Foreign aid and oil taxes: helping the poor in oil-rich countries*

RUXANDA BERLINSCHI
LICOS, Centre for Institutions and Economic Performance, Catholic University of Leuven, Belgium. Email: ruxanda.berlinschi@econ.kuleuven.be.

JULIEN DAUBANES (Corresponding author)
CER-ETH at ETHZ, Swiss Federal Institute of Technology Zurich, Zürichbergstrasse 18, 8092 Zurich, Switzerland. Tel: +41 44 632 24 27. Fax: +41 44 632 13 62. Email: jdaubanes@ethz.ch.

Appendix

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Proof of Proposition 1

1. Let us prove that production and consumption variables grow at the rate $g = (1-\alpha)g_A - \alpha\rho$.

The production functions (1) imply $Y_N/Y_S = (L_N/\varphi L_S)^{1-\alpha}(R_N/R_S)^{\alpha}$. The first-order conditions (8) imply $R_N/R_S = (Y_N/(1+\theta_N))/(Y_S/(1+\theta_S))$. Then, $Y_N/Y_S = (L_N/\varphi L_S)((\theta_S+1)/(\theta_N+1))^{\alpha/(1-\alpha)}$. As L_N and L_S are inelastic and θ_N and θ_S are constant, then $g_{Y_N} = g_{Y_S} = g_Y$. The world resource constraint (7) implies $g_Y = g_C$. The Ramsey-Keynes conditions (15) then give $g_Y = r - \rho$. Using the first-order conditions (8) and $g_{Y_N} = g_{Y_S}$, one obtains $g_{R_N} = g_{R_S} = g_R$. The Hotelling rule (10) can be rewritten using (8) like $g_R = g_Y - r$. Then. $g_Y = r - \rho$ above implies $g_R = -\rho$. Finally, the production functions (1) give $g_Y = (1-\alpha)g_A + \alpha g_R$. Using $g_R = -\rho$, it yields $g_Y = (1-\alpha)g_A - \alpha\rho$. Overall, $g_Y = (1-\alpha)g_A - \alpha\rho = g_{Y_N} = g_{Y_S} = g_C = g_{C_N} = g_{C_S}$.

2. This immediately follows from $Y_N/Y_S = (L_N/\varphi L_S)((\theta_S+1)/(\theta_N+1))^{\alpha/(1-\alpha)}$ proven above.

3. Let us show that $C_N(t) = (1-\frac{\alpha}{\theta_N+1})Y_N(t) - F(t)$, $C_{SP}(t) = (1-\alpha)Y_S(t) + F(t)$ and $C_{SR}(t) = (\frac{\alpha}{\theta_N+1})\, Y_N(t) + \alpha Y_S(t)$.

Let us first develop the instantaneous budget constraints of the three groups. From the first-order conditions (9), $w_N = (1-\alpha)Y_N/L_N$ and $w_S = (1-\alpha)Y_S/L_S$. From the first-order conditions (8), $p\theta_N R_N = \alpha Y_N \theta_N/(\theta_N+1)$, $p\theta_S R_S = \alpha Y_S\theta_S/(\theta_S+1)$ and $p(R_N+R_S) = \alpha Y_N/(\theta_N+1) + \alpha Y_S/(\theta_S+1)$. By substituting these revenues into (11), (12) and (13) and rearranging, the instantaneous budget constraints become $C_N/L_N + \dot{B}_N/L_N = (1-\alpha/(\theta_N+1))Y_N/L_N + rB_N/L_N - F/L_N$, $C_{SP}/L_S + \dot{B}_{SP}/L_S = (1-\alpha)Y_S/L_S + rB_{SP}/L_S + F/L_S$ and

$C_{SR} + \dot{B}_{SR} = \alpha Y_N/(\theta_N + 1) + \alpha Y_S + rB_{SR}$.

Moreover, we have shown above that $g_Y = r - \rho$ and $g_Y = (1-\alpha)g_A - \alpha\rho$. These equations imply $r = (1-\alpha)(g_A + \rho)$.

