

Supplemental Appendix to “Accounting for Skewed or
One-Sided Measurement Error in the Dependent
Variable”

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Table A1. Monte Carlo Results (One-Sided Measurement Error): $E[\sigma_v^2]/E[\sigma_{u_i}^2] = 1$.

Design	Bias				Mean Absolute Error				Mean Squared Error				Coverage Rate			
	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS
I. One-Sided Homoskedastic Errors (N=100)																
β_0	-1.331	-0.096	-0.234	-0.698	1.331	0.344	0.447	1.639	1.791	0.241	0.365	19.381	0.000	0.929	0.763	0.160
β_1	0.000	0.003	-0.001	0.095	0.115	0.114	0.170	0.194	0.021	0.021	0.047	0.125	0.951	0.937	0.918	0.959
β_2	0.007	0.009	-0.003	0.111	0.120	0.122	0.181	0.197	0.022	0.023	0.053	0.158	0.950	0.925	0.892	0.942
II. One-Sided Heteroskedastic Errors (N=100)																
β_0	-1.215	0.068	-0.125	-0.421	1.215	0.348	0.383	1.591	1.501	0.211	0.254	19.046	0.000	0.849	0.757	0.208
β_1	-0.416	-0.326	-0.077	-0.234	0.416	0.328	0.193	0.365	0.198	0.130	0.066	0.258	0.196	0.367	0.843	0.535
β_2	0.299	0.245	0.070	0.346	0.301	0.251	0.180	0.371	0.115	0.087	0.051	0.289	0.498	0.600	0.869	0.596
III. One-Sided Heteroskedastic Errors (N=100)																
β_0	-1.093	0.438	-0.484	1.724	1.093	0.522	0.614	2.962	1.238	0.370	0.490	162.642	0.003	0.518	0.401	0.439
β_1	-0.493	-0.323	-0.211	0.294	0.493	0.327	0.387	0.516	0.290	0.134	0.240	1.330	0.165	0.424	0.524	0.885
β_2	0.118	0.113	0.136	0.360	0.166	0.154	0.188	0.371	0.042	0.037	0.054	0.446	0.855	0.807	0.826	0.783
I. One-Sided Homoskedastic Errors (N=10,000)																
β_0	-1.325	-0.003	-0.003	-0.378	1.325	0.029	0.029	1.756	1.755	0.001	0.001	5.161	0.000	0.950	0.950	0.069
β_1	0.001	0.001	0.000	0.030	0.011	0.011	0.016	0.040	0.000	0.000	0.000	0.007	0.948	0.957	0.951	0.970
β_2	-0.001	-0.001	-0.001	0.027	0.011	0.011	0.018	0.042	0.000	0.000	0.000	0.008	0.947	0.950	0.947	0.963
II. One-Sided Heteroskedastic Errors (N=10,000)																
β_0	-1.207	0.187	-0.017	1.476	1.207	0.187	0.042	2.180	1.458	0.036	0.017	34.333	0.000	0.000	0.936	0.736
β_1	-0.423	-0.339	-0.005	0.077	0.423	0.339	0.020	0.238	0.179	0.115	0.003	0.148	0.000	0.000	0.941	0.792
β_2	0.302	0.242	0.005	-0.045	0.302	0.242	0.019	0.168	0.092	0.059	0.002	0.070	0.000	0.000	0.936	0.803
III. One-Sided Heteroskedastic Errors (N=10,000)																
β_0	-1.031	0.600	-0.618	-0.530	1.031	0.600	0.618	0.585	1.068	0.372	0.428	0.413	0.000	0.000	0.000	0.273
β_1	-0.441	-0.294	-0.242	0.077	0.441	0.294	0.418	0.158	0.195	0.087	0.236	0.043	0.000	0.000	0.001	0.612
β_2	0.118	0.101	0.137	0.253	0.118	0.101	0.138	0.253	0.014	0.010	0.031	0.068	0.004	0.000	0.291	0.000

Notes: Results based on 1000 repetitions. OLS = Ordinary Least Squares. SF1 = Homoskedastic Stochastic Frontier. SF2 = Heteroskedastic Stochastic Frontier. NLLS = Nonlinear Least Squares. Coverage rates based on 95% confidence interval. In Design 1, the one-sided measurement error is homoskedastic. In Designs 2 and 3, the one-sided measurement error is heteroskedastic. In Design 2, the heteroskedasticity function is linear in the covariates and the SF2 and NLLS models use the correct functional form for the heteroskedasticity. In Design 3, the heteroskedasticity function is nonlinear in the covariates and the SF2 and NLLS models use a linear functional form for the heteroskedasticity. See text for further details.

