Online Appendix for "The Structure of Political Choices: Distinguishing Between Constraint and Multidimensionality"

William Marble* Matthew Tyler†

December 22, 2020

*Ph.D. Candidate, Department of Political Science, Stanford University. wpmarble@stanford.edu
†Ph.D. Candidate, Department of Political Science, Stanford University. mdtyler@stanford.edu

Contents

A Technical Appendix: `MultiScale` Algorithm

The Parametric Model

N voters and J binary questions to vote on. The vote matrix is $Y \in \{0,1\}^{N \times J}$. For some $D \in \mathbb{N}$, Let $\alpha_j \in \mathbb{R}$, $\beta_j \in \mathbb{R}^D$ and $\gamma_i \in \mathbb{R}^D$ for each $j = 1, \ldots, J$ and $i = 1, \ldots, N$. We assume the following latent variable model generates the binary vote matrix Y.

$$\begin{aligned} y_{ij} &= I(s_{ij} > 0) \\ s_{ij} &= \alpha_j + \beta_j^\intercal \gamma_i + \epsilon_{ij}, \quad \epsilon_{ij} \overset{\text{ind.}}{\sim} N(0,1), \end{aligned}$$

where we have assumed $\sigma = 1$, since it is not identified. Note that for $\theta = (\{\alpha_j\}_{j=1}^J, \{\beta_j\}_{j=1}^J, \{\gamma_i\}_{i=1}^N)$, this implies the reduced form likelihood

$$p(Y \mid \theta) = \prod_{i=1}^N \prod_{j=1}^J \left[\Phi(\alpha_j + \beta_j^\intercal \gamma_i)\right]^{y_{ij}} \left[1 - \Phi(\alpha_j + \beta_j^\intercal \gamma_i)\right]^{1-y_{ij}}.$$

Let $R \in \{0,1\}^{N \times J}$ denote the matrix of observation statuses. That is $r_{ij} = 1$, if the (i,j)th cell of Y is observed and $r_{ij} = 0$ if it is missing. We assume the data are missing at random, $P(R \mid Y_{\text{obs}}, Y_{\text{mis}}, \theta, \omega) = P(R \mid Y_{\text{obs}}, \theta, \omega)$, and that the parameters ω that determine R are distinct from the structural voting parameters θ, meaning that we can ignore the likelihood of R (Section 6.2, Little and Rubin, 2014). The resulting (ignorable) likelihood is

$$p(Y_{\text{obs}} \mid \theta) = \prod_{i=1}^N \prod_{j=1}^J \left\{\left[\Phi(\alpha_j + \beta_j^\intercal \gamma_i)\right]^{y_{ij}} \left[1 - \Phi(\alpha_j + \beta_j^\intercal \gamma_i)\right]^{1-y_{ij}}\right\}^{r_{ij}}.$$

We assume standard priors on θ; specifically,

$$\xi(\theta) = \prod_{j=1}^J N\left(\begin{bmatrix}\alpha_j \\ \beta_j\end{bmatrix}; \mu_{ab}, \Sigma_{ab}\right) \prod_{i=1}^N N\left(\gamma_i; \mu_\gamma, \Sigma_\gamma\right),$$

where $\mu_{ab} \in \mathbb{R}^{D+1}$, $\mu_\gamma \in \mathbb{R}^d$ and $\Sigma_{ab} \in \mathbb{R}^{(D+1)\times(D+1)}$, $\Sigma_\gamma \in \mathbb{R}^{D\times D}$ are positive definite matrices.

The Algorithm

We consider just the log likelihood to illustrate how we extend the algorithm of Imai, Lo and Olmsted (2016). Let $m_{ij} = \alpha_j + \beta_j^\top \gamma_i$ and S_{obs} be the values of S that correspond to the Y observed values Y_{obs}. The complete-data (complete here is with respect to S_{obs}, not the values of Y for which $r_{ij} = 0$) log likelihood is given by

$$
\begin{aligned}
&\log p(Y_{\text{obs}}, S_{\text{obs}} \mid \theta) \\
&= \log \prod_{i=1}^{N} \prod_{j=1}^{J} \Big[N(s_{ij} \mid m_{ij}, 1) \Big]^{r_{ij}\left(I(y_{ij}=1)I(s_{ij}\geq 0)+I(y_{ij}=0)I(s_{ij}<0) \right)} \\
&= \sum_{i=1}^{N} \sum_{j=1}^{J} r_{ij} \Big(I(y_{ij} = 1) I(s_{ij} \geq 0) + I(y_{ij} = 0) I(s_{ij} < 0) \Big) \log N(s_{ij}; m_{ij}, 1).
\end{aligned}
$$

But this is the same complete-data log likelihood found in Imai, Lo and Olmsted (2016, Appendix A), except for the insistence on only using the observed data as observations. Therefore we can take their update equations and restrict ourselves to only using observed data. Specifically, iterate between

