SI: Balance as a Pre-Estimation Test for Time Series Analysis

Mark Pickup[*]
Paul M. Kellstedt[†]

September 30, 2021

[*]Department of Political Science, Simon Fraser University, 8888 University Drive, Burnaby, BC V5A 1S6, Canada, e-mail: mark.pickup@sfu.ca

[†]Department of Political Science, Texas A&M University, College Station, TX 77843, USA, e-mail: kellstedt@tamu.edu

1 Data Availability Statement

For Dataverse replication materials, see https://doi.org/10.7910/DVN/G0XXSE (Pickup and Kellstedt, 2021).

2 $I(2)$ variables and multiple cointegration

As an example of an $I(2)$ process, consider price levels. The change in the price of something may be $I(1)$. This is to say, the first difference of the series is $I(1)$ and would have to be differenced a second time to be stationary, so the original price level series is $I(2)$. As a further example of an $I(2)$ process, consider the stock and flow of immigrants. Flow is the change in the stock, so if stock is $I(d)$ then flow is $I(d-1)$. If one's theoretical expectation is that the flow of immigrates into a country is $I(1)$, the stock would be $I(2)$.

A concept rarely discussed in political science is "multiple cointegration." In multiple cointegration, two or more variables with the same order of integration add up to a lower order of integration and then the resulting process combines with one or more other variables with the same order of integration to add up to an even lower order of integration. For example: $X_1 \sim I(2)$ and $X_2 \sim I(2)$ combine to produce $Y_1 \sim I(1)$. And then, $Y_1 \sim I(1)$ and $Y_2 \sim I(1)$ combine to produce $Z \sim I(0)$. If this occurs, the combination of X_1 and X_2 and Y_2 is of order $I(0)$. For example, if prices and wages are both $I(2)$ but cointegrate to produce an $I(1)$ variable, that new process may cointegrate with an $I(1)$ public liberal policy appetite variable to produce an $I(0)$ process. Multiple cointegration is a possibility but such relationships have never been explicitly proposed in political science (to our knowledge).

3 An example of theorizing that leads to testable implications about integration and cointegration

A rare example of a theoretical model that (at least partially) carefully considers the equilibrium properties of key variables can be found in a careful reading of Stimson *et al.* (1995). The article examines the responsiveness of government policy to public mood. The relationship between Mood and Policy is explicitly called an equilibrating process (p 562, note 4) and implicitly described as one: "Public sentiment [Mood] shifts. Political actors sense the shift. And then they alter their policy behavior at the margin" (pg. 543). Although it is not stated explicitly, Policy is described as though it is not self-correcting (pg 548), implying it is instead an I(1) process. At the same time it is described as correcting in relation to Mood: "Elected politicians, we believe, sense the mood of the moment, assess its trend, and anticipate its consequences for future elections" (pg 545), and "Thus when public policy drifts away from the public's demand for policy, the representation system acts as a control mechanism to keep policy on course" (pg 544). This all implies a cointegrating relationship between Mood and Policy. This defines the nature of the equilibrium between the two, and implies a balanced theoretical model of Mood and Policy. Focusing on whether the theoretical model is balanced has the added benefit of forcing the

researcher to consider *how* balance is achieved (e.g., is cointegration required?). Consider that Stimson *et al.* (1995) could have also described a relationship between two stationary variables. Mood and Policy would then each have their own stationary equilibrium: a change in Mood would be expected to cause a change in Policy but Mood would be expected to shortly return to its equilibrium and Policy to follow. This paints a different picture of government responsiveness than that described by Stimson *et al.* (1995). It suggests that public mood is subject to short-term perturbations from an equilibrium to which government policy will respond. The theoretical model described by Stimson *et al.* (1995) instead implies that public mood can wander around with little to restrict it and policy will chase after it. Our long-term expectations for public mood and policy are very different under these two theoretical models.

4 Examples

We now discuss two influential articles using time series data and show how they could have benefitted from applying the concept of balance theoretically and empirically.