Table A2. Monte Carlo Results (One-Sided Measurement Error): $E[\sigma_v^2]/E[\sigma_u^2] = 2$.

Design	Bias				Mean Absolute Error				Mean Squared Error				Coverage Rate			
	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS
I. One-Sided Homoskedastic Errors (N=100)																
β_0	-0.944	-0.104	-0.202	-0.268	0.944	0.392	0.435	1.346	0.905	0.252	0.290	12.990	0.000	0.908	0.669	0.191
β_1	-0.004	-0.002	-0.004	0.095	0.102	0.102	0.155	0.187	0.016	0.016	0.039	0.150	0.958	0.936	0.898	0.958
β_2	0.007	0.008	0.002	0.098	0.103	0.106	0.159	0.177	0.017	0.018	0.042	0.119	0.950	0.936	0.896	0.950
II. One-Sided Heteroskedastic Errors (N=100)																
β_0	-0.865	0.037	-0.125	0.363	0.865	0.390	0.395	1.744	0.765	0.226	0.231	66.837	0.000	0.848	0.677	0.246
β_1	-0.298	-0.260	-0.098	-0.120	0.298	0.263	0.194	0.315	0.107	0.086	0.062	0.321	0.339	0.457	0.778	0.683
β_2	0.217	0.195	0.086	0.275	0.221	0.203	0.174	0.318	0.065	0.058	0.047	0.275	0.610	0.659	0.853	0.699
III. One-Sided Heteroskedastic Errors (N=100)																
β_0	-0.778	0.355	-0.335	1.502	0.778	0.505	0.471	2.384	0.630	0.336	0.285	126.463	0.005	0.574	0.432	0.457
β_1	-0.346	-0.261	-0.218	0.233	0.347	0.266	0.332	0.407	0.147	0.090	0.179	0.851	0.277	0.472	0.551	0.906
β_2	0.085	0.086	0.091	0.267	0.132	0.131	0.155	0.282	0.027	0.027	0.037	0.223	0.875	0.852	0.865	0.831
I. One-Sided Homoskedastic Errors (N=10,000)																
β_0	-0.936	-0.004	-0.005	0.105	0.936	0.036	0.036	1.562	0.877	0.002	0.002	5.776	0.000	0.953	0.953	0.074
β_1	0.000	0.000	0.000	0.029	0.010	0.009	0.017	0.040	0.000	0.000	0.000	0.007	0.950	0.950	0.951	0.961
β_2	0.000	-0.001	-0.002	0.032	0.010	0.010	0.018	0.042	0.000	0.000	0.000	0.008	0.954	0.950	0.946	0.963
II. One-Sided Heteroskedastic Errors (N=10,000)																
β_0	-0.854	0.174	-0.004	1.438	0.854	0.174	0.032	1.947	0.729	0.032	0.002	31.126	0.000	0.009	0.954	0.738
β_1	-0.299	-0.267	0.001	0.071	0.299	0.267	0.014	0.193	0.090	0.071	0.000	0.110	0.000	0.000	0.953	0.796
β_2	0.214	0.191	0.001	-0.039	0.214	0.191	0.014	0.136	0.046	0.036	0.000	0.052	0.000	0.000	0.950	0.795
III. One-Sided Heteroskedastic Errors (N=10,000)																
β_0	-0.728	0.564	-0.509	-0.374	0.728	0.564	0.509	0.410	0.533	0.330	0.273	0.203	0.000	0.000	0.000	0.282
β_1	-0.312	-0.237	-0.372	0.055	0.312	0.237	0.418	0.113	0.098	0.057	0.197	0.021	0.000	0.000	0.002	0.630
β_2	0.083	0.078	0.047	0.179	0.083	0.078	0.049	0.179	0.007	0.006	0.006	0.034	0.006	0.001	0.635	0.001

Notes: See Table A1 and text for further details.

Table A3. Monte Carlo Results (One-Sided Measurement Error): $E[\sigma_v^2]/E[\sigma_{u_i}^2] = 5$.

