APPENDIX B: Economic Modeling Addendum

The following equations represent the structure of the model for egg type *i* (*te*: table eggs; *pe*: processed eggs; *s*: shell eggs) for time *t*:

Shell Egg Supply:

$$S_t = a_{s,t}(w_t, P_{s,t}, r_{PE,t}) \tag{1}$$

Allocation of Shell Eggs:

$$S_t = (a_{s,TE,t}q_{TE,t} + a_{s,PE,t}q_{PE,t})\phi \tag{2}$$

Retail Demand:

$$D_{TE,t} = D_{TE,t}(P_{TE,t}, P_{PE,t})\, \gamma_{TE,t} \tag{3}$$

$$D_{PE,t} = D_{PE,t}(P_{TE,t}, P_{PE,t})\, \gamma_{PE,t} \tag{4}$$

Net Export Demand:

$$(X_{TE,t} - M_{TE,t}) = (X_{TE,t} - M_{TE,t})(P^w_{TE,t})\, \delta_{TE,t} \tag{5}$$

$$(X_{PE,t} - M_{PE,t}) = (X_{PE,t} - M_{PE,t})(P^w_{PE,t})\, \delta_{PE,t} \tag{6}$$

Stock Demand:

$$I_{TE,t} = I_{TE,t}(P_{s,t}) \tag{7}$$

$$I_{PE,t} = I_{PE,t}(P_{s,t}) \tag{8}$$

Egg Processing Capacity:

$$k_{TE,t} = a_{k,TE,t}q_{TE,t} \tag{9}$$

$$k_{PE,t} = a_{k,PE,t}q_{PE,t} \tag{10}$$

Retail Price Equations:

$$P_{TE,t} = P_{TE,t}(w_t, P_{s,t}, r_{TE,t}) \tag{11}$$

$$P_{PE,t} = P_{PE,t}(w_t, P_{s,t}, r_{PE,t}) \tag{12}$$

World Price Equations:

$$P^{w}_{TE,t} = P_{TE,t} + t_{TE,t} \quad (13)$$

$$P^{w}_{PE,t} = P_{PE} + t_{PE} \quad (14)$$

Market Clearing Equation:

$$q_{TE,t} + I_{TE,t-1} = (X_{TE,t} - M_{TE,t}) + D_{TE,t} + I_{TE,t} \quad (15)$$

$$q_{PE,t} + I_{PE,t-1} = (X_{PE,t} - M_{PE,t}) + D_{PE,t} + I_{PE,t} \quad (16)$$

Price (P_i) is represented by egg type i (*te*: table eggs; *pe*: processed eggs) for both domestic and world reference prices (P^{w}_{i}). Quantity of shell egg supply (S) are diverted to processing eggs demanded (q_i), which are then vertically linked to final use through the retail demand (D_i), ending stocks ($I_{i,t}$), or exports (X_i). World reference prices are linked by domestic prices plus transportation costs (c_i). Assuming zero profits in production, price of eggs is derived from the cost of inputs used in production (w), the price of shell eggs (P_s), and returns to capital (r). Per-unit derived demand are separated into the factors of production: shell eggs ($a_{s,i}$), capital ($a_{k,i}$), and labor ($a_{l,i}$) for egg type of the i. Exogenous shocks can be applied at various levels of the model through supply shocks (ϕ), trade shocks ($\delta_{i,t}$), or shocks to demand preferences ($\gamma_{i,t}$).

Market clearing conditions for table and processed eggs, Eq. 15 and 16, insure that the market clears such that net exports (exports (X_i) minus imports (M_i)), domestic consumption (D_i), and ending stocks (I_i) in the current period (t) should equal production ($q_{i,t}$) plus begging stocks ($I_{i,t-1}$) which are the ending stocks from the previous period.