4.1 Example: How public opinion responds to the U.S. Supreme Court

In a provocative study, Ura (2014) uses a single-equation ECM on U.S. data to examine the relationship between the public's aggregate level of liberalism ("Mood") and the liberalism of Supreme Court decisions. Competing theories about the dynamics of Mood suggest that, on the one hand, Supreme Court liberalism will nudge public opinion in a conservative direction—"backlash," in Ura's phrase—or, on the other hand, that Supreme Court liberalism will encourage the public to become more liberal itself—Ura calls this "legitimation." Using annual data, Ura creates a cumulative index of the liberalism of Supreme Court decisions, and measures Mood with Stimson's (1991) well known Policy Mood index. Controls in the model include effects for the cumulative liberalism of overall government policy, inflation, and unemployment. The theoretical model is not specified (separately from the empirical model) but generally it is:

$$Mood_t \sim f(Mood_{t-1}, CourtDecisions_t, Policy_t, Inflation_t, Unemployment_t, \epsilon_t) \quad (1)$$

where $f()$ is some function. The empirical model Ura estimates (see his Table 1) is:

$$\begin{aligned}\Delta Mood_t = \alpha_0 + \alpha_1 Mood_{t-1} + \beta_1 Court_{t-1} + \beta_2 \Delta Court_t \\ + \beta_3 Policy_{t-1} + \beta_4 \Delta Policy_t + \beta_5 Unemployment_{t-1} + \beta_6 \Delta Unemployment_t \\ + \beta_7 Inflation_{t-1} + \beta_8 \Delta Inflation_t + \epsilon_t \quad (2)\end{aligned}$$

He finds (Table 1, p 118) that:

> The data indicate significant relationships between Supreme Court decision making and public mood in both the short run and the long run, yet the directions of these two effects are different. The short-run relationship between

> changes in cumulative Supreme Court liberalism and public mood is negative, which is consistent with the thermostatic response hypothesis. The long-run relationship between changes in cumulative Supreme Court liberalism and public mood is positive, which is consistent with the legitimizing response hypothesis. Together, these results point to a complex interaction between the Supreme Court and the mass public characterized by short-term backlash against Supreme Court decisions in public mood followed by a long-run movement in public opinion toward the ideological position taken up by the Court.

In other words, in equation 2, β_2 is negative, but β_1 is positive.

In keeping with the common practice, Ura does not investigate balance, either as a theoretical or empirical matter. Instead, and again following conventions in the literature, he writes (pp 116-7):

> Though ECMs were originally developed for investigating cointegrated time series, DeBoef and Keele (2008) note that they may also be applied in a variety of time-series contexts in the absence of cointegration with either stationary or nonstationary data.

As a theoretical matter, Ura's left-hand-side variable, $Mood_t$, could be $I(0)$ or $I(1)$. If $Mood_t$ is $I(1)$, the right-hand side variables must contain at least one $I(1)$ process (rule iv). If *Mood* is $I(0)$, to achieve balance, the $I(1)$ right-hand-side variables must co-integrate (rule ii).

Ura says little about the order of integration of the left-hand side of the equation. As for the right-hand side, his key variable of interest is the *Court* variable, which he describes (p 116) in the following way:

> I construct a cumulative measure of liberalism in the Supreme Court's decisions by rescaling the net number of liberal decisions in each period as its deviation from the mean value of the annual Supreme Court liberalism series and taking the sum of the series at each point in time.

Because the measure is cumulative—a sum—it is theoretically most natural to assume that the series is $I(1)$, though (as noted) Ura does not comment on this.

The *Policy* measure is constructed similarly. Ura begins by extending Mayhew's (1991) list of major laws, noting (p 116) that:

> ... these values are used to construct a measure of cumulative policy liberalism produced by Congress and the president by scoring each year's policy outputs as the difference between its value and the mean of the annual series and then taking the sum of the resulting series at each point in time.

Again, a cumulative measure would likely be $I(1)$, from a theoretical perspective. The same would likely be true for $Inflation$, which most scholars treat as $I(1)$. *Unemployment* has debatable stationarity properties as well.

Collectively, this raises important questions about how, theoretically, these right-hand-side variables might combine to yield a balanced model. Because no argument is made about whether or not these variables might be cointegrated—either with one another, or (importantly) with the *Mood* variable, which in lagged form is also on the right-hand side—these questions go unanswered, missing an opportunity to flesh out the theoretical model.