Design	Bias				Mean Absolute Error				Mean Squared Error				Coverage Rate			
	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS
I. One-Sided Homoskedastic Errors (N=100)																
β_0	-0.595	-0.064	-0.114	0.179	0.595	0.411	0.379	1.167	0.366	0.217	0.190	21.929	0.000	0.877	0.564	0.230
β_1	-0.005	-0.005	-0.001	0.089	0.091	0.092	0.139	0.168	0.012	0.013	0.031	0.114	0.960	0.943	0.903	0.962
β_2	0.011	0.011	0.004	0.102	0.095	0.099	0.145	0.169	0.014	0.015	0.034	0.111	0.949	0.934	0.899	0.949
II. One-Sided Heteroskedastic Errors (N=100)																
β_0	-0.550	0.017	-0.104	0.077	0.550	0.420	0.367	0.948	0.315	0.217	0.175	12.377	0.000	0.833	0.566	0.251
β_1	-0.187	-0.175	-0.095	-0.066	0.192	0.183	0.175	0.213	0.049	0.045	0.046	0.105	0.624	0.624	0.810	0.789
β_2	0.140	0.133	0.093	0.203	0.155	0.151	0.159	0.234	0.034	0.033	0.041	0.128	0.754	0.749	0.843	0.775
III. One-Sided Heteroskedastic Errors (N=100)																
β_0	-0.493	0.241	-0.153	1.391	0.493	0.480	0.333	1.965	0.260	0.292	0.147	118.033	0.018	0.680	0.538	0.479
β_1	-0.220	-0.187	-0.101	0.186	0.224	0.195	0.211	0.326	0.066	0.051	0.074	0.606	0.524	0.619	0.720	0.920
β_2	0.058	0.061	0.081	0.198	0.109	0.111	0.135	0.228	0.019	0.019	0.030	0.152	0.892	0.874	0.874	0.889
I. One-Sided Homoskedastic Errors (N=10,000)																
β_0	-0.592	-0.016	-0.025	0.496	0.592	0.065	0.072	1.401	0.351	0.008	0.011	6.556	0.000	0.958	0.938	0.079
β_1	0.000	0.000	0.002	0.026	0.009	0.009	0.021	0.040	0.000	0.000	0.001	0.008	0.955	0.956	0.936	0.960
β_2	0.000	0.000	-0.002	0.032	0.009	0.009	0.022	0.041	0.000	0.000	0.001	0.008	0.953	0.949	0.939	0.959
II. One-Sided Heteroskedastic Errors (N=10,000)																
β_0	-0.540	0.139	-0.008	1.137	0.540	0.140	0.043	1.516	0.292	0.023	0.004	23.424	0.000	0.330	0.943	0.686
β_1	-0.189	-0.183	-0.001	0.049	0.189	0.183	0.015	0.146	0.036	0.034	0.001	0.069	0.000	0.000	0.954	0.751
β_2	0.135	0.130	0.003	-0.020	0.135	0.130	0.016	0.098	0.018	0.017	0.001	0.025	0.000	0.000	0.939	0.745
III. One-Sided Heteroskedastic Errors (N=10,000)																
β_0	-0.461	0.504	-0.210	-0.230	0.461	0.504	0.210	0.264	0.214	0.264	0.047	0.087	0.000	0.000	0.000	0.298
β_1	-0.197	-0.170	0.030	0.037	0.197	0.170	0.100	0.074	0.039	0.029	0.017	0.010	0.000	0.000	0.049	0.690
β_2	0.053	0.053	0.104	0.114	0.053	0.053	0.105	0.114	0.003	0.003	0.012	0.014	0.015	0.008	0.100	0.002

Notes: See Table A1 and text for further details.

Table A4. Monte Carlo Results (One-Sided Measurement Error): $E[\sigma_v^2]/E[\sigma_u^2] = 10$.