Next, solving the instantaneous budget constraints as first-order differential equations in B_N, B_{SP} and B_{SR}, one obtains the following intertemporal budget constraints satisfied for all $T \geq 0$:

$B_N(T)e^{-rT} + \int_0^T C_N(t)e^{-rt}dt = (1 - \alpha/(\theta_N + 1)) \int_0^T Y_N(t)e^{-rt}dt - \int_0^T F(t)e^{-rt}dt + B_N(0)$,

$B_{SP}(T)e^{-rT} + \int_0^T C_{SP}(t)e^{-rt}dt = (1-\alpha) \int_0^T Y_S(t)e^{-rt}dt + \int_0^T F(t)e^{-rt}dt + B_{SP}(0)$ and

$B_{SR}(T)e^{-rT} + \int_0^T C_{SR}(t)e^{-rt}dt = \frac{\alpha}{\theta_N+1} \int_0^T Y_N(t)e^{-rt}dt + +\alpha \int_0^T Y_S(t)e^{-rt}dt + B_{SR}(0)$.

The no-Ponzi-game conditions (14) write $\lim_{T\to\infty} B_i(T)e^{-rT} = 0,\ i = N, SP, SR$. Thus, taking the limit as T goes to infinity and using the assumption $B_N(0) = B_{SP}(0) = B_{SR}(0) = 0$, we obtain

$$\int_0^\infty C_N(t)e^{-rt}dt = \left(1 - \frac{\alpha}{\theta_N+1}\right) \int_0^\infty Y_N(t)e^{-rt}dt - \int_0^\infty F(t)e^{-rt}dt.$$

$$\int_0^\infty C_{SR}(t)e^{-rt}dt = \frac{\alpha}{\theta_N+1} \int_0^\infty Y_N(t)e^{-rt}dt + \alpha \int_0^\infty Y_S(t)e^{-rt}dt \text{ and}$$

$$\int_0^\infty C_{SP}(t)e^{-rt}dt = (1-\alpha) \int_0^\infty Y_S(t)e^{-rt}dt + \int_0^\infty F(t)e^{-rt}dt.$$

We know that C_N, C_{SP} C_{SR} Y_N, Y_S and F grow at the same rate $r - \rho$. Note that for any variable X such that $g_X = r - \rho$, we have $\int_0^\infty X(t)e^{-rt}dt = \int_0^\infty X(0)e^{(r-\rho)t}e^{-rt}dt = X(0)\int_0^\infty e^{-\rho t}dt = X(0)/\rho$. Therefore, the intertemporal budget constraints imply

$$C_N(0) = (1 - \frac{\alpha}{\theta_N+1})Y_N(0) - F(0),$$

$$C_{SP}(0) = (1-\alpha)Y_S(0) + F(0),$$

$$C_{SR}(0) = \left(\frac{\alpha}{\theta_N+1}\right) Y_N(0) + \alpha Y_S(0), \text{ and}$$

$$C_N(t) = (1 - \frac{\alpha}{\theta_N+1})Y_N(t) - F(t) \equiv C_N(\theta_N, \theta_S, F)(t),$$

$C_{SR}(t) = \left(\frac{\alpha}{\theta_N+1}\right) Y_N(t) + \alpha Y_S(t) \equiv C_{SR}(\theta_N, \theta_S)(t)$ and

$C_{SP}(t) = (1-\alpha) Y_S(t) + F(t) \equiv C_{SP}(\theta_N, \theta_S, F)(t)$.

These expressions imply that for given $Y_N(t)$ and $Y_S(t)$, θ_N has a positive effect on $C_N(t)$, a negative effect on $C_{SR}(t)$ and no effect on $C_{SP}(t)$ while θ_S has no effect on the consumption levels.

Proof of Proposition 2

From the above expressions of the consumption functions, note that $C_N(\theta_N, \theta_S, F)(t) = C_N(\theta_N, \theta_S, 0)(t) - F(t)$ and $C_{SP}(\theta_N, \theta_S, F)(t) = C_{SP}(\theta_N, \theta_S, 0)(t) + F(t)$.

a. Let us show that the governments' optimization problems reduce to the constrained maximization of date 0 utilities.

From Proposition 1, C_N, C_{SP}, C_{SR} and F grow at the same rate $g = (1-\alpha) g_A + \alpha g_R$. Then, $C_N(\theta_N, \theta_S, 0)(t) = C_N(\theta_N, \theta_S, 0)(0) e^{gt}$, $C_{SP}(\theta_N, \theta_S, 0)(t) = C_{SP}(\theta_N, \theta_S, 0)(0) e^{gt}$, $C_{SR}(\theta_N, \theta_S)(t) = C_{SR}(\theta_N, \theta_S)(0) e^{gt}$ and $F(t) = F(0) e^{gt}$.