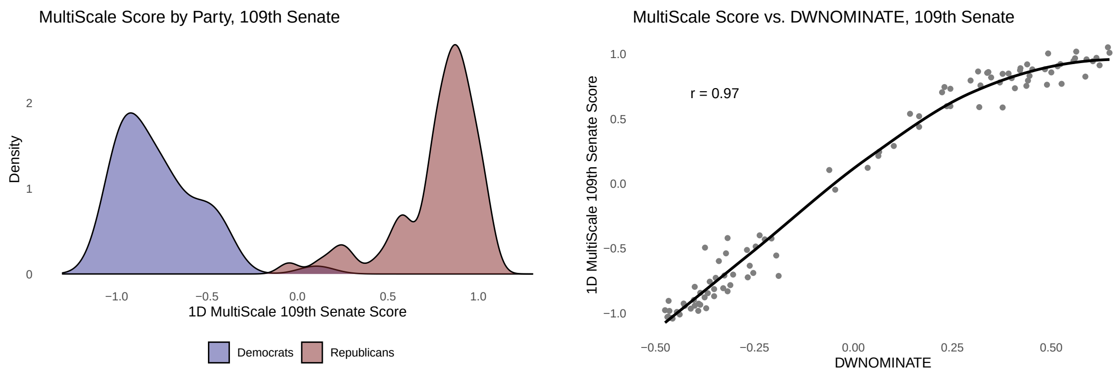
$$
s_{ij} \leftarrow m_{ij} + (2y_{ij} - 1) \frac{\phi(m_{ij})}{\Phi\left((2y_{ij} - 1) m_{ij}\right)}
$$

$$
\gamma_i \leftarrow \left(\Sigma_\gamma^{-1} + \sum_{j=1}^{J} r_{ij} \beta_j \beta_j^\top \right)^{-1} \left(\Sigma_\gamma^{-1} \mu_\gamma + \sum_{j=1}^{J} r_{ij} \beta_j (s_{ij} - \alpha_j) \right)
$$

$$
\begin{bmatrix} \alpha_j \\ \beta_j \end{bmatrix} \leftarrow \left(\Sigma_{ab}^{-1} + \sum_{i=1}^{N} r_{ij} \begin{bmatrix} 1 \\ \gamma_i \end{bmatrix} \begin{bmatrix} 1 \\ \gamma_i \end{bmatrix}' \right)^{-1} \left(\Sigma_{ab}^{-1} \mu_{ab} + \sum_{i=1}^{N} r_{ij} s_{ij} \begin{bmatrix} 1 \\ \gamma_i \end{bmatrix} \right).
$$

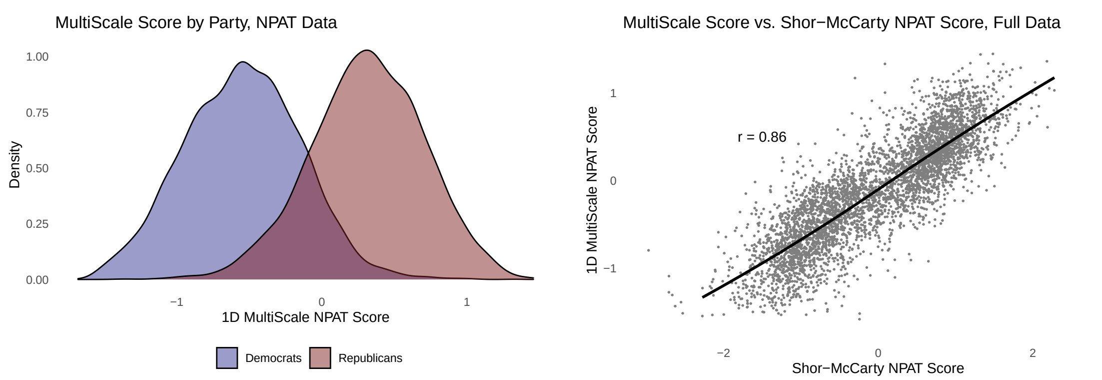
B External Validation for the MultiScale Algorithm

In the following figures, we plot several validation measure for the `MultiScale` algorithm. They show that for several data sources, the `MultiScale` estimates correlate highly with other measures of ideology.



(a) (b)

Figure 1: Comparison of `MultiScale` ideal points among Senators in the 109th Congress. The left-hand side shows that Democrats are almost universally to the left of Republicans. The right-hand side shows that `MultiScale` scores are highly correlated with DW-NOMINATE scores.



(a) (b)

Figure 2: Comparison of `MultiScale` ideal points among politicians using NPAT data. The left-hand side shows that Democrats are consistently to the left of Republicans. The right-hand panel shows the correlation between `MultiScale` scores estimated with the NPAT data to the Shor-McCarty NPAT scores.