Design	Bias				Mean Absolute Error				Mean Squared Error				Coverage Rate			
	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS
I. One-Sided Homoskedastic Errors (N=100)																
β_0	-0.424	0.019	-0.041	0.459	0.424	0.390	0.337	1.123	0.190	0.190	0.157	25.854	0.015	0.854	0.527	0.289
β_1	0.000	0.001	0.011	0.119	0.087	0.089	0.129	0.187	0.012	0.012	0.027	0.185	0.952	0.939	0.918	0.951
β_2	0.008	0.007	0.016	0.091	0.092	0.095	0.137	0.167	0.013	0.014	0.031	0.086	0.951	0.930	0.903	0.947
II. One-Sided Heteroskedastic Errors (N=100)																
β_0	-0.388	0.077	-0.053	0.498	0.388	0.399	0.318	1.094	0.161	0.201	0.134	39.153	0.041	0.823	0.525	0.344
β_1	-0.132	-0.127	-0.087	-0.014	0.145	0.143	0.160	0.200	0.030	0.030	0.039	0.186	0.734	0.726	0.826	0.858
β_2	0.102	0.099	0.093	0.176	0.127	0.127	0.152	0.209	0.024	0.024	0.037	0.113	0.821	0.810	0.848	0.843
III. One-Sided Heteroskedastic Errors (N=100)																
β_0	-0.353	0.202	-0.049	0.847	0.353	0.450	0.290	1.264	0.137	0.260	0.120	26.807	0.112	0.725	0.608	0.572
β_1	-0.157	-0.141	-0.063	0.149	0.166	0.154	0.170	0.267	0.038	0.033	0.047	0.288	0.685	0.717	0.807	0.932
β_2	0.042	0.044	0.065	0.167	0.100	0.103	0.130	0.202	0.015	0.016	0.028	0.118	0.911	0.891	0.867	0.913
I. One-Sided Homoskedastic Errors (N=10,000)																
β_0	-0.419	-0.051	-0.089	0.501	0.419	0.129	0.156	1.143	0.176	0.032	0.043	4.703	0.000	0.922	0.733	0.091
β_1	0.000	0.000	0.004	0.023	0.008	0.008	0.025	0.032	0.000	0.000	0.001	0.005	0.948	0.949	0.917	0.961
β_2	0.000	0.000	-0.002	0.028	0.008	0.008	0.025	0.036	0.000	0.000	0.001	0.007	0.954	0.951	0.923	0.955
II. One-Sided Heteroskedastic Errors (N=10,000)																
β_0	-0.382	0.078	-0.089	0.695	0.382	0.134	0.124	1.037	0.146	0.025	0.031	11.614	0.000	0.685	0.688	0.612
β_1	-0.134	-0.132	-0.046	0.012	0.134	0.132	0.058	0.114	0.018	0.018	0.008	0.039	0.000	0.000	0.693	0.673
β_2	0.095	0.094	0.035	0.005	0.095	0.094	0.044	0.080	0.009	0.009	0.004	0.017	0.000	0.000	0.689	0.662
III. One-Sided Heteroskedastic Errors (N=10,000)																
β_0	-0.326	0.450	-0.151	-0.159	0.326	0.450	0.151	0.192	0.107	0.213	0.024	0.050	0.000	0.001	0.030	0.305
β_1	-0.139	-0.128	0.020	0.027	0.139	0.128	0.067	0.055	0.020	0.017	0.008	0.006	0.000	0.000	0.292	0.775
β_2	0.037	0.038	0.075	0.081	0.037	0.038	0.076	0.081	0.002	0.002	0.006	0.007	0.075	0.060	0.092	0.005

Notes: See Table A1 and text for further details.

Table A5. Monte Carlo Results: Total Outcomes.

Design	Bias				Mean Absolute Percentage Error			
	Observed	SF1	SF2	NLLS	Observed	SF1	SF2	NLLS
I. One-Sided Homoskedastic Errors (N=100)								
$E[\sigma_v^2]/E[\sigma_{u_i}^2] = 1$	-132.09	-8.63	-22.43	-68.98	138.59	34.87	45.21	173.68
$E[\sigma_v^2]/E[\sigma_{u_i}^2] = 2$	-93.56	-9.56	-19.52	-26.00	97.67	40.08	44.06	142.51
$E[\sigma_v^2]/E[\sigma_{u_i}^2] = 5$	-59.25	-6.13	-11.19	18.08	61.28	42.11	38.00	122.94
$E[\sigma_v^2]/E[\sigma_{u_i}^2] = 10$	-41.98	2.34	-3.77	46.09	43.69	40.50	35.24	123.27
II. One-Sided Heteroskedastic Errors (N=100)								
$E[\sigma_v^2]/E[\sigma_{u_i}^2] = 1$	-120.70	7.57	-12.08	-41.55	126.80	36.43	38.87	168.00
$E[\sigma_v^2]/E[\sigma_{u_i}^2] = 2$	-85.43	4.70	-11.62	37.16	90.06	40.77	39.84	184.69
$E[\sigma_v^2]/E[\sigma_{u_i}^2] = 5$	-54.05	2.61	-9.73	8.33	56.77	44.13	37.37	101.46
$E[\sigma_v^2]/E[\sigma_{u_i}^2] = 10$	-38.29	8.23	-4.96	50.25	39.97	42.01	32.99	110.44
III. One-Sided Heteroskedastic Errors (N=100)								
$E[\sigma_v^2]/E[\sigma_{u_i}^2] = 1$	-109.37	43.87	-48.48	171.32	113.13	53.13	63.13	312.70
$E[\sigma_v^2]/E[\sigma_{u_i}^2] = 2$	-77.39	35.99	-33.18	149.78	80.34	51.96	48.13	252.13
$E[\sigma_v^2]/E[\sigma_{u_i}^2] = 5$	-49.04	24.40	-14.91	138.46	50.90	49.53	34.08	210.06
$E[\sigma_v^2]/E[\sigma_{u_i}^2] = 10$	-34.87	20.67	-4.50	84.46	36.43	46.93	29.44	130.03
I. One-Sided Homoskedastic Errors (N=10,000)								
$E[\sigma_v^2]/E[\sigma_{u_i}^2] = 1$	-13245.34	-26.55	-30.28	-3777.28	132.54	2.80	2.81	175.79
$E[\sigma_v^2]/E[\sigma_{u_i}^2] = 2$	-9363.96	-37.86	-45.65	1054.40	93.70	3.50	3.51	156.38
$E[\sigma_v^2]/E[\sigma_{u_i}^2] = 5$	-5923.38	-161.39	-250.36	4965.25	59.26	6.36	7.07	140.24
$E[\sigma_v^2]/E[\sigma_{u_i}^2] = 10$	-4187.72	-505.63	-890.61	5014.81	41.89	12.87	15.58	114.25
II. One-Sided Heteroskedastic Errors (N=10,000)								
$E[\sigma_v^2]/E[\sigma_{u_i}^2] = 1$	-12071.90	1867.43	-171.39	14765.10	120.81	18.69	4.11	218.17
$E[\sigma_v^2]/E[\sigma_{u_i}^2] = 2$	-8536.36	1736.58	-38.91	14381.46	85.43	17.38	3.10	194.56
$E[\sigma_v^2]/E[\sigma_{u_i}^2] = 5$	-5399.14	1389.82	-79.91	11366.56	54.04	14.04	4.27	151.58
$E[\sigma_v^2]/E[\sigma_{u_i}^2] = 10$	-3817.81	780.25	-889.31	6949.96	38.18	13.40	12.29	103.33
III. One-Sided Heteroskedastic Errors (N=10,000)								
$E[\sigma_v^2]/E[\sigma_{u_i}^2] = 1$	-10303.98	5997.57	-6174.13	-5296.71	103.14	60.02	61.80	58.49
$E[\sigma_v^2]/E[\sigma_{u_i}^2] = 2$	-7281.07	5643.49	-5093.58	-3741.88	72.84	56.45	50.97	41.03
$E[\sigma_v^2]/E[\sigma_{u_i}^2] = 5$	-4608.65	5041.64	-2103.20	-2304.26	46.12	50.45	21.05	26.40
$E[\sigma_v^2]/E[\sigma_{u_i}^2] = 10$	-3261.62	4499.10	-1512.41	-1589.54	32.64	45.02	15.14	19.24