Our theoretical expectation is that *Mood* is an $I(1)$ process. There are shocks that dissipate for some individuals but not others, resulting in the combination of $I(0)$ and $I(1)$ processes, which add up to an $I(1)$ process.[1] We also expect that policy and court decisions are $I(1)$, if not unemployment and inflation. By rule (iv) of section 4 (of the main text), the theoretical model is balanced: Both the left and right hand side of the theoretical model (equation 1) are $I(1)$ processes. This also implies that if *Mood* has a long-run relationship with *Court* and *Policy*, it is a cointegrating relationship. The government and Supreme Court make decisions that accumulate over time. *Mood* changes in response to each of these decisions and these changes accumulate over time. The responses may cause *Mood* to go up or to go down, but there is no equilibrium—that is, long-run mean—to which public liberalism will converge in the long run. If *Mood* is $I(0)$, though, then in order to achieve balance, the (seemingly) $I(1)$ variables *Court* and *Policy*, (and perhaps $Unemployment$ and $Inflation$, depending on their univariate properties), must be co-integrated. These two scenarios are quite distinct. In the first case ($Mood \sim I(1)$), we expect *Mood* to exhibit permanent changes in response to court and policy decisions that accumulated over time. In the second case ($Mood \sim I(0)$), we expect *Mood* to exhibit temporary deviations from its equilibrium in response to court and policy decisions temporarily departing from their joint equilibrium. Perhaps one of these two distinct scenarios happens. If so, it reveals something substantively important that has gone undetected; if not, then the model is unbalanced and inappropriate. Particularly because the series in Ura's analyses are short, and because tests for unit roots and stationarity have low power, our view is that theorizing about balance becomes even more important.

As an empirical matter, Ura must assume that the residuals are $I(0)$ (among other features) in order to use OLS standard errors. This requires $I(0)$ balance. We have obtained Ura's replication files and conducted tests to determine whether Ura's models were appropriate.[2] Considering first the univariate properties of the various series, Dickey-Fuller tests fail to reject the null hypothesis that *Mood*, *Court* and *Policy* are unit root processes (with or without drift).[3] However, the result for *Mood* allowing for drift is borderline (P-value = 0.054). Given our theoretical expectation that *Mood* is $I(1)$, we are inclined to concur with the Dickey-Fuller results.

If *Mood* is $I(1)$, the first difference of *Mood* on the left hand side of the GECM is $I(0)$.

[1] Such a combination might result in a fracionally integrated process, but this is not necessarily true, as Granger (1980) shows. Fractional integration will result if the original processes that are being combined contain particular distributions, but otherwise it will result in an $I(1)$ process.

[2] For Dataverse replication materials, see https://doi.org/10.7910/DVN/G0XXSE (Pickup and Kellstedt, 2021).

[3] The P-values allowing for drift are: 0.0540, 0.2385 and 0.1911 respectively. We can reject the null of the unit root process for $Unemployment$ (P-value = 0.0059) and $Inflation$ (P-value = 0.0028), suggesting that they are stationary.

Becuase the GECM has several $I(1)$ right-hand-side variables as identified above, in order to achieve $I(0)$ balance, the $I(1)$ right-hand-side variables (including $Mood_{t-1}$) must cointegrate, or else the model is unabalanced. Hence, we employ a Johansen rank test. Using the maximum-eigenvalue statistic, we can reject the null hypothesis that the rank of the variables on the right-hand side of equation 2 is zero, against the alternative that it is one (maximum-eigenvalue statistic = 44.07; 5% critical value = 33.46). On that basis, we reject the null hypothesis that *Mood* is $I(1)$ and no cointegrating equation exists between its lag and the other right-hand side variables. We cannot reject the null hypothesis that the rank is one against the alternative that it is two (maximum-eigenvalue statistic = 19.25; 5% critical value = 27.07). This test indicates that there is a single cointegrating equation—there is a linear combination of the lag of *Mood* and the right hand side variables that is stationary. On this basis, our theoretical expectations regarding the order of integration and cointegration of the variables are met. By rule (ii), the empirical model (equation 2) is balanced, and $I(0)$ balanced. The test statistics used by Ura will have the expected distributions and will be valid.

This exercise of ensuring equation balance, while essential, is admittedly post-hoc, in the sense that we did not theorize ex ante about a cointegrating relationship between the *Mood*, *Policy*, and *Court* variables. And yet such a relationship exists—one that is more complicated (and arguably richer) than Ura's theorized investigation between *Mood* and *Court*. The cointegrating relationship contains additional variables, which presumably would be of interest to substantive experts in the area. The equilibrating relationship involving the U.S. Supreme Court includes not only mass opinion, as Ura theorized, but also policymaking activity from the other branches of the national government.