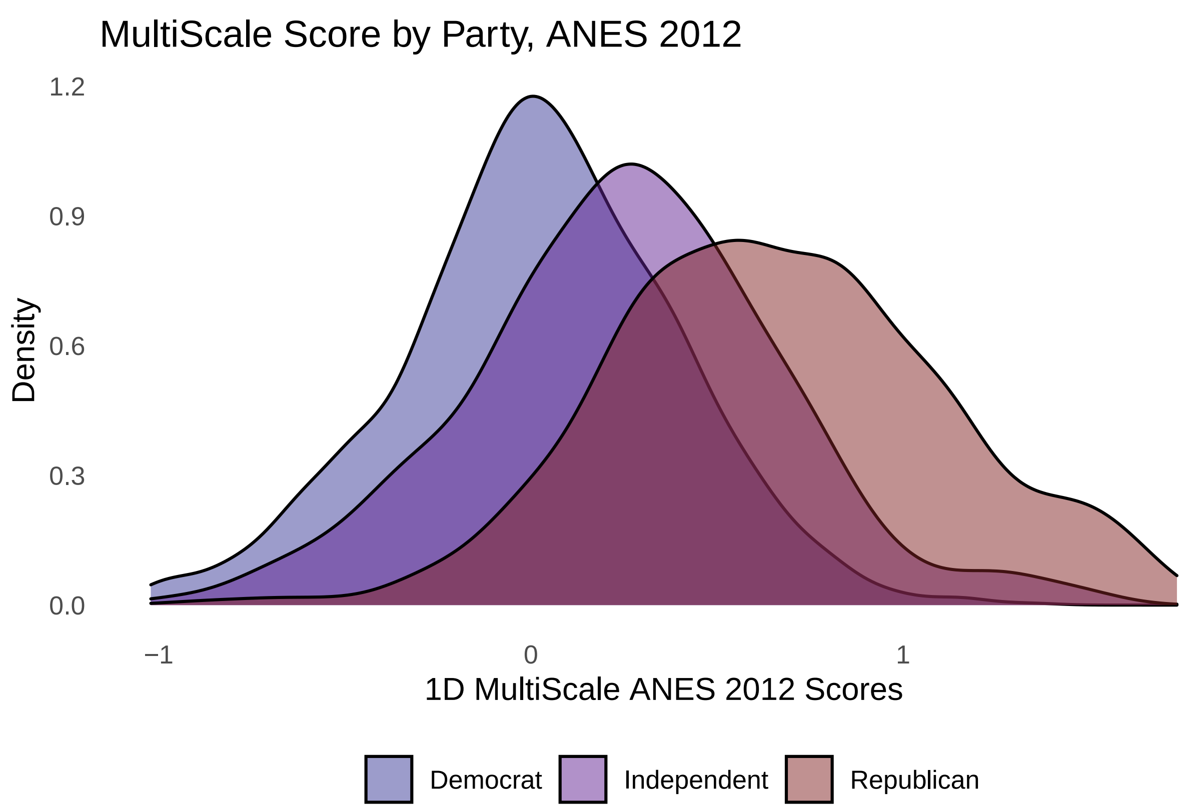


Figure 3: Comparison of `MultiScale` ideal points from the ANES by party.

C Simulation Study of Cross-Validation Estimator

To illustrate that our proposed method of out-of-sample validation can accurately recover the latent dimensionality of political choices, we conduct a small simulation study.

We simulate data sets according to the spatial voting model laid out in Section 2 in the main text. First, we fix the number of actors and choices (N and J, in the notation of the paper). Then, we simulate a series of choice matrices Y^D generated according to a $D \in \{0, \ldots, 5\}$ dimensional ideal point model. In particular, the voter surplus for voter i on choice j is modeled as

$$s_{ij} = \alpha_j + \beta_j^{D^\top} \gamma_i^D + \epsilon_{ij} \tag{1}$$

$$\epsilon_{ij} \overset{\text{ind.}}{\sim} N(0, 1) \tag{2}$$

$$Y_{ij}^D = I(s_{ij} > 0), \tag{3}$$

where we explicitly denote the dimensionality with superscripts. We draw α_j independent standard normal (separately for each dimension) and γ_i^D from a multivariate standard normal. To ensure that all D dimensions are in fact relevant to the choice, we restrict each element of β_j^D to be either -1 or 1, chosen randomly.

For each simulated data set, we run the cross-validation procedure outlined in the text. We estimate the predictive error associated with estimating models that assume 0 to 5 dimensions. If the cross-validation procedure can correctly measure the dimensionality of the data, the accuracy should be maximized when we estimate a model that assumes the same dimensionality that the data were generated with.