Therefore, the North and South governments' optimization problems become respectively $\max_{\theta_N, F(0)} \left[L_N \ln\left(\frac{C_N(\theta_N,\theta_S,0)(0)-F(0)}{L_N}\right) + \delta L_S \ln\left(\frac{C_{SP}(\theta_N,\theta_S,0)(0)+F(0)}{L_S}\right)\right] \int_0^\infty e^{-\rho t} dt + [L_N + \delta L_S] \int_0^\infty gt\, e^{-\rho t} dt$, subject to $F(0) \geq 0$, and $\max_{\theta_S} \ln\left(C_{SR}(\theta_N, \theta_S)(0)\right) \int_0^\infty e^{-\rho t} dt + \int_0^\infty gt\, e^{-\rho t} dt$. Since neither the second terms of these sums nor the factor $\int_0^\infty e^{-\rho t} dt$ include the control variables, the problems are respectively equivalent to $\max_{\theta_N, F(0)} \left[L_N \ln\left(\frac{C_N(\theta_N,\theta_S,0)(0)-F(0)}{L_N}\right) + \delta L_S \ln\left(\frac{C_{SP}(\theta_N,\theta_S,0)(0)+F(0)}{L_S}\right)\right]$, subject to $F(0) \geq 0$, and $\max_{\theta_S} \ln(C_{SR}(\theta_N, \theta_S)(0))$.

b. Let us develop the expressions of $C_N(\theta_N, \theta_S, 0)(0)$, $C_{SP}(\theta_N, \theta_S, 0)(0)$ and $C_{SR}(\theta_N, \theta_S)(0)$.

We have shown in Proposition 1 that $C_N(\theta_N, \theta_S, 0)(0) = (1 - \frac{\alpha}{\theta_N+1})Y_N(0)$, $C_{SP}(\theta_N, \theta_S, 0)(0) = (1-\alpha)Y_S(0)$ and $C_{SR}(\theta_N, \theta_S)(0) = \left(\frac{\alpha}{\theta_N+1}\right)Y_N(0) + \alpha Y_S(0)$. Let us compute $Y_N(0)$ and $Y_S(0)$ as functions of θ_N and θ_S. The production functions (1) imply $Y_N(0) = (A_0 L_N)^{1-\alpha} R_N(0)^\alpha$ and $Y_S(0) = (A_0 \varphi L_N)^{1-\alpha} R_S(0)^\alpha$. Let us thus compute $R_N(0)$ and $R_S(0)$. From Proof of Proposition 1, $g_R = -\rho$. Note now that the stock of resource is asymptotically exhausted because extraction costs are nil. Then, from (3), $Q_0 = \int_0^\infty R(t)dt = \int_0^\infty R(0)e^{-\rho t}dt = R(0)/\rho$. Thus, we have $R(0) = Q_0\rho$. Moreover, the first-order conditions (8) imply $R_N/R_S = (Y_N/(1+\theta_N))/(Y_S/(1+\theta_S))$. Using (1), we then find $Y_N/Y_S = (L_N/\varphi L_S)((\theta_S+1)/(\theta_N+1))^{\alpha/(1-\alpha)}$. This, in turn, implies $R_N(0)/R_S(0) = ((\theta_S+1)/(\theta_N+1))^{1/(1-\alpha)} L_N/(\varphi L_S)$, which, using $R_N(0) + R_S(0) = R(0)$, results in

$R_N(0) = \rho Q_0/\left[1 + (\varphi L_S/L_N)((\theta_N+1)/(\theta_S+1))^{1/(1-\alpha)}\right]$ and

$R_S(0) = \rho Q_0/\left[1 + (L_N/\varphi L_S)((\theta_S+1)/(\theta_N+1))^{1/(1-\alpha)}\right]$.