Fully Differentiated Model

The model is fully differentiated such that all variables are represented as percentage changes. The term E is used to represent the relative change operator (i.e., dln). To fully represent changes that could occur in per unit factor usage, a vector of the three per unit derived demand for the factors of production included in this model $a_{s,i}$, $a_{k,i}$, and $a_{l,i}$ are also differentiated to link changes in these usage of these factors and their respective prices through elasticities of substitution under constant returns to scale between shell eggs and capital ($\sigma_{s,k|i}$) as well as between labor and capital ($\sigma_{l,k|i}$) (Paarlberg et al. 2008).

$$Ea_{s,TE,t} - Ea_{k,TE,t} = -\sigma_{s,k|TE}(EP_{s,t} - Er_{TE,t}) \tag{17}$$

$$Ea_{s,PE,t} - Ea_{k,PE,t} = -\sigma_{s,k|PE}(EP_{s,t} - Er_{PE,t}) \tag{18}$$

$$Ea_{l,TE,t} - Ea_{k,TE,t} = -\sigma_{l,k|TE}(Ew_{t} - Er_{TE,t}) \tag{19}$$

$$Ea_{l,PE,t} - Ea_{k,PE,t} = -\sigma_{l,k|PE}(Ew_{t} - Er_{PE,t}) \tag{20}$$

Additionally, movement around the unit isoquant can be shown to be a function of the changes to the per unit factor demands and represented by the following equations:

$$\theta_{s,TE}\, Ea_{s,TE,t} + \theta_{l,TE}\, Ea_{l,TE,t} + \theta_{k,TE}\, Ea_{k,TE,t} = 0 \tag{21}$$

$$\theta_{s,PE}\, Ea_{s,PE,t} + \theta_{l,PE}\, Ea_{l,PE,t} + \theta_{k,PE}\, Ea_{k,PE,t} = 0 \tag{22}$$

Equation 21 and 22 represents an adding up condition that dictates changes to the per-unit derived demand multiplied by its respective unit revenue share ($\theta_{s,i}$, $\theta_{k,i}$, and $\theta_{l,i}$)should sum to zero for both table and processed eggs.

Table B1 presents all 22 fully differentiated equations. Parameters included in the differentiated equations include: shell egg own-price elasticity (ε_s), own-price elasticities of demand ($\varepsilon_{i,i}$), cross-price elasticities of demand ($\varepsilon_{i,j}$), factor shares of production ($\lambda_{s,i}$), stock elasticities ($\varepsilon_{I,i}$), net export elasticities ($\varepsilon_{x-m,i}$), and transportation elasticity (c_i) for the respective egg types.

To provide context and further description, variables and mean data for 2014 are presented in Table B2.

