Supplementary material: Model assumptions in numerical examples

S1 Stand timber volume

We utilize the Fridh-Nilsson growth function (Fridh and Nilsson 1980) to depict the accumulation of timber volume[1]. The function has been parametrized to portray a generic Nordic forest. It has been previously applied in forest economic literature by e.g. Gong and Löfgren (2007, 2016) and in a textbook[2] by Amacher et al. (2009). Below we review its properties.

Timber volume, $v(a)$, is a function of stand age, a. The specific functional form is

$$v(a) = bc\alpha\left(1-\beta^{-\frac{a}{c}}\right)^{\gamma}, \qquad [S01]$$

where $b, c, \alpha, \beta, \gamma > 0$ are parameters. The parameters b and c are the Maximum Sustained Yield (MSY) and the MSY rotation[3], respectively. Hence, bc is the standing timber volume at the end of the MSY rotation. The parameter α expresses the maximum potential timber volume relative to bc (i.e. as $a \longrightarrow \infty$, $v(a) \longrightarrow bc\alpha$).

By giving these interpretations to b, c and α we constrain our choice of β and γ in two ways. The first constraint arises from the requirement that $v(c) = bc$. Thus, when $a = c$, it must hold that

$$\alpha(1-\beta^{-1})^{\gamma} = 1. \qquad [S02]$$

The second constraint arises from the fact that b is the MSY rotation. Letting $\tilde{v}(a) = v(a)/a$, it must therefore be that $\tilde{v}'(c) = 0$ (as the average annual yield is be maximized at rotation length c). Solving $\tilde{v}'$ at $a = c$ and simplifying we obtain

$$\gamma = \frac{\beta-1}{\ln\beta}. \qquad [S03]$$

Constraints [S02] and [S03] imply that the values of β and γ are fixed by our choice[4] of b, c and α. Hence, the growth function has only three effective parameters (all of which have straight-forward interpretations).

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[1] We use the continuous function to derive a vector of standing timber volumes by age class.

[2] However, the parameter values used in Amacher et al. do not adhere to the restrictions described in this section and, therefore, the same interpretations cannot be given to the parameters in their examples.

[3] MSY is the maximum mean annual timber yield that can be obtained from the stand over time. The MSY rotation is the rotation length that produces this yield level.

[4] Naturally, instead of choosing a value for α, we can choose a value for β or γ (choosing a value for any of the three shape parameters determines the values of the remaining two parameters). Here, we have chosen treat α as the actual shape parameter that determines the values of the two others, because α has a clear interpretation. In our numerical model, we determine α and γ in terms of an exogenously given β, simply because this offers an easier way to write constraints.

The parameter values were estimated by fitting the Fridh-Nilsson growth function to data from the Motti simulator (version 3.0). Motti is a stand-level forest simulator developed for analyzing forest management in Finnish growing conditions (Hynynen et al. 2002, Salminen et al. 2005).

Fits were estimated two Myrtillus type sites: Jyväskylä (in Central Finland) and Pudasjärvi (in Northern Finland). The parameter values obtained for Jyväskylä were $\alpha \approx 1.42$, $\beta = 15.44$, $\gamma \approx 5.28.$, $b = 7.98$ and $c = 70.96$. The parameter values obtained for Pudasjärvi were $\alpha \approx 1.68$, $\beta = 5.78$, $\gamma \approx 2.72$, $b = 4.01$ and $c = 109.18$. The former parametrization (Jyväskylä) is used in most of the numerical examples. The latter parametrization (Pudasjärvi) is used in the sensitivity analysis to reflect weaker growing conditions.

In the numerical examples, the market-level model is solved with a 5-year time-step. Hence, also forests are divided into age classes with a 5-year interval. Timber volume in each age-class is set equal to the volume at the lower limit of the age class. For example, the per hectare volume of stands in the just regenerated age class is $v_0 = v(0)$, and the volume of stands in the first age class is $v_1 = v(5)$.

S2 Carbon in timber and living biomass

In our model, the amount of carbon contained in timber and other biomass depend linearly on timber volume (see Equation 8). The parameter γ_v is timber carbon density. The parameter $\gamma_b$expresses the carbon content of other woody (non-timber) biomass proportional to the timber volume. Estimates for the parameter values were obtained as follows.