Finally, we have

$Y_N(0) = (A_0 L_N)^{1-\alpha}\left[\rho Q_0/\left[1 + (\varphi L_S/L_N)((\theta_N+1)/(\theta_S+1))^{1/(1-\alpha)}\right]\right]^\alpha$ and

$Y_S(0) = (A_0 \varphi L_S)^{1-\alpha}\left[\rho Q_0/\left[1 + (L_N/\varphi L_S)((\theta_S+1)/(\theta_N+1))^{1/(1-\alpha)}\right]\right]^\alpha$,

and then

$$C_N(\theta_N, \theta_S, 0)(0) = \left(1 - \frac{\alpha}{(\theta_N+1)}\right)(A_0 L_N)^{1-\alpha}\left[\frac{\rho Q_0}{\left[1+\left(\frac{\varphi L_S}{L_N}\right)\left(\frac{\theta_N+1}{\theta_S+1}\right)^{1/(1-\alpha)}\right]}\right]^\alpha,$$

$C_{SP}(\theta_N, \theta_S, 0)(0) = (1-\alpha)(A_0 \varphi L_S)^{1-\alpha} \left[\frac{\rho Q_0}{\left[1+\left(\frac{L_N}{\varphi L_S}\right)\left(\frac{\theta_S+1}{\theta_N+1}\right)^{1/(1-\alpha)}\right]} \right]^{\alpha}$ and

$C_{SR}(\theta_N, \theta_S)(0) =$

$$\frac{\alpha}{(\theta_N+1)} (A_0 L_N)^{1-\alpha} \left[\frac{\rho Q_0}{\left[1+\left(\frac{\varphi L_S}{L_N}\right)\left(\frac{\theta_N+1}{\theta_S+1}\right)^{1/(1-\alpha)}\right]} \right]^{\alpha} + \alpha (A_0 \varphi L_S)^{1-\alpha} \left[\frac{\rho Q_0}{\left[1+\left(\frac{L_N}{\varphi L_S}\right)\left(\frac{\theta_S+1}{\theta_N+1}\right)^{1/(1-\alpha)}\right]} \right]^{\alpha}.$$

Let us finally find an expression of the resource price $p(t)$. From (1) and (8), one has $R_N = \left(\frac{\alpha}{p(\theta_N+1)}\right)^{1/(1-\alpha)} A L_N$ and $R_S = \left(\frac{\alpha}{p(\theta_S+1)}\right)^{1/(1-\alpha)} \varphi A L_S$. Using now $R_N(t) + R_S(t) = R(t) = \rho Q_0 e^{-\rho t}$, one gets $p(t) = \left[\frac{A_0 e^{(x+\rho)t}}{\rho Q_0} \left(\left(\frac{\alpha}{\theta_N+1}\right)^{1/(1-\alpha)} L_N + \left(\frac{\alpha}{\theta_S+1}\right)^{1/(1-\alpha)} \varphi L_S \right)^{1-\alpha} \right]$.

c. Let us solve the South's maximization problem.

This program amounts to the maximization of $C_{SR}(\theta_N, \theta_S)(0)$ with respect to θ_S. After some simplifications, $\partial C_{SR}(\theta_N, \theta_S)(0)/\partial \theta_S < 0$ can be shown to be equivalent to $\left(\frac{L_N}{\varphi L_S}\right)^{1-\alpha} \left[\frac{1+(\varphi L_S/L_N)((\theta_N+1)/(\theta_S+1))^{1/(1-\alpha)}}{1+(L_N/\varphi L_S)((\theta_S+1)/(\theta_N+1))^{1/(1-\alpha)}} \right]^{-\alpha-1} \frac{\varphi L_S}{L_N} \left(\frac{(\theta_S+1)}{(\theta_N+1)}\right)^{-1/(1-\alpha)} <$ $\left(\frac{L_N}{\varphi L_S}\right)\left(\frac{\theta_S+1}{\theta_N+1}\right)^{1/(1-\alpha)} (\theta_N+1)$. This condition can still be reduced to $(\theta_N+1)^{-1}((\theta_S+1)/(\theta_N+1))^{\alpha/(1-\alpha)} < ((\theta_S+1)/(\theta_N+1))^{1/1-\alpha}$, which is equivalent to $(\theta_S+1) > 1$. Thus, ${\theta_S}^e = 0$ maximizes $C_{SR}(\theta_N, \theta_S)(0)$ for all θ_N.

d. Let us solve the North's optimization problem.