Table B1: Fully differentiated expanded model equations for U.S. egg industry

(B1)	$EP_{TE,t} = \theta_{l,TE}\, Ew_t + \theta_{s,TE}\, EP_{s,t} + \theta_{k,TE}\, Er_{TE,t}$
(B2)	$EP_{PE,t} = \theta_{l,PE}\, Ew_t + \theta_{s,PE}\, EP_{s,t} + \theta_{k,PE}\, Er_{PE,t}$
(B3)	$ES_t = E\phi_t + \lambda_{s,te}\, Eq_{te,t} + \lambda_{s,pe}\, Eq_{pe,t} + \lambda_{s,te}\, Ea_{s,te,t} + \lambda_{s,pe}\, Ea_{s,pe,t}$
(B4)	$ES_t = \varepsilon_s EP_{s,t}$
(B5)	$Ek_{TE,t} = Ea_{k,TE,t} + Eq_{TE,t}$
(B6)	$Ek_{PE,t} = Ea_{k,PE,t} + Eq_{PE,t}$
(B7)	$Ea_{s,TE,t} - Ea_{k,TE,t} = -\sigma_{s,k\|TE}(EP_{s,t} - Er_{TE,t})$
(B8)	$Ea_{l,TE,t} - Ea_{k,TE,t} = -\sigma_{l,k\|TE}(Ew_t - Er_{TE,t})$
(B9)	$\theta_{s,TE}\, Ea_{s,TE,t} + \theta_{l,TE}\, Ea_{l,TE,t} + \theta_{k,TE}\, Ea_{k,TE,t} = 0$
(B10)	$Ea_{s,PE,t} - Ea_{k,PE,t} = -\sigma_{s,k\|PE}(EP_{s,t} - Er_{PE,t})$
(B11)	$Ea_{l,PE,t} - Ea_{k,PE,t} = -\sigma_{l,k\|PE}(Ew_t - Er_{PE,t})$
(B12)	$\theta_{s,PE}\, Ea_{s,PE,t} + \theta_{l,PE}\, Ea_{l,PE,t} + \theta_{k,PE}\, Ea_{k,PE,t} = 0$
(B13)	$q_{TE,t}Eq_{\,TE,t} + I_{TE,t-1}EI_{\,TE,t-1} = \left(X_{TE,t} - M_{TE,t}\right)E\left(X_{TE,t} - M_{TE,t}\right) + D_{TE,t}ED_{TE,t} + I_{TE,t}EI_{\,TE,t}$
(B14)	$q_{PE,t}Eq_{\,PE,t} + I_{PE,t-1}EI_{\,PE,t-1} = \left(X_{PE,t} - M_{PE,t}\right)E\left(X_{PE,t} - M_{PE,t}\right) + D_{PE,t}ED_{PE,t} + I_{PE,t}EI_{\,PE,t}$
(B15)	$EI_{TE,t} = \varepsilon_{I,TE}EP_{s,t}$
(B16)	$EI_{PE,t} = \varepsilon_{I,PE}EP_{s,t}$
(B17)	$ED_{TE,t} = E\gamma_{TE,t} + \varepsilon_{TE,TE}\, EP_{TE,t} + \varepsilon_{TE,PE}\, EP_{PE,t}$
(B18)	$ED_{PE,t} = E\gamma_{PE,t} + \varepsilon_{PE,PE}\, EP_{PE,t} + \varepsilon_{PE,TE}\, EP_{TE,t}$
(B19)	$E\left(X_{TE,t} - M_{TE,t}\right) = \delta_{TE,t} + \varepsilon_{x-m,TE}EP^{w}_{TE,t}$
(B20)	$E(X_{PE,t} - M_{PE,t}) = \delta_{PE,t} + \varepsilon_{x-m,PE}EP^{w}_{PE,t}$
(B21)	$P^{w}_{TE,t}EP^{w}_{TE,t} = P_{TE,t}\, EP_{TE,t} + c_{TE,t}Et_{TE,t}$
(B22)	$P^{w}_{PE,t}EP^{w}_{PE,t} = P_{PE,t}\, EP_{PE,t} + c_{PE,t}Et_{PE,t}$

Note: E is used to denote dln, TE denotes table eggs, PE denotes processed eggs, *t* denotes time

Table B2: Variable definitions and summary of data used in U.S. Egg Model, 2014

Parameters	Description	Unit	Minnesota Mean	Rest of the United States Mean
S	Shell egg supply[d]	millions of dozens	53	1,786
D_{TE}	Retail demand: table eggs[b,d]	millions of dozens	21	1,203
D_{PE}	Retail demand: processed eggs[a,d]	millions of equivalent dozens	9	516
$(X_{TE} - M_{TE})$	Net exports: table eggs[a,d]	millions of dozens	13	208
$(X_{PE} - M_{PE})$	Net exports: processed eggs[a,d]	millions of equivalent dozens	6	83
I_{TE}	Beginning stocks: table eggs[a,d]	millions of dozens	0.5	16
I_{PE}	Beginning stocks: processed eggs[a,d]	millions of equivalent dozens	0.2	6.7
q_{TE}	Egg Production: table eggs[a,d]	millions of dozens	38	1,250
q_{PE}	Egg Production: processed eggs[a,d]	millions of equivalent dozens	16	536
$P_{s,t}$	Shell egg price[a]	$/dozen	0.72	0.72
$P_{TE,t}$	Table egg price[a]	$/dozen	2.00	2.00
$P_{PE,t}$	Processed egg price[a]	$/equivalent dozen	4.63	4.63
$P_{TE,t}^{w}$	World table egg price[b,d]	$/dozen	2.02	2.02
$P_{PE,t}^{w}$	World processed egg price[b,d]	$/equivalent dozen	4.64	4.64

Source:
[a] Agricultural Marketing Service (2015)
[b] Economic Research Service (2013; 2015),
[c] National Agricultural Statistics Service (2014; 2016),
[d] World Agricultural Supply and Demand Estimates (USDA-ERS 2016)

Model Parameter Calculations Descriptions

The discussion below provides further explanation for the calculated parameters. All parameters are reported in Table B3 below.