A biomass function for estimating stemwood mass (i.e. timber mass), $m(v)$, based on standing timber volume, v, was obtained from Lehtonen et al. (2004). The function is

$$m(v) = \alpha v^{\beta}, \qquad \text{[S04]}$$

where $\alpha = 0.3278$ and $\beta = 1.0298$ are parameters[5]. The density of stems changes slightly as trees grow [S04]. However, in our analytical model we make the simplifying assumption that the relationship between volume and mass is linear and independent of timber volume. For consistency, we therefore also apply the same assumption in our numerical examples.

The linear mass-volume relation is calculated using [S04] for an average stand and use it for all stand volumes. A stand with a timber volume of 250 m^3ha^{-1} was used as a benchmark[6] in the calculations. The mass of 250 m^3ha^{-1} is 96,60 tha^{-1} [S04]. The carbon share of dry Norway spruce stemwood biomass is 0.5243 (Nurmi, 1997). Thus, the timber carbon density, γ_v , is

[5] The parameter values in Lehtonen et al. (2004) are given in Table 7.

[6] The functions in Table 7 in Lehtonen et al. (2004) were developed using data from stands between 10 and 250m^3ha^{-1}. Thus, 250 m^3ha^{-1} is the upper bound at which the functions are still guaranteed to work. However, the geometry of the trees changes little after the stand has reached a volume of 250 m^3ha^{-1}, so the derived coefficients provide a fairly good approximation also for stands with higher volume per hectare.

$$\gamma_v = \frac{0.5243\times 96.60\ \mathrm{tha}^{-1}}{250\ \mathrm{m}^3\mathrm{ha}^{-1}} \approx 0.2026\ \mathrm{tm}^{-3}. \qquad [S05]$$

Likewise, total biomass per hectare can be estimated based on timber volume, using a function of the same form as in [S04]. However, in this case, $\alpha = 1.0233$ and $\beta = 0.9511$. When the timber volume is 250 m^3ha^{-1}, the total biomass of the stand is 195.29 tha^{-1} [S04]. Assuming the same carbon share as for stemwood, i.e. 0.5243, we obtain

$$\gamma_b = \frac{0.5243\times 195.29\ \mathrm{tha}^{-1}}{250\ \mathrm{m}^3\mathrm{ha}^{-1}} - 0.2026\ \mathrm{tm}^{-3} \approx 0.2070\ \mathrm{tm}^{-3}. \qquad [S06]$$

S3 Soil carbon emissions

Logging residues are composed of foliage, branches, bark, stumps and roots. The amount of each type of residue generated at harvest was estimated using biomass functions of the form [S04] obtained from Lehtonen et al. (2004). Again, a 250 m^3ha^{-1} stand was used as a benchmark. The utilized parameter values and the components' estimated biomass shares (of total logging residues) are provided in Table S1.

The decomposition of each logging residue component was modelled separately using the Yasso07 soil carbon model (Tuomi et al. 2011A, Tuomi et al. 2011B). The chemical compositions the residue types were obtained from Repo et al. (2012). The decomposition was modelled in the current climatic conditions[7] of Hämeenlinna in Southern Finland.

Table S1: Biomass Expansion Factor function parameter values and logging residue shares of different tree compartments.

Tree compartment	α	β	$m(v)^*$	Share of logging residues	Assumed diameter (cm)
Foliage	0.2283	0.7718	16.19	0.162	0.1
Branches (live)	0.2358	0.8642	27.85	0.278	3
Branches (dead)	0.0160	0.9141	2.49	0.025	3
Bark	0.0596	0.9221	9.69	0.097	1
Stump	0.0528	0.9750	11.50	0.115	30
Roots (coarse)	0.0606	1.0810	23.71	0.237	5
Roots (fine)	0.1200	0.7707	8.46	0.084	1
Total			99.89	1.000	

* when $v = 250m^3ha^{-1}$

The decomposition of mixed residues was estimated by weighting the decomposition of each component according to its share of the generated logging residues (Table S1). The remaining

[7] The climate in the Yasso model is described by three parameter values, (1) mean annual temperature (°C) (2) temperature amplitude (i.e. the difference between the mean temperatures of the coldest and warmest months divided by two), and precipitation (mm). In our calculations we assumed the values (1) 4.5 °C, (2) 11.9 °C, and (3) 615 mm, respectively.