Taking as given the South's dominant strategy ${\theta_S}^e = 0$, the North solves the program $\max_{\theta_N, F(0)} \left[L_N \ln\left(\frac{C_N(\theta_N,0,0)(0)-F(0)}{L_N}\right) + \delta L_S \ln\left(\frac{C_{SP}(\theta_N,0,0)(0)+F(0)}{L_S}\right) \right]$, subject to $F(0) \geq 0$. The associated Lagrangian function writes $\mathcal{L} = L_N \ln\left(\frac{C_N(\theta_N,0,0)(0)-F(0)}{L_N}\right) + \delta L_S \ln\left(\frac{C_{SP}(\theta_N,0,0)(0)+F(0)}{L_S}\right) + \pi F(0)$, where π is the multiplier associated to the constraint. The first-order conditions are the following:

(C1) $\partial\mathcal{L}/\partial\theta_N = 0,$

(C2) $\partial\mathcal{L}/\partial F(0) = 0,$

(C3) $\pi F(0) = 0,$

(C4) $F(0) \geq 0,$

(C5) $\pi \geq 0.$

- **Case 1:** $F(0) = 0.$

(C1) becomes $\frac{L_N}{C_N(\theta_N,0,0)(0)}\frac{\partial C_N(\theta_N,0,0)(0)}{\partial\theta_N} + \frac{\delta L_S}{C_{SP}(\theta_N,0,0)(0)}\frac{\partial C_{SP}(\theta_N,0,0)(0)}{\partial\theta_N} = 0.$

Let us show that this has a unique solution $\theta_N^e(\delta) > 0$.

Using the expressions of $C_N(\theta_N,0,0)(0)$ and $C_{SP}(\theta_N,0,0)(0)$ computed in b and after some simplifications, (C1) is equivalent to $Z1(\theta_N) = 0$, where

$Z1(\theta_N) \equiv \frac{(\theta_N+1)^{-1/(1-\alpha)}}{\theta_N+1-\alpha} + \frac{(\varphi L_S/L_N)}{\theta_N+1-\alpha} - \frac{(\varphi L_S/L_N)}{1-\alpha} + \left(\frac{\delta L_S}{L_N}\right)\frac{1}{1-\alpha}(\theta_N+1)^{-1/(1-\alpha)}$. For any $\theta_N \leq \alpha - 1$, $C_N(\theta_N,0,0)(0)$ is negative and the objective is not defined. Let us then look for solutions on $\theta_N > \alpha - 1$. On this set, one can check that $Z1'(\theta_N) < 0$, $Z1(0) > 0$ and $\lim_{\theta_N\to+\infty} Z1(\theta_N) < 0$. These properties imply that for all $\delta \geq 0$, there exists a unique $\theta_N^e(\delta)$ such that $Z1(\theta_N^e(\delta)) = 0$. Moreover, $\theta_N^e(\delta) > 0$. In this case, (C2) and (C5) require $\delta \leq (C_{SP}(\theta_N^e(\delta),0,\ 0)(0)/L_S)/(C_N(\theta_N^e(\delta),0,\ 0)(0)/L_N)$. As $\frac{\partial C_{SP}(\theta_N,0,0)(0)}{\partial\theta_N} > 0$ for all θ_N, (C1) implies that θ_N^e is such that $\frac{\partial C_N(\theta_N^e(\delta),0,0)(0)}{\partial\theta_N} < 0$.

Let us show that $\theta_N^e(\delta)$ is increasing in δ.

$Z1(\theta_N) = 0$ can be rearranged to give $\delta = \frac{1-\alpha}{L_S/L_N}\left[-\frac{1}{\theta_N+1-\alpha} + \frac{\varphi L_S/L_N}{1-\alpha}(\theta_N+1)^{1/(1-\alpha)} - \frac{\varphi L_S}{L_N}\frac{(\theta_N+1)^{1/(1-\alpha)}}{\theta_N+1-\alpha}\right]$. Here, it is straightforward that the first term into brackets is increasing in θ_N.

The derivative of the second and third terms is of the same sign as $\frac{(\theta_N+1)^{\alpha/(1-\alpha)}}{(\theta_N+1-\alpha)^2}\left(\left(\frac{\theta_N+1-\alpha}{1-\alpha}\right)^2 - \left(\frac{\theta_N+1-\alpha}{1-\alpha}\right)\right) + (\theta_N+1)^{1/(1-\alpha)}$, which is positive for all $\theta_N > 0$. Thus, the solution ${\theta_N}^e(\delta)$ is increasing in $\delta \geq 0$.

In particular, let us denote by θ_0 the solution for $\delta = 0$, i.e. $\theta_0 \equiv \theta_N^e(0)$. Its definition is then $\frac{\partial C_N(\theta_N,0,0)(0)}{\partial \theta_N} = 0$. Hence, $\frac{\partial C_N(\theta_N,0,0)(0)}{\partial \theta_N} < 0$ if and only if $\theta_N > \theta_0$. Note that $\theta_N^e(\delta) \geq \theta_0$ for all $\delta \geq 0$.

