Online Supplement

"Taking Distributions Seriously: On the Interpretation of the Estimates of Interactive Nonlinear Models"

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Appendix A. Relevant Articles in the Top Three Journals (January 2006 – January 2020)

Relevant Articles in the Top Three Journals (January 2006 – January 2020)

Journal	AJPS	APSR	JOP	Cumulative
Articles with Non-linear Models				
Total	303	166	430	899
w/ interaction	139	66	203	408 (45% of above)
w/ interaction between continuous variables	43	20	70	133 (33% of above)
Articles with Some Form of Logistic or Probit Regression				
Total	257	143	356	756
w/ interaction	119	58	174	351 (46% of above)
w/ interaction between continuous variables	34	18	58	110 (31% of above)
w/ a graphical interpretation of the interaction	31	14	49	94 (86% of above)
Plots $MEM_x(z)$	3	6	16	25 (27% of above)

Appendix B. When Does the Statistical Significance of a Marginal Effect Depend on X?

No Non-linear Functions of X in the Linear Component of the Model

For a model with the prediction

$$h(x, z) = f(\hat{\beta}_0 + \hat{\beta}_1 x + \hat{\beta}_2 z + \hat{\beta}_3 xz)$$

the estimate of the marginal effect of x at (x, z) is

$$h_X(x, z, \hat{\boldsymbol{\beta}}) = (\hat{\beta}_1 + \hat{\beta}_3 z) f'(\hat{\beta}_0 + \hat{\beta}_1 x + \hat{\beta}_2 z + \hat{\beta}_3 xz)$$

Since for any inverse link function $f'() > 0$, $h_X(x, z, \hat{\boldsymbol{\beta}}) = 0$ if and only if $\hat{\beta}_1 + \hat{\beta}_3 z = 0$. Thus, if $h_X(\bar{x}, z, \hat{\boldsymbol{\beta}}) = 0$ then $h_X(\tilde{x}, z, \hat{\boldsymbol{\beta}}) = 0$ for any $\tilde{x}$.

The Linear Component of the Model Includes a Non-linear Function of X

Accordingly, if the linear component of the model specification includes a non-linear function of X, e.g., a cubic polynomial, the statistical significance of the marginal effect may depend on X. For the following model:

$$h(x, z) = f(\hat{\beta}_0 + \hat{\beta}_1 s(X) + \hat{\beta}_2 Z + \hat{\beta}_3 s(X) Z)$$

the estimate of the marginal effect of x at (x, z) is

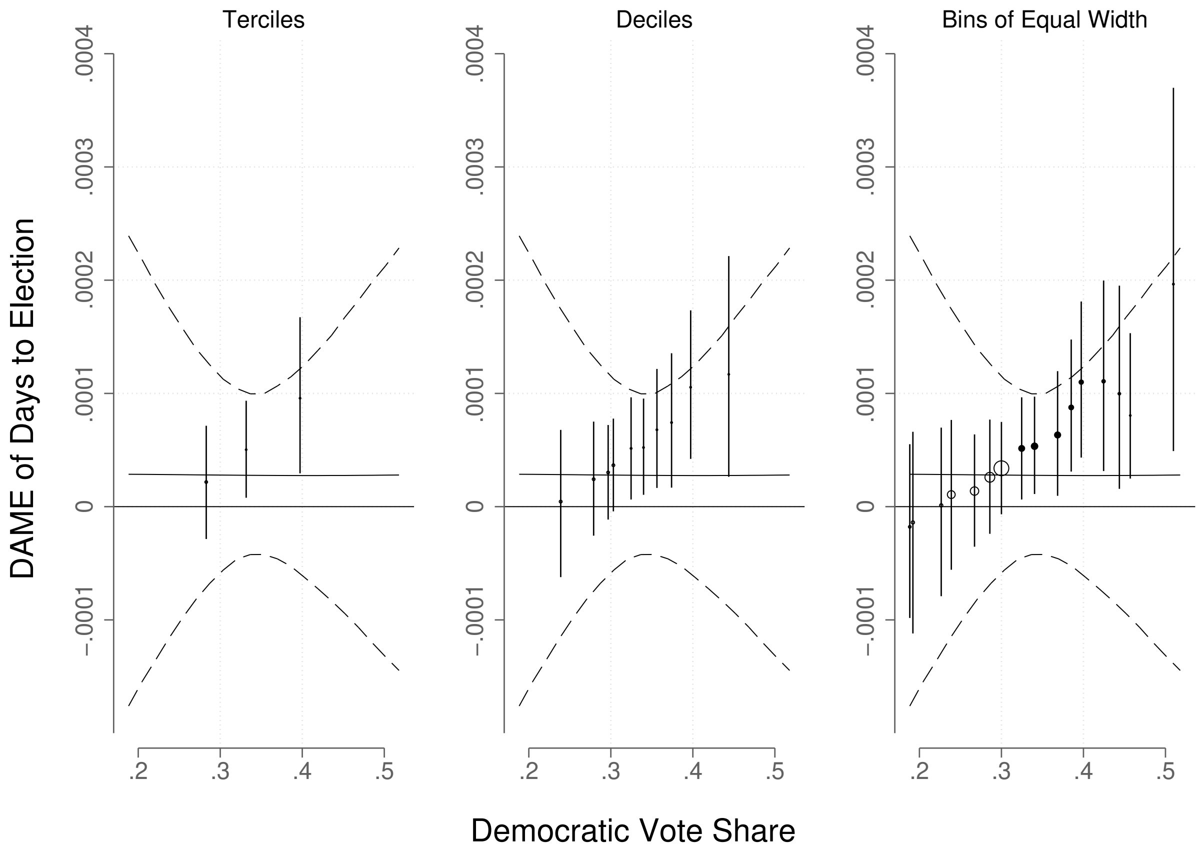
$$h_X(x, z, \hat{\boldsymbol{\beta}}) = (\hat{\beta}_1 s'(x) + \hat{\beta}_3 s'(x) z) f'(\hat{\beta}_0 + \hat{\beta}_1 s(x) + \hat{\beta}_2 z + \hat{\beta}_3 s(x) z)$$