We repeat this exercise twice. First, we simulate data sets that are approximately the same size as the Senate data used in the main text, with $N = 102$ and $J = 645$. Second, we simulate data sets approximately the same size as the ANES, with $N = 1,000$ and $J = 20$.

The results are shown in Figure 4. The left-hand panel shows the results for the Senate-sized data and the right-hand panel shows the results for the ANES-sized data. In all cases, the estimated dimensionality is the same as the true dimensionality, providing evidence that the

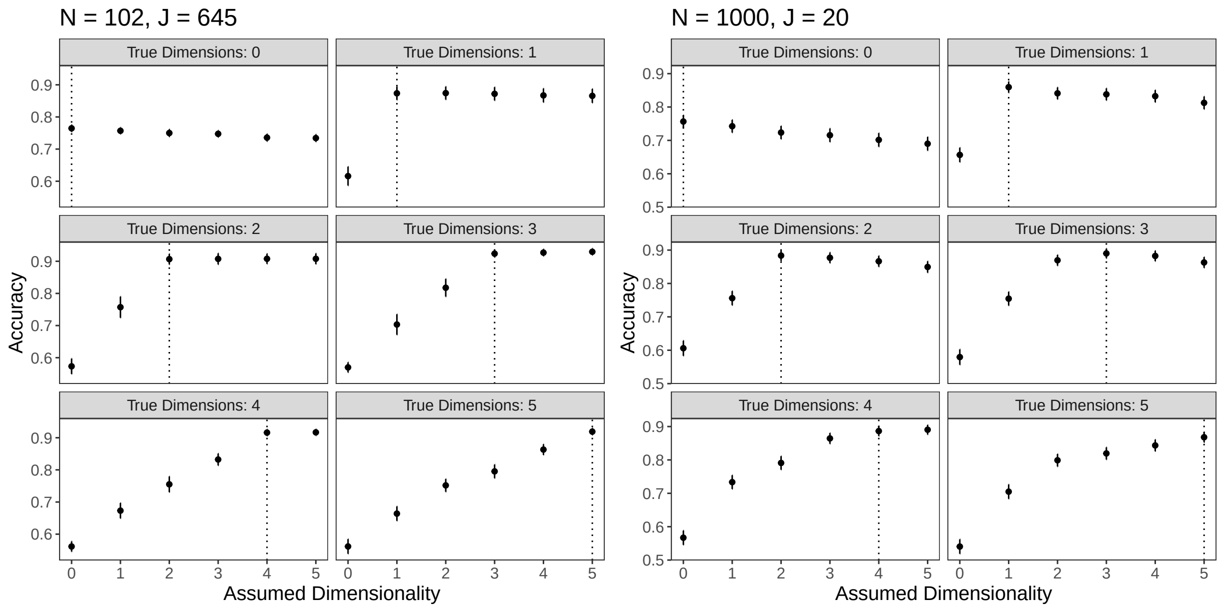


Figure 4: Simulation results. Left-hand panel shows Senate-sized data; right-hand panel shows ANES-sized data. Dotted vertical lines indicate the dimensionality of the true data-generating process.

validation strategy proposed in the paper can accurately recover the dimensionality of the data-generating process.

D Data Appendix

In this Appendix, we present the items used from each of the data sources referenced in the main text. For items that had more than two responses categories, we binarize them by splitting them at the mean (treating ordinal variables as cardinal). We could split them at the median with substantively similar results. Table 1 shows the number of rows and columns in each data source, along with patterns of missingness and number of questions for which we observe $K = 1, 2, \ldots, 7$ responses. The subsequent tables list and describe the variables used from each data source.

Data source	Rows	Columns	% Missing	$K = 1$	$K = 2$	$K = 3$	$K = 4$	$K = 5$	$K = 6$	$K = 7$
109th Senate	102	645	4.5	101	544	0	0	0	0	0
NPAT	12,794	225	79.1	0	225	0	0	0	0	0
Legislator Survey	225	31	5.5	0	19	0	0	0	1	11
2012 ANES	2,052	27	10.2	0	6	9	1	7	0	4
2012 CCES Roll Calls	54,068	10	3.2	0	10	0	0	0	0	0

Table 1: Description of data sources used in the main analyses. The columns labeled $K = 1, 2, \ldots, 7$ show the number of questions with K observed responses. For example, $K = 1$ means the responses to a given item were unanimous, $K = 2$ means there were two observed responses for a given item, and so on.

