T}},\ \forall k=1,\ldots,J \tag{8}$$

Similar to the case of affirmative sensitive responses, a crude upper/lower bound for the level of support for the sensitive item is given by the solution to the following linear system:

$$\begin{aligned}
&\max / \min_{(p_{kL},p_{kN},p_{kT})_{k=0}^{J}} \textstyle\sum_{k=0}^{J} p_{kL}+p_{kT} \\
&\text{s.t. } c_0 = p_{0N}+p_{0L}+p_{0T},\ \ldots,\ c_J = p_{JN}+p_{JL}+p_{JT} \\
&\quad t_0 = p_{0T},\ \ldots,\ t_J = p_{J-1,N}+p_{J-1,L}+p_{J,T},\ t_{J+1} = p_{JN}+p_{JL} \\
&\quad p_{kL}/(p_{kL}+p_{kT}) \leq (p_{0N}+p_{0L})/(p_{0N}+p_{0L}+p_{0T}),\ \forall k=1,\ldots,J
\end{aligned} \tag{9}$$

The confidence set for the interval estimate can be constructed in the same way as before.

For the mappings in Table A.4 and the relaxed liars assumption (8), they are mirror-symmetric to the case of affirmative sensitive responses (Table 3 and Inequality (1)). For ease of exposition, in the text I consider proportions of different types of respondents conditional on the number of control items answered negatively (instead of affirmatively). For example, answering negatively to all control items is equivalent to answering affirmatively to zero control items.

## A.6 No Design Effect

While relaxing the no liars assumption, I maintain the assumption of no design effect. No design effect is more likely to be satisfied if respondents consider the items on the list one by one and "do not evaluate items on the list relative to one another" (Imai, 2011, p. 409). While allowing the possibility of liars in my analysis, no design effect still rules out inter-item behavior like concealing preference for the sensitive item by lying about nonsensitive items (e.g., by choosing zero item). However, there is no consensus on how no design effect is likely to be violated in list experiments, which makes it difficult, if at all possible, to develop techniques robust to such violations.

Meanwhile, some researchers (Holbrook and Krosnick, 2010; Ahlquist et al., 2014; Kiewiet de Jonge and Nickerson, 2014; Frye et al., 2016) use placebo list experiments to detect violations of no design effects, where they replace the sensitive item with a nonsensitive item with prevalence either known or estimable. Placebo experiments allow, to some extent, a comparison of the average latent response to control items under control and treatment. If there is no significant difference, researchers may have more confidence in no design effect and focus on potential violations of no liars.

## A.7 Summary of List Experiments in Section 3.2

Table A.5: Summary of List Experiments in Section 3.2

| | Sensitive behavior or attitude | $N_C$ | Yes/No | N | Mode |
|---|---|---|---|---|---|
| Kuklinski et al. (1997) | A black family moving in next door | 3 | Yes | 1213 | telephone |
| | Black leaders asking for affirmative action | 3 | Yes | 1171 | telephone |
| Heerwig and McCabe (2009) | Supporting a black presidential candidate | 3 | No | 1044 | online |
| Redlawsk et al. (2010) | First black president | 4 | Yes | 1395 | telephone |
| Gonzalez-Ocantos et al. (2012) | Vote buying in Nicaragua | 4 | Yes | 995 | face-to-face |
| Glynn (2013) | Black person becoming president | 4 | Yes | 1762 | online |
| Ahlquist et al. (2014) | Voter impersonation (wave 1) | 4 | Yes | 995 | online |
| | Voter impersonation (wave 2) | 4 | Yes | 3000 | online |
| Blair et al. (2014) | Support for ISAF among Pashtun men | 3 | Yes | 1836 | face-to-face |
| Meng et al. (2014) | Officials' receptivity to formal participation in China | 3 | No | 883 | private |
| | Officials' receptivity to Internet participation in China | 3 | No | 868 | private |
| Gonzalez-Ocantos et al. (2015) | Vote buying in Honduras | 4 | Yes | 993 | face-to-face |
| Imai et al. (2015) | Vote buying in Mexico | 3 | Yes | 1120 | face-to-face |
| Malesky et al. (2015) | Bribery in Vietnam (domestic firms) | 3 | Yes | 16236 | NA |
| | Bribery in Vietnam (foreign-invested enterprises) | 3 | Yes | 3570 | NA |
| Frye et al. (2016) | Support for Putin (historical list, March wave) | 3 | No | 1599 | telephone |
| | Support for Putin (contemporary list, March wave) | 3 | No | 1598 | telephone |
| Lax et al. (2016) | Support for same-sex marriage | 4 | No | 1878 | online |
| | Support for employment non-discrimination laws | 4 | No | 1187 | online |
| Rosenfeld et al. (2016) | Vote for an anti-abortion referendum (MS 2011) | 4 | Yes | 1319 | telephone |
| Coppock (2017) | Support for Trump | 3 | Yes | 5290 | online |
| Eady (2017) | Women's competence in politics | 4 | No | 22372 | online |

Note: The table summarizes basic information about list experiments in published studies: the sensitive behaviors or attitudes measured (column 2), the number of control items $N_C$ (column 3), whether an affirmative ('yes') or negative ('no') response to the sensitive item is considered sensitive (column 4), the sample size $N$ (column 5), and the mode of the survey (column 6).

# References

Ahlquist, John S., Kenneth R. Mayer, and Simon Jackman, "Alien Abduction and Voter Impersonation in the 2012 U.S. General Election: Evidence from a Survey List Experiment," *Election Law Journal: Rules, Politics, and Policy*, 2014, *13* (4), 460–475.

de Jonge, Chad P. Kiewiet and David W. Nickerson, "Artificial Inflation or Deflation? Assessing the Item Count Technique in Comparative Surveys," *Political Behavior*, 2014, *36* (3), 659–682.

Frye, Timothy, Scott Gehlbach, Kyle L. Marquardt, and Ora John Reuter, "Is Putin's popularity real?," *Post-Soviet Affairs*, 2016, *33* (1), 1–15.

Holbrook, Allyson L. and Jon A. Krosnick, "Social desirability bias in voter turnout reports: Tests using the item count technique," *Public Opinion Quarterly*, 2010, *74* (1), 37–67.

Imai, Kosuke, "Multivariate Regression Analysis for the Item Count Technique," *Journal of the American Statistical Association*, 2011, *106* (494), 407–416.