Notes: OLS = Ordinary Least Squares. SF1 = Homoskedastic Stochastic Frontier. SF2 = Heteroskedastic Stochastic Frontier. NLLS = Nonlinear Least Squares. Values represent the bias and mean absolute percentage error of the sum of the observed and predicted dependent variables relative to the sum of the actual dependent variable, where the sum is computed over the observations in the simulated data. See Table A1 and text for further details.

Table A6. Monte Carlo Results (Skewed Measurement Error): $E[\sigma_v^2]/E[\sigma_{u_i}^2] = 1$.

Design	Bias				Mean Absolute Error				Mean Squared Error				Coverage Rate			
	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS
I. Skewed Homoskedastic Errors (N=100)																
β_0	-1.720	-1.121	-1.115	-1.225	1.720	1.123	1.117	1.856	2.973	1.495	1.443	14.073	0.000	0.741	0.275	0.129
β_1	-0.004	-0.003	-0.014	0.059	0.097	0.099	0.153	0.152	0.015	0.015	0.037	0.062	0.948	0.935	0.909	0.971
β_2	0.001	0.002	-0.012	0.089	0.088	0.089	0.149	0.156	0.012	0.013	0.034	0.116	0.960	0.948	0.924	0.966
II. Skewed Heteroskedastic Errors (N=100)																
β_0	-1.552	-0.831	-1.015	0.801	1.552	0.836	1.018	3.203	2.431	0.951	1.197	121.780	0.000	0.876	0.238	0.260
β_1	-0.618	-0.590	-0.390	-0.290	0.618	0.590	0.391	0.620	0.398	0.364	0.202	0.827	0.002	0.002	0.458	0.358
β_2	0.438	0.423	0.282	0.401	0.438	0.423	0.292	0.515	0.206	0.192	0.116	0.505	0.037	0.055	0.568	0.334
III. Skewed Heteroskedastic Errors (N=100)																
β_0	-1.358	0.009	-0.960	2.958	1.358	0.414	0.967	4.029	1.901	0.314	1.033	271.803	0.003	0.782	0.101	0.618
β_1	-0.677	-0.537	-0.566	0.605	0.677	0.537	0.607	0.758	0.499	0.316	0.559	2.499	0.014	0.076	0.396	0.902
β_2	0.176	0.185	0.184	0.544	0.206	0.203	0.212	0.545	0.061	0.058	0.066	0.604	0.706	0.615	0.711	0.557
I. Skewed Homoskedastic Errors (N=10,000)																
β_0	-1.716	-1.035	-1.052	-0.568	1.716	1.035	1.052	2.123	2.944	1.076	1.120	6.628	0.000	0.000	0.000	0.070
β_1	-0.001	-0.001	-0.003	0.031	0.010	0.009	0.022	0.043	0.000	0.000	0.001	0.009	0.952	0.950	0.937	0.964
β_2	0.000	0.000	-0.002	0.028	0.010	0.010	0.023	0.040	0.000	0.000	0.001	0.007	0.947	0.945	0.935	0.962
II. Skewed Heteroskedastic Errors (N=10,000)																
β_0	-1.547	-0.653	-1.008	0.677	1.547	0.653	1.008	1.081	2.392	0.428	1.018	3.855	0.000	0.000	0.000	0.839
β_1	-0.611	-0.591	-0.297	0.038	0.611	0.591	0.297	0.160	0.374	0.349	0.088	0.051	0.000	0.000	0.000	0.832
β_2	0.437	0.422	0.211	-0.028	0.437	0.422	0.211	0.116	0.191	0.178	0.045	0.027	0.000	0.000	0.000	0.834
III. Skewed Heteroskedastic Errors (N=10,000)																
β_0	-1.289	0.269	-0.933	-0.688	1.289	0.269	0.933	0.688	1.673	0.103	0.889	0.508	0.000	0.017	0.000	0.011
β_1	-0.637	-0.501	-0.492	0.077	0.637	0.501	0.492	0.127	0.407	0.253	0.456	0.025	0.000	0.000	0.148	0.215
β_2	0.170	0.164	0.186	0.357	0.170	0.164	0.186	0.357	0.030	0.027	0.050	0.131	0.000	0.000	0.026	0.000