Broockman (2016) State Legislator Variables

Variable	Description
iq_vouchers	The government should provide parents with vouchers to send their children to any school they choose, be it private, public, or religious. (Binary)
iq_medicalpot	Allow doctors to prescribe marijuana to patients. (Binary)
iq_taxesover250k	Increase taxes for those making over $250,000 per year. (Binary)
iq_overturnroe	Overturn Roe v. Wade. (Binary)
iq_privitsocialsec	Allow workers to invest a portion of their payroll tax in private accounts that they can manage themselves. (Binary)
iq_gaymarriage	Same-sex couples should be allowed to marry. (Binary)
iq_unihealth	Implement auniversal health care program to guarantee coverage to all Americans, regardless of income. (Binary)
iq_medlawsuits	Limit the amount of punitive damages that can be awarded in medical malpractice lawsuits. (Binary)

iq_guncontrol	There should be strong restrictions on the purchase and possession of guns. (Binary)
iq_illegalim	Illegal immigrants should not be allowed to enroll in government food stamp programs. (Binary)
iq_enda	Include sexual orientation in federal anti-discrimination laws. (Binary)
iq_affaction	Prohibit the use of affirmative action by state colleges and universities. (Binary)
iq_unfunding	The US should contribute more funding and troops to UN peacekeeping missions. (Binary)
iq_fundarts	The government should not provide any funding to the arts. (Binary)
iq_dealthpenalty	I support the death penalty in my state. (Binary)
iq_repealcapgainstax	Repeal taxes on interest, dividends, and capital gains. (Binary)
iq_epaprohibit	Prohibit the EPA from regulating greenhouse gas emissions. (Binary)
iq_birthcontrolmandate	Health insurance plans should be required to fully cover the cost of birth control. (Binary)
iq_subsidizeloans	The federal government should subsidize student loans for low income students. (Binary)
eq_guns	Which statement comes closest to describing your views on gun control? (1-7 scale)
eq_health	Which statement comes closest to describing your views on the issue of health care? (1-7 scale)
eq_immigration	Which statement comes closest to describing your views on immigration? (1-7 scale)
eq_taxes	Which statement comes closest to describing your views on taxes? (1-7 scale)
eq_abortion	Which statement comes closest to describing your views on abortion? (1-7 scale)
eq_environment	Which statement comes closest to describing your views on pollution and the environment? (1-7 scale)
eq_medicare	Which statement comes closest to describing your views on Medicare, the government's program for covering the elderly's health care costs? (1-7 scale)
eq_gays	Which statement comes closest to describing your views on rights for gays and lesbians? (1-7 scale)
eq_affirmativeaction	Which statement comes closest to describing your views on affirmative action in higher education? (1-7 scale)
eq_unions	Which statement comes closest to describing your views on unions? (1-7 scale)
eq_education	Which statement comes closest to describing your views on public funding for private school education? (1-7 scale)

eq_contraception_version2	Which statement comes closest to describing your views on birth control? (1-7 scale)

2012 ANES Variables, from Hill and Tausanovitch (2015)

Variable	Description
VCF0806	R Placement: Government Health Insurance Scale
VCF0809	R Placement: Guaranteed Jobs and Income Scale
VCF0823	R Opinion: Better off if U.S. Unconcerned with Rest of World
VCF0830	R Placement: Aid to Blacks Scale
VCF0838	R Opinion: By Law, When Should Abortion Be Allowed
VCF0839	R Placement: Government Services/Spending Scale
VCF0843	R Placement: Defense Spending Scale
VCF0867a	R Opinion: Affirmative Action in Hiring/Promotion [2 of 2]
VCF0876a	R Opinion Strength: Law Against Homosexual Discrimination
VCF0877a	R Opinion Strength: Favor/Oppose Gays in Military
VCF0878	R Opinion: Should Gays/Lesbians Be Able to Adopt Children
VCF0879a	R Opinion: U.S. Immigrants Should Increase/Decrease [2 of 2]
VCF0886	R Opinion: Federal Spending- Poor/Poor People
VCF0887	R Opinion: Federal Spending- Child Care
VCF0888	R Opinion: Federal Spending- Dealing with Crime
VCF0889	R Opinion: Federal Spending- Aids Research/Fight Aids
VCF0894	R Opinion: Federal Spending- Welfare Programs
VCF9013	R Opinion: Society Ensure Equal Opportunity to Succeed
VCF9014	R Opinion: We Have Gone Too Far Pushing Equal Rights
VCF9015	R Opinion: Big Problem that Not Everyone Has Equal Chance
VCF9037	R Opinion: Government Ensure Fair Jobs for Blacks
VCF9040	Blacks Should Not Have Special Favors to Succeed
VCF9047	R Opinion: Federal Spending- Improve/Protect Environment
VCF9048	R Opinion: Federal Spending- Space/Science/Technology
VCF9049	R Opinion: Federal Spending- Social Security
VCF9131	R Opinion: Less Government Better OR Government Do More
VCF9132	R Opinion: Govt Handle Economy OR Free Market Can Handle
VCF9133	R Opinion: Govt Too Involved in Things OR Problems Require

