An Analysis of US Greenhouse Gas Cap-and-Trade Proposals using a Forward-Looking Economic Model

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APPENDIX – The Forward-looking Model in MCP Formulation

In the forward-looking dynamic EPPA model, the optimization problem is converted into a market equilibrium formulation using the mixed complementarity problem (MCP) algorithm (Mathiesen, 1985; Rutherford, 1995), and solved numerically using the General Algebraic Modeling System (GAMS) software (Brooke *et al*., 1998). MCP problems are written and solved through three types of inequalities: the zero profit, market clearance and income balance conditions. The equilibrium structure of the EPPA model, defined by these three conditions has been documented by Paltsev *et al*. (2005).

To illustrate the structure of the dynamic model in MCP format, we simplify the notation and remove the subscript r (and the associated complication of including international trade in the representation of the model). The MCP formulation relies on the duality of the consumption and production theories to define unit expenditure and unit cost functions, demand functions for goods, intermediate inputs and primary factors.

The constrained minimization of total expenditure generates:

- $D_{jt}^{C}(p_{jt})$, the compensated demand for good *j*, where p_{jt} are the prices of goods *j*;
- $E_{t}^{C}(p_{it}, p_{jt})$, the unit cost of aggregate consumption at time *t*, as a function of the prices of goods *i* and *j* (*i* and *j* representing the same set of goods).

The minimization of total capital expenditure, subject to the production of one unit of aggregated investment generates:

- $D_{jt}^{I}(p_{jt})$, the compensated demand for good *j* for investments purposes;
- $E_{t}^{I}(p_{it}, p_{jt})$, the unit cost of aggregate investment at time *t*.

On the production side of the economy, the minimization of total costs of production, subject to the production (y_{it}) of one unit of sectoral output generates:

- $D_{ijt}^{ID}(p_{jt}, y_{it})$, the compensated demand for intermediate input *j* at sector *i*;
- $D_{it}^{F}(p_{t}^{F}, y_{it})$, the compensated demand for primary factors labor (L) and energy resources (R) at sector *i*;
- $D_{it}^{K}(r_{t}^{K}, y_{it})$, the compensated demand for capital services at sector *i*;
- $E_{it}^{Y}(p_{jt}, p_{t}^{F})$, the unit cost function for sector *i*, where p_{t}^{F} represents the price of factors.

Using these functions, we write the MCP conditions: zero profits, market clearance and income balance. The zero profit condition means that profit should be equal to zero for any sector (including the "sectors" that produce aggregate demand and the aggregate investment) that produces a positive quantity of output or, if profit is negative, there is no production at all. The zero profit condition can be represented by the following relation for every sector:

$$profit \geq 0, output \geq 0, output * (-profit) = 0. \quad \text{(A1)}$$

The market clearance condition implies that a positive price exists for any good with supply equal to demand, or the price will be zero if the good has an excess of supply. This condition can be represented by the relation for every good and factor:

$$supply - demand \geq 0, price \geq 0, price * (supply - demand) = 0. \quad \text{(A2)}$$

The income balance condition means that total expenditure should be equal to the total value of endowments for each agent.

We can use these MCP conditions to write the dynamic EPPA model as following. The zero profit conditions are:

a) the aggregate price index is equal to the unit cost of aggregate consumption:

$$p_t = E_t^C(p_{it}, p_{jt}). \quad \text{(A3)}$$

b) the price of capital in the next period is equal to the unit cost of aggregate investment:

$$p_{t+1}^K = E_t^I(p_{it}, p_{jt}). \quad \text{(A4)}$$

c) the price of capital at time t must be equal to its returns (per unit of capital) plus the price of capital at next period, accounting for the depreciation (zero profit for capital accumulation):

$$p_t^K = r_t^K + (1-\delta)p_{t+1}^K. \quad \text{(A5)}$$

d) the price of output from sector *i* is equal to its unit cost:

$$p_{it} = E_{it}^Y(p_{jt}, p_t^F), \quad \text{(A6)}$$

The market clearance conditions are:

a) Supply equals demand in each commodity market:

$$y_{it} = \sum_j D_{ijt}^{ID}(p_{jt}, y_{it}) + D_{it}^{C}(p_{it}) + D_{it}^{I}(p_{it}), \tag{A7}$$

b) Supply (F_t^F) equals demand in each primary factor market:

$$F_t^F = \sum_j D_{jt}^F(p_t^F, y_{jt}). \tag{A8}$$

c) Capital accumulation (perpetual inventory equation):

$$K_{t+1} = I_t + (1-\delta)K_t. \tag{A9}$$

The total of the stream of incomes over the agent's lifetime is given by:

$$M = p_0^K K_0 + \sum_{F,t} p_t^F F_t^F - p_{T+1}^K K_{T+1}. \tag{A10}$$

The equations from (A3) to (A10) represent an abstract version of the model in MCP. Other practical issues in the implementation of the dynamic version of EPPA are addressed in the paper.