Notes: Results based on 1000 repetitions. OLS = Ordinary Least Squares. SF1 = Homoskedastic Stochastic Frontier. SF2 = Heteroskedastic Stochastic Frontier. NLLS = Nonlinear Least Squares. Coverage rates based on 95% confidence interval. In Design 1, the skewed measurement error is homoskedastic. In Designs 2 and 3, the skewed measurement error is heteroskedastic. In Design 2, the heteroskedasticity function is linear in the covariates and the SF2 and NLLS models use the correct functional form for the heteroskedasticity. In Design 3, the heteroskedasticity function is nonlinear in the covariates and the SF2 and NLLS models use a linear functional form for the heteroskedasticity. See text for further details.

Table A7. Monte Carlo Results (Skewed Measurement Error): $E[\sigma_v^2]/E[\sigma_{u_i}^2] = 2$.

Design	Bias				Mean Absolute Error				Mean Squared Error				Coverage Rate			
	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS
I. Skewed Homoskedastic Errors (N=100)																
β_0	-1.155	-0.666	-0.624	-0.442	1.155	0.685	0.644	1.573	1.347	0.644	0.566	23.157	0.000	0.996	0.600	0.150
β_1	-0.004	-0.003	-0.009	0.073	0.086	0.088	0.144	0.151	0.012	0.012	0.032	0.077	0.964	0.953	0.908	0.964
β_2	0.005	0.005	0.001	0.096	0.083	0.084	0.148	0.159	0.011	0.011	0.033	0.118	0.959	0.934	0.904	0.962
II. Skewed Heteroskedastic Errors (N=100)																
β_0	-1.036	-0.501	-0.627	0.545	1.036	0.555	0.647	2.180	1.089	0.466	0.524	65.630	0.000	0.986	0.435	0.248
β_1	-0.435	-0.423	-0.314	-0.237	0.435	0.423	0.321	0.446	0.203	0.193	0.145	0.428	0.032	0.042	0.506	0.385
β_2	0.313	0.307	0.228	0.364	0.313	0.307	0.245	0.405	0.109	0.105	0.083	0.330	0.171	0.179	0.611	0.401
III. Skewed Heteroskedastic Errors (N=100)																
β_0	-0.897	0.150	-0.606	1.679	0.897	0.461	0.618	2.407	0.838	0.332	0.440	67.034	0.011	0.712	0.223	0.626
β_1	-0.483	-0.405	-0.538	0.403	0.483	0.405	0.564	0.541	0.258	0.183	0.419	1.139	0.052	0.145	0.281	0.919
β_2	0.120	0.130	0.090	0.367	0.151	0.152	0.144	0.370	0.034	0.034	0.032	0.259	0.805	0.756	0.878	0.749
I. Skewed Homoskedastic Errors (N=10,000)																
β_0	-1.154	-0.699	-0.710	-0.164	1.154	0.699	0.710	1.629	1.333	0.509	0.525	4.231	0.000	0.017	0.006	0.081
β_1	-0.001	-0.001	-0.001	0.028	0.009	0.009	0.027	0.039	0.000	0.000	0.001	0.006	0.942	0.938	0.906	0.955
β_2	0.000	0.000	0.000	0.026	0.009	0.009	0.027	0.036	0.000	0.000	0.001	0.006	0.952	0.952	0.899	0.964
II. Skewed Heteroskedastic Errors (N=10,000)																
β_0	-1.035	-0.377	-0.676	0.862	1.035	0.377	0.676	1.135	1.072	0.146	0.459	10.436	0.000	0.000	0.000	0.872
β_1	-0.432	-0.426	-0.218	0.050	0.432	0.426	0.218	0.146	0.187	0.182	0.048	0.056	0.000	0.000	0.000	0.850
β_2	0.309	0.304	0.155	-0.036	0.309	0.304	0.155	0.106	0.095	0.093	0.024	0.029	0.000	0.000	0.000	0.857
III. Skewed Heteroskedastic Errors (N=10,000)																
β_0	-0.852	0.417	-0.625	-0.428	0.852	0.417	0.625	0.429	0.731	0.197	0.396	0.202	0.000	0.000	0.000	0.028
β_1	-0.450	-0.379	-0.668	0.053	0.450	0.379	0.668	0.091	0.204	0.144	0.467	0.013	0.000	0.000	0.017	0.287
β_2	0.120	0.121	0.040	0.252	0.120	0.121	0.041	0.252	0.015	0.015	0.003	0.065	0.000	0.000	0.381	0.000