2012 CCES Variables

Variable	Description
CC332A	roll-call votes - Ryan Budget Bill
CC332B	roll-call votes - Simpson-Bowles Budget Plan
CC332C	roll-call votes - Middle Class Tax Cut Act
CC332D	roll-call votes - Tax Hike Prevention Act
CC332E	roll-call votes - Birth Control Exemption
CC332F	roll-call votes - U.S.-Korea Free Trade Agreement
CC332G	roll-call votes - Repeal Affordable Care Act
CC332H	roll-call votes - Keystone Pipeline
CC332I	roll-call votes - Affordable Care Act of 2010
CC332J	roll-call votes - End Don't Ask, Don't Tell

Matched roll-call votes, Senate

We extracted data on roll-call votes corresponding to the CCES questions from `voteview.com`. Roll-call votes are on final passage, where applicable. In the case of issues that were voted on multiple times, we take the vote closest to the 2012 election.

Issue	Congress	Vote Number
Affordable Care Act	111th	396
Repeal Don't Ask, Don't Tell	111th	281
Tax Hike Prevention Act	111th	276
Ryan budget	112th	277
Middle Class Tax Cut Act	112th	184
US-Korea Free Trade Agreement	112th	161
Affordable Care Act Repeal	112th	9
Birth Control Exemption	112th	24
Keystone Pipeline	113th	280

E Alternative Measure of Fit

In this appendix we replicate the plots from the main text, except instead of accuracy we use the average likelihood of the observed responses. Working with the likelihood is slightly less interpretable, but has the advantage of being able to distinguish between correct classifications that are "just barely" correct (e.g., 51% likelihood of observed response) and correct classifications that have a higher degree of confidence (e.g., 95% likelihood of observed response).[1] The substantive conclusions remain unchanged.

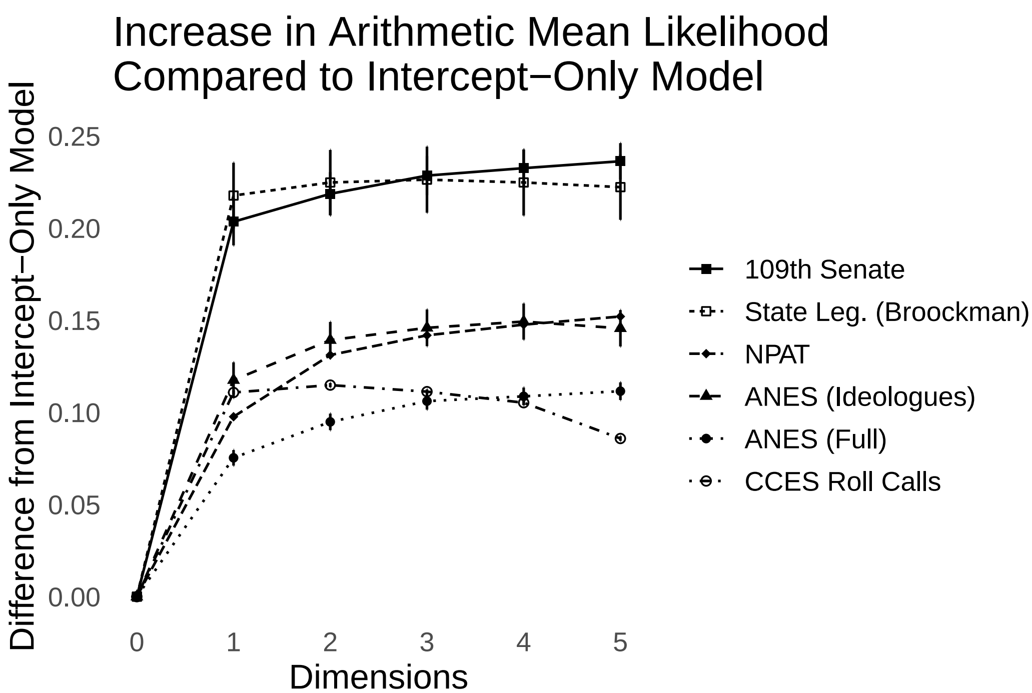


Figure 5: Increase in average cross-validation likelihood of observed response over an intercept-only model. Error bars show 95% confidence intervals clustered by respondent.

[1]Technically, the likelihood is a proper scoring rule while accuracy is not, meaning that the likelihood is maximized by the true model. Given that the substantive conclusions drawn are not sensitive to the use of accuracy and it is more easily interpretable, we focus on that in the main text.