which has the same sign as $(\hat{\beta}_1 s'(x) + \hat{\beta}_3 s'(x) z)$.

In this case, $h_X(\bar{x}, z, \hat{\boldsymbol{\beta}}) = 0$ does not guarantee that $h_X(\tilde{x}, z, \hat{\boldsymbol{\beta}}) = 0$ for some $\tilde{x} \neq \bar{x}$.

Appendix C. Using Bins of Z to Identify Heterogeneity of the Effect of X

Figure C.1. The Effect of Election Proximity on Party Discipline



Data source: Arceneaux, K., M. Johnson, R. Lindstädt, and R. J. Vander Wielen. 2016. "The Influence of News Media on Political Elites: Investigating Strategic Responsiveness in Congress." *American Journal of Political Science* 60 (1): 5–29.

Appendix D. Marginal Effects Formulas for Most Popular Link Functions

In what follows, m denotes the linear component of the respective model. In models with an interaction of X and Z – those including $\beta_{(X)}X + \beta_{(Z)}Z + \beta_{XZ}XZ$– it is:

$$m = \hat{\beta}_0 + \hat{\beta}_{(X)}x + \hat{\beta}_{(Z)}z + \hat{\beta}_{(XZ)}xz + \hat{\beta}_{(W_1)}w_1 + \hat{\beta}_{(W_2)}w_2 + \cdots ,$$

where W_1, W_2, ... are all covariates other than X and Z.

In models with an interaction of Z and a quadratic polynomial of X –those including $\beta_{(X)}X + \beta_{(X^2)}X^2 + \beta_{(Z)}Z + \beta_{(XZ)}XZ + \beta_{(X^2Z)}X^2Z$– it is:

$$m = \hat{\beta}_0 + \hat{\beta}_{(X)}x + \hat{\beta}_{(X^2)}x^2 + \hat{\beta}_{(Z)}z + \hat{\beta}_{(XZ)}xz + \hat{\beta}_{(X^2Z)}x^2z + \hat{\beta}_{(W_1)}w_1 + \hat{\beta}_{(W_2)}w_2 + \cdots$$

Link Function: logit (in logistic regressions)

• Expected value of the dependent variable:

$$h(m) = \text{logit}^{-1}(m) = \frac{1}{1 + \exp(-m)} = \frac{\exp(m)}{1 + \exp(m)}$$

$\text{logit}^{-1}(m)$ can be computed using the `plogis()` function in R and `logistic()` function in Stata

• Marginal effect of X in models with an interaction of X and Z:

$$\begin{aligned}\text{ME}_X(x, z, m) &= (\hat{\beta}_{(X)} + \hat{\beta}_{(XZ)}z)\frac{\exp(m)}{(1 + \exp(m))^2} \\ &= (\hat{\beta}_{(X)} + \hat{\beta}_{(XZ)}z) \cdot \text{logit}^{-1}(m) \cdot (1 - \text{logit}^{-1}(m))\end{aligned}$$

$\text{logit}^{-1}(m)$ can be computed using the `plogis()` function in R and `logistic()` function in Stata • Marginal effect of X in models with an interaction of Z and a quadratic polynomial of X:

$$\begin{aligned}\text{ME}_X(x,z,m) &= (\hat{\beta}_{(X)} + \hat{\beta}_{(X^2)}x + \hat{\beta}_{(XZ)}z + \hat{\beta}_{(X^2Z)}xz)\frac{\exp(m)}{(1+\exp(m))^2} \\ &= (\hat{\beta}_{(X)} + \hat{\beta}_{(X^2)}x + \hat{\beta}_{(XZ)}z + \hat{\beta}_{(X^2Z)}xz) \cdot \text{logit}^{-1}(m) \cdot (1 - \text{logit}^{-1}(m))\end{aligned}$$