Notes: See Table A6 and text for further details.

Table A8. Monte Carlo Results (Skewed Measurement Error): $E[\sigma_v^2]/E[\sigma_{u_i}^2] = 5$.

Design	Bias				Mean Absolute Error				Mean Squared Error				Coverage Rate			
	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS
I. Skewed Homoskedastic Errors (N=100)																
β_0	-0.657	-0.230	-0.242	-0.196	0.657	0.412	0.386	0.917	0.442	0.229	0.204	9.187	0.000	0.947	0.582	0.198
β_1	-0.002	-0.002	-0.006	0.069	0.082	0.084	0.132	0.135	0.011	0.011	0.027	0.068	0.953	0.944	0.910	0.966
β_2	0.005	0.006	0.009	0.077	0.081	0.082	0.132	0.145	0.010	0.011	0.029	0.068	0.953	0.944	0.915	0.962
II. Skewed Heteroskedastic Errors (N=100)																
β_0	-0.578	-0.156	-0.253	0.094	0.578	0.395	0.381	0.991	0.347	0.199	0.183	7.778	0.004	0.934	0.504	0.275
β_1	-0.278	-0.274	-0.242	-0.153	0.278	0.275	0.256	0.289	0.088	0.087	0.090	0.172	0.254	0.256	0.537	0.581
β_2	0.203	0.201	0.179	0.263	0.204	0.203	0.205	0.286	0.052	0.051	0.061	0.156	0.558	0.545	0.678	0.624
III. Skewed Heteroskedastic Errors (N=100)																
β_0	-0.488	0.230	-0.241	1.648	0.489	0.504	0.308	2.074	0.258	0.327	0.124	115.293	0.052	0.673	0.575	0.645
β_1	-0.306	-0.273	-0.271	0.270	0.306	0.274	0.311	0.401	0.109	0.089	0.152	0.787	0.238	0.340	0.521	0.915
β_2	0.075	0.082	0.083	0.252	0.112	0.114	0.134	0.263	0.020	0.020	0.029	0.226	0.886	0.854	0.861	0.880
I. Skewed Homoskedastic Errors (N=10,000)																
β_0	-0.656	-0.390	-0.352	0.475	0.656	0.390	0.352	1.467	0.431	0.186	0.145	6.511	0.000	0.704	0.362	0.082
β_1	-0.001	-0.001	-0.002	0.031	0.009	0.009	0.028	0.042	0.000	0.000	0.001	0.009	0.950	0.950	0.885	0.942
β_2	0.000	0.000	0.001	0.028	0.008	0.008	0.027	0.039	0.000	0.000	0.001	0.008	0.950	0.949	0.895	0.958
II. Skewed Heteroskedastic Errors (N=10,000)																
β_0	-0.581	-0.193	-0.363	1.203	0.581	0.195	0.363	1.376	0.338	0.064	0.134	20.096	0.000	0.965	0.000	0.882
β_1	-0.274	-0.273	-0.144	0.060	0.274	0.273	0.144	0.140	0.075	0.075	0.021	0.066	0.000	0.000	0.000	0.855
β_2	0.195	0.194	0.103	-0.039	0.195	0.194	0.103	0.099	0.038	0.038	0.011	0.032	0.000	0.000	0.000	0.854
III. Skewed Heteroskedastic Errors (N=10,000)																
β_0	-0.465	0.508	-0.279	-0.196	0.465	0.508	0.279	0.201	0.219	0.275	0.079	0.047	0.000	0.000	0.002	0.111
β_1	-0.285	-0.257	-0.356	0.034	0.285	0.257	0.356	0.058	0.082	0.067	0.165	0.006	0.000	0.000	0.173	0.434
β_2	0.076	0.078	0.045	0.160	0.076	0.078	0.046	0.160	0.006	0.006	0.005	0.026	0.007	0.000	0.596	0.000