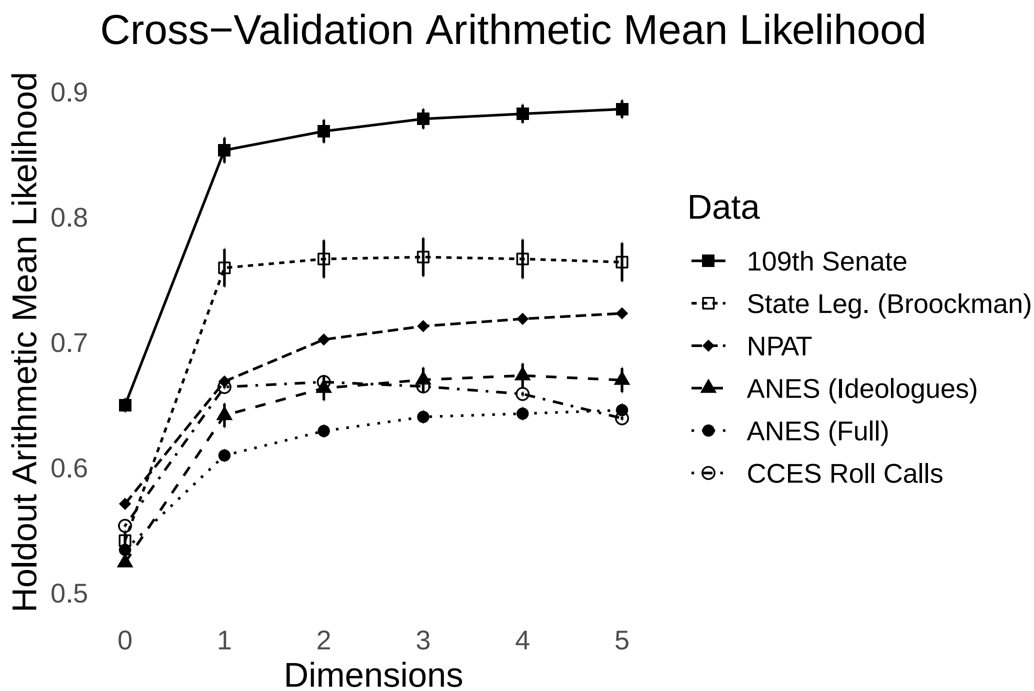


Figure 6: Average cross-validation likelihood of observed response. Error bars show 95% confidence intervals clustered by respondent.

F Constructing the Matched CCES-Senate Sample

In Section 6.3, we compare Senate roll call votes to the CCES responses of a subset of respondents who are demographically similar to Senators. Here, we describe the matching process.

We begin with data on the demographics Senators collected by Carnes (2013) and the Congressional Research Service (Petersen, 2012). Compared to the general public, Senators skew much richer, highly educated, older, white, and male. Even with the large sample size afforded by the CCES, it is difficult to find a sample that matches the demographics of Senators exactly.

Still, we create a more similar sample, we first subset the CCES to respondents whose family incomes exceed $150,000, who identify as either Democrats or Republicans, who are at least 35 years old, and who have at least a college degree. This leaves us with a sample of around 1,200. Then, within this sample, we use a raking procedure to construct weights that target the demographics of Senators on the following variables: age (in 10-year bins), sex, education (college or graduate school), and party. We use the `R` package `anesrake` to construct the weights (Pasek, 2018). We then sample 500 respondents in proportion to their weights to create a matched sample. Table 3 compares the distribution of age, party, education, and sex in the Senate to the matched sample.

Variable	Senate (%)	CCES (%)	Difference
Age			
30-39	0.3	0.2	-0.1
40-49	11.3	15.0	3.7
50-59	32.0	33.0	1.0
60-69	33.7	33.2	-0.5
70+	22.7	18.6	-4.1
Party			
Democrat	47.7	51.2	3.5
Republican	52.3	48.8	-3.5
Education			
Post-graduate	53.0	56.0	3.0
No post-graduate	47.0	44.0	-3.0
Sex			
Male	85.3	80.2	-5.1
Female	14.7	19.8	5.1

Table 3: Balance table comparing demographics of Senators in 108th-110th Congresses to the matched CCES sample. All CCES respondents reported a family income of at least $150,000, are at least 35 years old, and have at least a college degree. Senate demographics drawn from Carnes (2013) and Petersen (2012).

G Predicting Dyadic Agreement

Another way of demonstrating the limited gains from fitting a higher-dimensional model is by examining agreement scores between pairs of respondents and relating them to estimated distance between their ideal points in D dimensions. Intuitively, respondents who tend to respond to the same questions should be ideologically similar, and thus the distance between their ideal points should be small. If a higher-dimensional model provides a better description of survey responses, we should see a meaningfully different relationship between agreement scores and estimated ideal points across different choices of D.