4.2 Example: Media coverage and economic performance

In a recent example, Soroka *et al.* (2015) examine the relationship between both the volume and tone of media coverage of the economy and various aspects of economic performance. They ask, in effect: How does economic performance and consumer confidence shape economic news? For our purposes, we focus on the two models presented in Table 6. One model is for volume of media coverage and the other is for tone. They test if these aspects of media coverage are influenced by consumer confidence (both prospective and retrospective), and the leading index of economic indicators (LEI). The implied theoretical model is:

$$Media_t \sim f(Media_{t-1}, LEI_t, Pros(confidence)_t, Retro(confidence)_t, \varepsilon_t) \tag{3}$$

and the empirical model is:

$$\begin{aligned}\Delta Media_t = \alpha_0 + \alpha_1 Media_{t-1} + \beta_1 LEI_{t-1} + \beta_2 \Delta LEI_t \\ + \beta_3 Pros(confidence)_{t-1} + \beta_4 \Delta Pros(confidence)_t + \beta_5 Retro(confidence)_{t-1} \\ + \beta_6 \Delta Retro(confidence)_t + \epsilon_t\end{aligned} \tag{4}$$

As in the previous example, Soroka *et al.* (2015) do not discuss the possibility of their theoretical model being balanced. Past work suggests $Media_t$ is $I(0)$. On that basis, for

the theoretical model to be balanced, each of the right-hand side variables must be $I(0)$ or each variable of a higher order of integration must cointegrate to an $I(0)$ process.

While Soroka *et al.* (2015) do not discuss their theoretical expectations for the variables, they do empirically test their univariate properties. They determine that the dependent variable is $I(0)$ stationary while the two consumer confidence variables are $I(0)$ stationary and the single economic variable is $I(1)$ nonstationary (see their Appendix B). Considering the empirical model, three $I(0)$ and one $I(1)$ variable cannot cointegrate to produce an $I(0)$ process, so the three independent variables and the lag of the dependent variable produce a $I(1)$ process. By rule (ii), the empirical model is not balanced, and therefore not $I(0)$ balanced. The left-hand side is $I(0)$ and the right-hand side is $I(1)$. As it stands, the model is incorrect. Either there is something missing from the model or beliefs regarding the order of integration of the variables are incorrect.

Unlike the previous example, these authors pre-tested their data for unit roots, and drew conclusions about the $I(0)$ and $I(1)$ properties of their univariate series. This represents a positive step for the community of scholars to be able to evaluate the models considered. And yet, as we have shown above, a key table in the article does not meet the requirements to have balanced equations. It should be noted that in response to our critique, these authors have since written an erratum that addresses the issue. Specifically, the authors applied additional tests of integration to the variables and, keeping the principle of balance in mind, came to the conclusion that the economic variable is, in fact, $I(0)$ stationary. This then balanced their empirical models (rule i)—both the left-hand-side and right-hand-side are $I(0)$. Importantly, it also leads to a different conclusion regarding the relationship between the economy and media tone. While the original results suggested that media tone with a constant long-run equilibrium was being driven (in part) by a variable without any such long-run equilibria (an impossibility), or that the economic variable cointegrated with some unknown variable to drive media tone, the new results suggest that media tone is driven by variables which also have their own constant long-run equilibria.

References

GRANGER, C. W. J. (1980). Long memory relationships and the aggregation of dynamic models. *Journal of Econometrics*, **514**, 227–238.

MAYHEW, D. R. (1991). *Divided We Govern: Party Control, Lawmaking, and Investigations, 1946–1990*. New Haven, CT: Yale University Press.

PICKUP, M. and KELLSTEDT, P. (2021). Replication Data for: Balance as a Pre-Estimation Test for Time Series Analysis.

SOROKA, S. N., STECULA, D. A. and WLEZIEN, C. (2015). It's (change in) the (future) economy, stupid: Economic indicators, the media and public opinion. *American Journal of Political Science*, **59** (2), 457–474.

STIMSON, J. A. (1991). *Public opinion in America: Moods, cycles, and swings*, vol. 14. Westview Press.

—, MACKUEN, M. B. and ERIKSON, R. S. (1995). Dynamic representation. *The American Political Science Review*, **89** (3), 543–565.

URA, J. D. (2014). Backlash and legitimation: macro political responses to supreme court decisions. *American Journal of Political Science*, **58** (1), 110–126.