$\text{logit}^{-1}(m)$ can be computed using the `plogis()` function in R and `logistic()` function in Stata

Link Function: probit (in probit regressions)

• Expected value of the dependent variable:

$$h(m) = \Phi(m) = (2\pi)^{-0.5} \int_{-\infty}^{m} \exp(-0.5u^2)du$$

$\Phi(m)$ can be computed using the `pnorm()` function in R and `normal()` function in Stata

• Marginal effect of X in models with an interaction of X and Z:

$$\begin{aligned}\text{ME}_X(x,z,m) &= (\hat{\beta}_{(X)} + \hat{\beta}_{(XZ)}z)\phi(m) \\ &= (\hat{\beta}_{(X)} + \hat{\beta}_{(XZ)}z)(2\pi)^{-0.5}\exp(-0.5m^2)\end{aligned}$$

$\phi(m)$ can be computed using the `dnorm()` function in R and `normalden()` function in Stata

• Marginal effect of X in models with an interaction of Z and a quadratic polynomial of X:

$$\begin{aligned}\text{ME}_X(x,z,m) &= (\hat{\beta}_{(X)} + \hat{\beta}_{(X^2)}x + \hat{\beta}_{(XZ)}z + \hat{\beta}_{(X^2Z)}xz)\phi(m) \\ &= (\hat{\beta}_{(X)} + \hat{\beta}_{(X^2)}x + \hat{\beta}_{(XZ)}z + \hat{\beta}_{(X^2Z)}xz)(2\pi)^{-0.5}\exp(-0.5m^2)\end{aligned}$$

$\phi(m)$ can be computed using the `dnorm()` function in R and `normalden()` function in Stata

Link Function: log (in Poisson, negative binomial, and gamma GLMs)

- Expected value of the dependent variable:

$$h(m) = \exp(m)$$

- Marginal effect of X in models with an interaction of X and Z:

$$\text{ME}_X(x, z, m) = (\hat{\beta}_{(X)} + \hat{\beta}_{(XZ)} z) \exp(m)$$

- Marginal effect of X in models with an interaction of Z and a quadratic polynomial of X:

$$\text{ME}_X(x, z, m) = (\hat{\beta}_{(X)} + \hat{\beta}_{(X^2)} x + \hat{\beta}_{(XZ)} z + \hat{\beta}_{(X^2 Z)} xz) \exp(m)$$

Link function: linear/identity (as in Gaussian models)

- Expected value of the dependent variable:

$$h(m) = m$$

- Marginal effect of X in models with an interaction of X and Z:

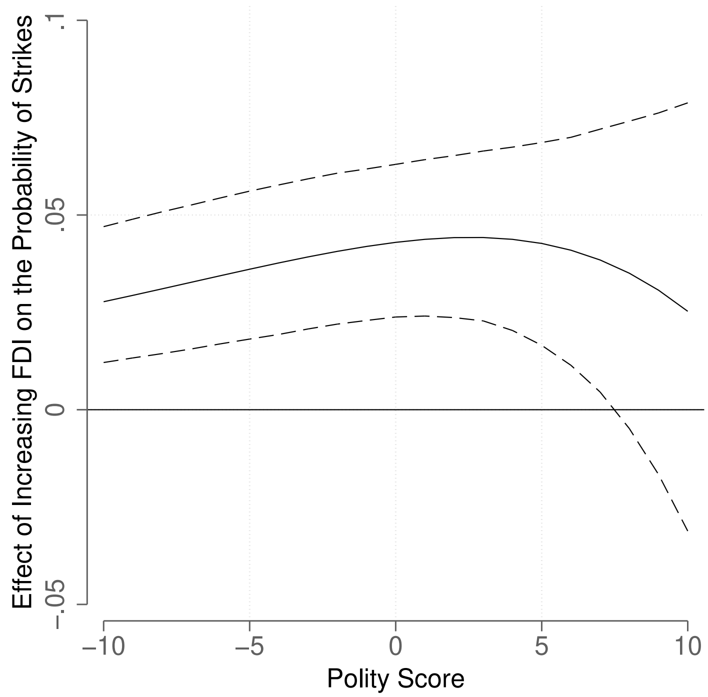
$$\text{ME}_X(x, z, m) = \hat{\beta}_{(X)} + \hat{\beta}_{(XZ)} z$$

- Marginal effect of X in models with an interaction of Z and a quadratic polynomial of X:

$$\text{ME}_X(x, z, m) = \hat{\beta}_{(X)} + \hat{\beta}_{(X^2)} x + \hat{\beta}_{(XZ)} z + \hat{\beta}_{(X^2 Z)} xz$$

Appendix E. The effect of a Binary Variable

Figure E.1. The Effect of Increasing FDI Inflows on the Probability of Strikes at Different Levels of Polity Score



Data source: Robertson, G. B., and E. Teitelbaum. 2011. “Foreign Direct Investment, Regime Type, and Labor Protest in Developing Countries.” *American Journal of Political Science* 55 (3): 665–677.