Notes: See Table A6 and text for further details.

Table A9. Monte Carlo Results (Skewed Measurement Error): $E[\sigma_v^2]/E[\sigma_{u_i}^2] = 10$.

Design	Bias				Mean Absolute Error				Mean Squared Error				Coverage Rate			
	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS	OLS	SF1	SF2	NLLS
I. Skewed Homoskedastic Errors (N=100)																
β_0	-0.402	-0.008	-0.044	-0.003	0.402	0.361	0.321	0.651	0.172	0.163	0.133	7.147	0.026	0.893	0.536	0.267
β_1	0.001	0.001	0.005	0.065	0.082	0.083	0.122	0.134	0.010	0.011	0.024	0.060	0.961	0.942	0.925	0.972
β_2	0.006	0.006	0.015	0.080	0.078	0.080	0.125	0.141	0.010	0.010	0.025	0.074	0.951	0.940	0.920	0.963
II. Skewed Heteroskedastic Errors (N=100)																
β_0	-0.348	0.047	-0.040	0.458	0.348	0.358	0.290	0.981	0.132	0.163	0.115	27.100	0.081	0.879	0.616	0.382
β_1	-0.199	-0.197	-0.176	-0.101	0.201	0.199	0.203	0.236	0.050	0.049	0.060	0.189	0.526	0.524	0.692	0.743
β_2	0.142	0.141	0.144	0.211	0.149	0.149	0.179	0.234	0.030	0.030	0.047	0.133	0.737	0.737	0.769	0.771
III. Skewed Heteroskedastic Errors (N=100)																
β_0	-0.288	0.265	-0.054	1.397	0.291	0.471	0.222	1.658	0.098	0.303	0.077	49.763	0.232	0.707	0.726	0.743
β_1	-0.216	-0.199	-0.154	0.224	0.218	0.202	0.212	0.342	0.060	0.052	0.075	0.540	0.473	0.530	0.707	0.918
β_2	0.055	0.058	0.076	0.189	0.098	0.099	0.126	0.212	0.015	0.016	0.026	0.126	0.919	0.906	0.888	0.918
I. Skewed Homoskedastic Errors (N=10,000)																
β_0	-0.406	-0.198	-0.203	0.526	0.406	0.217	0.216	1.135	0.165	0.071	0.068	4.821	0.000	0.986	0.620	0.122
β_1	-0.001	-0.001	0.001	0.024	0.008	0.008	0.024	0.034	0.000	0.000	0.001	0.005	0.950	0.950	0.904	0.951
β_2	0.000	0.000	0.001	0.028	0.008	0.008	0.024	0.037	0.000	0.000	0.001	0.007	0.947	0.947	0.904	0.953
II. Skewed Heteroskedastic Errors (N=10,000)																
β_0	-0.352	-0.097	-0.220	1.205	0.352	0.164	0.220	1.328	0.124	0.043	0.050	18.736	0.000	0.953	0.053	0.861
β_1	-0.194	-0.194	-0.171	0.049	0.194	0.194	0.171	0.129	0.038	0.038	0.036	0.054	0.000	0.000	0.000	0.828
β_2	0.138	0.138	0.122	-0.029	0.138	0.138	0.122	0.092	0.019	0.019	0.018	0.030	0.000	0.000	0.014	0.818
III. Skewed Heteroskedastic Errors (N=10,000)																
β_0	-0.271	0.523	-0.127	-0.078	0.271	0.523	0.127	0.094	0.075	0.287	0.017	0.012	0.000	0.000	0.015	0.334
β_1	-0.202	-0.190	-0.060	0.025	0.202	0.190	0.062	0.043	0.041	0.036	0.016	0.003	0.000	0.000	0.709	0.589
β_2	0.054	0.055	0.088	0.113	0.054	0.055	0.089	0.113	0.003	0.003	0.009	0.013	0.010	0.005	0.113	0.000

Notes: See Table A6 and text for further details.