share of the carbon contained in logging residues, remaining in organic matter s years after harvest is shown in Figure S1. This share, σ, as a function of time measured in 5 year periods, s, is described by the exponential fit:

$$\sigma(s) = \sum_{i=1}^{3} \beta_i e^{-\eta_i s} \qquad [S07]$$

where $\beta_1 = 0.52$, $\beta_2 = 0.34$, $\beta_3 = 0.14$, $\eta_1 = 0.2373$, $\eta_2 = 0.0295$, and $\eta_3 = 0.0051$. For the discrete time-model, we define

$$\begin{gathered} \delta_j^S = \sigma(j) - \sigma(j+1)\ \forall j < 40\ and \\ \delta_j^S = \sigma(j)\ when\ j = 40, \end{gathered} \qquad [S08]$$

where j =0,1,…,40 soil carbon vintages. To limit the time horizon of the decay (in our calculations), we assume that all carbon remaining in the soils is released after 40 periods (200 years). In practice, this remainder is fairly small (see Fig. S1).

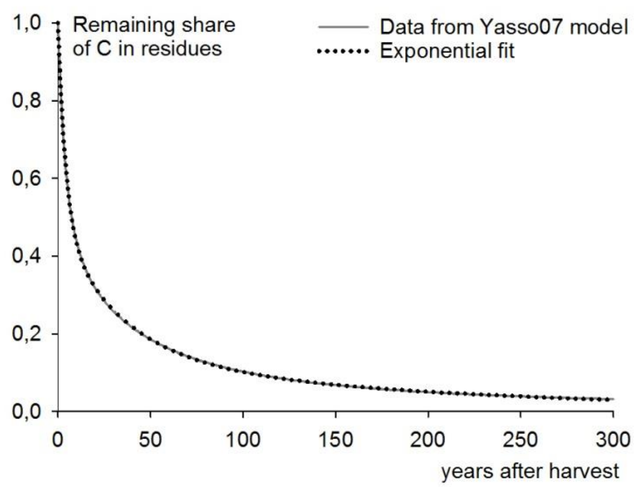

Figure S1 Decomposition of mixed logging residues modelled using Yasso07.

S4 Emissions from wood processing and product carbon stocks

We assume that half of the carbon contained in timber is released immediately during wood processing. The rest is stored in wood products.

In 2015, 40% (8.5 Mm^3) of the Finnish forest industries' domestic spruce consumption in was pulpwood and 60% (12.7 Mm^3) was logs (Natural Resources Institute Finland, 2016). We use these figures to depict the harvest shares allocated to the production of paper products (made of pulpwood) and solidwood products (made of logs). According to the IPCC, the default half-lives[8] of paper and solidwood products are 2 years and 30 years, respectively (IPCC 2006).

[8] These values can be found in Section 12.2.2 in IPCC (2006).

Hence, we obtain the decay rates for paper and solidwood products, $\lambda_p = \ln 2/2 \approx 0.3466$ and $\lambda_s = \ln 2/30 \approx 0.0231$.

We assume that the product carbon stock has 41 vintages, distinguished by j =0,1,…,40. The emissions from each vintage are:

$$\delta_j^P = \left(0.4e^{-\lambda_p j} + 0.6e^{-\lambda_s j}\right) - \left[0.4e^{-\lambda_p (j+1)} + 0.6e^{-\lambda_s (j+1)}\right] \forall j < 40 \; and \quad \delta_j^P = 0.4e^{-\lambda_p j} + 0.6e^{-\lambda_s j} when\ j = 40. \quad [S09]$$

S5 Albedo's warming power

We model stand summer albedo, A, as a function of living biomass, B, so that

$$A(B(a)) = \alpha + \beta e^{-\gamma B(a)}, \quad [S10]$$

where $\alpha = 0.09132355$, $\beta = 0.10644404$, $\gamma = 0.03216253$ and biomass, $B(a)$, is approximated by

$$B(a) = \frac{\gamma_v + \gamma_b}{0.5243} v(a), \quad [S11]$$

where 0.5243 is the stemwood share of dry Norway spruce stemwood biomass (see Section S2).