Unit Revenue Shares

Unit revenue shares were derived from published information (Bell 2001) and supplemented with industry expert opinions. The revenue shares were calculated as a percentage of the price going to different factors of production. These were aggregated into three main categories: shell eggs (breaker eggs), costs of processing, and margin. The most likely values were requested when eliciting expert opinion. In order to compare prices across multiple products, the processed egg products were calculated using a shell egg equivalent. Where necessary the shell egg equivalent conversion factor used was 1.4 pounds per dozen shell eggs.

Egg prices used to calculate the unit revenue shares were the 2014 mean prices as reported by USDA–AMS (2015). Table eggs are the average 2014 prices of Grade A, white, large eggs. For processed eggs, an aggregate processed egg price was calculated using the AMS reported prices in conjunction with expert opinion for the diversion of eggs into the three processed egg categories: liquid, frozen, or dried. Each product and product price was converted to a shell egg equivalent. The average tended toward the average frozen processed egg prices.

Costs used in calculations are general values for either table or processed eggs. All transportation costs are assumed to be 10 cents per dozen. Table egg costs consist of 22 cents per dozen processing costs for washing, sanitizing, and cartoning the eggs plus transportation costs. Processed egg costs are estimated as 24 cents per equivalent dozen, which include transportation, processing, and pasteurizing. The processing costs include the costs associated with breaking eggs. Eggs are typically pasteurized as egg products, so that a shell egg equivalent had to be used to estimate the cost on a per-egg basis.

Margin on eggs was assumed to be the difference between the end prices of products minus the aforementioned costs. This reduced the sensitive nature of eliciting opinion on margin values from producers. This margin would accrue to the processor and retailer as well as any additional middle steps including wholesaler, warehousing, or additional transportation outside of Bell's transportation costs.

Product- Specific Elasticities

The nature of the economic egg model diverts eggs into either table or processed eggs. In order to parameterize the model appropriately, elasticities specific to end product were needed for most accurate results. These were not found in current literature or government sources. Available elasticities were generalized egg elasticities that included all egg products (including hatching eggs). In order to facilitate modeling endeavors, stock, net export, and price elasticities were calculated by end product used in the model (i.e., individually for table and processed eggs). This provides a means to differentiate market responses by product type, instead of assuming similar responses by type of product. All elasticities were calculated by use of double log models, which assume constant elasticities.

Stock Elasticities

To estimate the stock elasticities, individual models were calculated using ordinary least squares for ending stocks for table and processed eggs as a function of the price of shell eggs during the current time period *t*. The values were logged assuming constant elasticities.

$$EI_t = \beta_1 + \varepsilon_{I,i} EP_{s,t} \tag{23}$$

Net Export Elasticities

Net export elasticities had to be calculated for both egg product types due to limitation of the available net exports elasticities. Elasticities in the relevant literature were aggregated for all egg products and were separated into import and export elasticities. To calculate the net export elasticities, a simple ordinary least squares regression was estimated for net exports for both table and processed eggs using world price for the respective egg type as the regressor. All variables were logged such that the resulting coefficients are the elasticities. Below are the equations used in these calculations.

$$E(X_{TE} - M_{TE}) = \beta_1 + \varepsilon_{x-m,TE} EP_{TE}^{w} \tag{24}$$

$$E(X_{PE} - M_{PE}) = \beta_1 + \varepsilon_{x-m,PE} EP_{PE}^{w} \tag{25}$$

Price Elasticities

For similar reasons as above, the elasticities for the individual egg type had to be calculated to better parameterize the U.S. Egg Model. These again were estimated with ordinary least squares on a double log specified model. The quantity, or D_i, is regressed on the own and cross prices to better estimate the price elasticities.