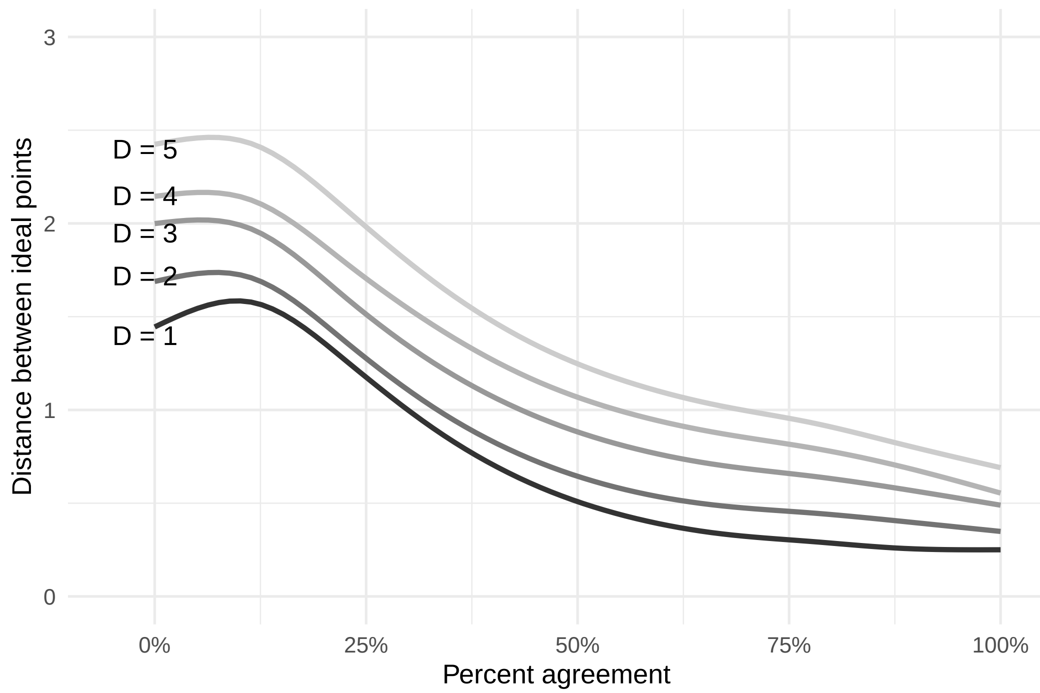


Figure 7: Relationship between ideal point distance and agreement score for each pair of 2012 ANES respondents. The reason for the non-monotonicity for low agreement scores is small sample size: respondents who answer 0 questions in the same way tend to have answered very few common questions, so those questions have relatively little influence on the ideal point estimates.

Formally, denote respondents $i = 1, \ldots, N$ and questions $j = 1, \ldots, J$. For the (undirected) respondent pair (i, k), denote the set of questions to which they both provide a response $\mathcal{S}_{ik}$. If the respondents provide the same response to question $j \in \mathcal{S}_{ik}$, then we say $x_{ik}^{j} = 1$, and 0 otherwise. The agreement score for (i, k) is then $y_{ik} = \frac{1}{|\mathcal{S}_{ik}|} \sum_{j \in \mathcal{S}_{ik}} x_{ij}^{j}$, which is simply the proportion of times that i and k answer in the same way.

We compute this quantity for each respondent pair in the 2012 ANES (for a total of 2 million pairs), and also compute the Euclidean distance between their estimated ideal points in $D = 1, \ldots, 5$ dimensions. Ideal points are estimated using the full data set, without any holdout data.

Figure 7 plots the distance between ideal points against the agreement score for each respondent pair. While there is an intercept shift, with respondents tending to be farther apart from each other in higher-dimensional space, the basic relationship between agreement scores and distance is the same across dimensions. This result further reinforces the idea that there is little additional insight gained in the public beyond one dimension.

References

Broockman, David E. 2016. "Approaches to Studying Policy Representation." *Legislative Studies Quarterly* 41(1):181–215.

Carnes, Nicholas. 2013. *White-Collar Government: The Hidden Role of Class in Economic Policy Making*. Chicago: University of Chicago Press.

Hill, Seth J. and Chris Tausanovitch. 2015. "A Disconnect in Representation? Comparison of Trends in Congressional and Public Polarization." *Journal of Politics* 77(4):1058–1075.

Imai, Kosuke, James Lo and Jonathan Olmsted. 2016. "Fast Estimation of Ideal Points with Massive Data." *American Political Science Review* 110(4):1–20.

Little, Roderick J A and Donald B Rubin. 2014. *Statistical Analysis with Missing Data.* 2nd ed. Hoboken, NJ: John Wiley & Sons.

Pasek, Josh. 2018. *anesrake: ANES Raking Implementation.* R package version 0.80.
URL: *https://CRAN.R-project.org/package=anesrake*

Petersen, R. Eric. 2012. "Representatives and Senators: Trends in Member Characteristics Since 1945.".
URL: *https://fas.org/sgp/crs/misc/R42365.pdf*