The fit in [S10] is derived in Rautiainen and Lintunen[9] (2017) based on data from Lukeš et al. (2013). According to Bright et al. (2011), the mean annual warming power of open shrub is 1.254 MW ha^{-1}, and that of spruce forest is 1.412 MW ha^{-1}. We use the value given for spruce forest to depict a closed-canopy spruce forest[10]. We assume that there is a linear inverse relationship between the summer albedo and the mean annual warming power of the stand, i.e.

$$w_a = \left[1.254 + (1.412 - 1.254)\frac{A(B(a)) - A(0)}{A(B(\infty)) - A(0)}\right] \text{MWha}^{-1}. \quad [S12]$$

The warming power of the stand starts from that of open-shrub (at age zero) and converges towards that of a closed-canopy forest as the stand ages. Note that the warming power in [S12] is measured locally and given in MWha^{-1}. In Fig. 2, the warming power has been converted to global warming power, measured in nW (i.e. loosely speaking, "the warming power of a single hectare is evenly distributed over the entire surface of the Earth"). The conversion between the local warming power of a given surface to units of global radiative forcing is outlined in e.g. Bright et al. (2011) and Rautiainen and Lintunen (2017).

[9] The fit is developed in Section S6 of the supplementary material to Rautiainen and Lintunen (2017).

[10] The value provided in Bright et al. (2011) is a regional average which includes a small amount of logged areas. Thus, using the value to depict closed-canopy forests leads to a slight underestimate of the warming power of spruce forests. The error is largest for old forests (where it is roughly 5%, compared to Lintunen et al. (unpublished)).

S6 Economic assumptions

Let ε denote the elasticity in the inverse demand function for timber. The elasticity is the inverse of the price elasticity of demand. In our numerical examples, we (usually[11]) assume $\varepsilon = 1$. This implies that the demand is unit elastic (i.e. a 1% increase in price decreases demand by 1%). Let p^c and h^c denote the calibration price and quantity. We assume $p^c = 55$ €m^{-3}, which roughly equals the price of spruce logs in Finland. The calibration quality-weighted quantity $q_t^c \approx 11.07$ is the amount harvested in every five year time period, when half of the land is allocated to forestry and the rotation is 10 periods (i.e. 50 years).

The timber price, p_t, is given by the function

$$p_t = p^c \left(\frac{q_t}{q^c}\right)^{-\varepsilon} \quad \text{[S13]}$$

As stated in section 2.2.3, the social utility from timber consumption, U_t, is obtained by integrating p_t, i.e $U_t = U(q_t) \coloneqq \int_{q_{min}}^{q_t} p_t(h)dh$, where q_{min} is an arbitrary positive lower bound of integration.[12] Thus,

$$U_t = \begin{cases} \frac{p^c q^c}{1-\varepsilon}\left[\left(\frac{q_t}{q^c}\right)^{1-\varepsilon} - 1\right] \; \forall \; \varepsilon_t \neq 1, and \\ p^c q^c \ln\frac{q_t}{q^c} \; when \;\; \varepsilon_t = 1. \end{cases} \quad \text{[S14]}$$

The inverse demand function and utility functions for food are of a similar form (simply substitute $\tilde{p}_t$, $\tilde{p}^c$, $\tilde{\varepsilon}$, $\tilde{h}_t$, and $\tilde{h}^c$ into the above formulae in place of p, p^c, ε, q_t, and q^c, , respectively. We assume $\tilde{p} = 1000$, $\tilde{h}_t^c = 0.5$ (i.e. $b = 1$ and $y = 0.5$), and $\tilde{\varepsilon} = 1$. The calibration prices and quantities have been selected so that in the calibration steady-state (i.e. initial state), in which half of the land is allocated to agriculture, the value of agricultural land is equated with forest BLV.

For simplicity, we assume that timber harvesting costs are included in the stumpage price and the agricultural harvesting costs are included in the (net) price of the yield. (Thus, the values $c_h = 0$ and $\tilde{c}_h = 0$ were used in the calculations). Forest regeneration costs are 1000 €ha^{-1}. The calibration constraint implies that agricultural production costs are $c_f = 986.56$. The above values are valid in the Jyväskylä growth conditions. For the Pudasjärvi growth conditions, we obtain initial state rotation of 60 years, which results in $q_t^c \approx 3.89$ and $c_f = 1166.5$.

[11]We explicitly state when other elasticities are used.

[12] As usual, the level of utility is of no interest but the changes in utility are. Therefore, the lower bound of integration is arbitrary. In the quantitative assessment we set $q_{min} = q^c$.

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