$$ED_{TE} = \beta_1 + \varepsilon_{TE,TE}\, EP_{TE} + \varepsilon_{TE,PE}\, EP_{PE} \tag{26}$$

$$ED_{PE} = \beta_1 + \varepsilon_{PE,PE}\, EP_{PE} + \varepsilon_{PE,TE}\, EP_{TE} \tag{27}$$

Table B3: Summary of parameters used in the economic model analysis and their sources

Parameters	Description	Value	Source
$\theta_{l, te}$	Unit revenue shares	0.160	Bell, (2001); Industry Expertise
$\theta_{s,te}$	Unit revenue shares	0.515	Bell, (2001); Industry Expertise
$\theta_{k,te}$	Unit revenue shares	0.325	Bell, (2001); Industry Expertise
$\theta_{l, pe}$	Unit revenue shares	0.164	Bell, (2001); Industry Expertise
$\theta_{s,pe}$	Unit revenue shares	0.532	Bell, (2001); Industry Expertise
$\theta_{k,pe}$	Unit revenue shares	0.304	Bell, (2001); Industry Expertise
$\lambda_{s, te}$	Factor Share	0.700	USDA – AMS (2015)
$\lambda_{s, pe}$	Factor Share	0.300	USDA – AMS (2015)
$\varepsilon_{y,te}$	Income Elasticity	0.346	USDA – ERS (2013)
$\varepsilon_{y,pe}$	Income Elasticity	0.346	USDA – ERS (2013)
$\varepsilon_{I, te}$	Stock Elasticity	-1.315	Author's Calculation
$\varepsilon_{I, pe}$	Stock Elasticity	-0.108	Author's Calculation
$\varepsilon_{x, te}$	Net Export Elasticity	0.590	Author's Calculation
$\varepsilon_{x,pe}$	Net Export Elasticity	0.250	Author's Calculation
$\varepsilon_{te,pe}$	Cross Price Elasticity	0.149	Author's Calculation
ε_{te}	Own Price Elasticity	-0.538	Author's Calculation
ε_{pe}	Own Price Elasticity	-0.801	Author's Calculation
$\sigma_{s,k: te}$	Substitution Elasticity	0.436	Ollinger, MacDonald, & Madison (2005)
$\sigma_{l,k: te}$	Substitution Elasticity	0.436	Ollinger, MacDonald, & Madison (2005)
$\sigma_{s,k: pe}$	Substitution Elasticity	0.436	Ollinger, MacDonald, & Madison (2005)
$\sigma_{l,k: pe}$	Substitution Elasticity	0.436	Ollinger, MacDonald, & Madison (2005)
ε_{s}	Egg Price Elasticity	-0.088	USDA – ERS (2013)
η_{s}	Raw Egg Supply Elasticity	1.000	USDA – ERS (2013)

Works Cited

Bell, D. 2001. “California Egg Processing, Transportation, and Marketing Costs - 2000.” An Egg Economics Update No. 239, University of California.

Ollinger, M., J.M. MacDonald, and M. Madison. 2005. “Technological change and economies of scale in US poultry processing.” *American Journal of Agricultural Economics* 87(1):116–129.

Paarlberg, P.L., A.H. Seitzinger, J.G. Lee, and K.H. Mathews. 2008. *Economic impacts of foreign animal disease*. United States Department of Agriculture, Economic Research Service.

U.S. Department of Agriculture, Agricultural Marketing Service. 2015. “USDA Egg Market News Report.” Market News

U.S. Department of Agriculture, Economic Research Service. 2013. “Income elasticities for food subcategories.” International Food Consumption Patterns

U.S. Department of Agriculture, Economic Research Service. 2015. “International Macroeconomic Data Set.”

U.S. Department of Agriculture, Economic Research Service. 2016. “Livestock & Meat Domestic Data: Total Poultry.”

U.S. Department of Agriculture, National Agricultural Statistics Service. 2014. “Poultry-Production and Value 2013 Summary.” USDA.

U.S. Department of Agriculture, National Agricultural Statistics Service. 2016. “USDA/NASS QuickStats Ad-hoc Query Tool.” United States Department of Agriculture.