- **Case 2:** $F(0) > 0$.

(C3) implies $\pi = 0$ and (C2) implies $F(0) = \frac{\delta L_S C_N(\theta_N,0,0)(0) - L_N C_{SP}(\theta_N,0,0)(0)}{L_N + \delta L_S}$. Replacing $F(0)$ in (C1) and simplifying, one gets $\frac{\partial C_N(\theta_N,0,0)(0)}{\partial \theta_N} + \frac{\partial C_{SP}(\theta_N,0,0)}{\partial \theta_N} = 0$.

Let us show that this equation has a unique solution $\underline{\theta} > 0$.

Using the expressions of $C_N(\theta_N, 0, 0)(0)$ and $C_{SP}(\theta_N, 0,0)(0)$ computed in b and simplifying, (C1) becomes $Z2(\theta_N) = 0$, where

$$Z2(\theta_N) \equiv (L_N/\,\varphi L_S)(\theta_N+1)^{-(1/(1-\alpha))} + (2-\alpha)/(1-\alpha) - (\theta_N+1)/(1-\alpha).$$

One can check that $Z2'(\theta_N) < 0$ for all θ_N, $\lim_{\theta_N \to +\infty} Z2(\theta_N) < 0$ and $Z2(0) > 0$. These properties imply that there exists a unique $\underline{\theta}$ such that $Z1(\underline{\theta}) = 0$. Moreover, $\underline{\theta} > 0$.

In this case, (C4) requires $\delta \geq \left(C_{SP}(\underline{\theta}, 0, 0)(0)/L_S\right)/\left(C_N(\underline{\theta}, 0, 0)(0)/L_N\right) \equiv \underline{\delta}$.

- Using Cases 1 and 2, let us check that the North's problem has a unique solution for all δ.

First, note that when $\delta = (C_{SP}(\theta_N^e(\delta), 0, 0)(0)/L_S)/(C_N(\theta_N^e(\delta), 0, 0)(0)/L_N)$, then $\theta_N^e(\delta) = \underline{\theta}$. Thus, as $\theta_N^e(\delta)$ is increasing in δ, $\delta > \frac{\left(\frac{C_{SP}(\underline{\theta},0,0)}{L_S}\right)}{\left(\frac{C_N(\underline{\theta},0,0)}{L_N}\right)} = \underline{\delta}$ is equivalent to $\theta_N^e(\delta) > \underline{\theta}$. Second, after some computations, one can show that $\frac{\frac{\partial C_N(\theta_N,0,0)(0)}{\partial\theta_N}}{\frac{\partial C_{SP}(\theta_N,0,0)(0)}{\partial\theta_N}} = 1 + \frac{L_N}{\varphi L_S}(\theta_N + 1)^{-1/(1-\alpha)} - \frac{\theta_N+1-\alpha}{1-\alpha}$,

which is decreasing in θ_N. Hence, $-\frac{\frac{\partial C_{SP}(\theta_N,0,0)(0)}{\partial\theta_N}}{\frac{\partial C_N(\theta_N,0,0)(0)}{\partial\theta_N}}$ is also decreasing in θ_N. Since $-\frac{\frac{\partial C_{SP}(\underline{\theta},0,0)(0)}{\partial\theta_N}}{\frac{\partial C_N(\underline{\theta},0,0)(0)}{\partial\theta_N}} = 1$, by definition of $\underline{\theta}$, then $-\frac{\frac{\partial C_{SP}(\theta_N,0,0)(0)}{\partial\theta_N}}{\frac{\partial C_N(\theta_N,0,0)(0)}{\partial\theta_N}} < 1$ if and only if $\theta_N > \underline{\theta}$.

- Assume $\delta > \frac{\left(\frac{C_{SP}(\underline{\theta},0,0)}{L_S}\right)}{\left(\frac{C_N(\underline{\theta},0,0)}{L_N}\right)} = \underline{\delta}$. Then, $\theta_N^e = \underline{\theta}$ and $F^e(0) = \frac{\delta L_S C_N(\underline{\theta},0,0)(0) - L_N C_{SP}(\underline{\theta},0,0)(0)}{L_N+\delta L_S}$ is a solution (from Case 2). Let us now show that $\theta_N^e = \theta_N^e(\delta)$ and $F^e(0) = 0$ is not a solution (i.e. we cannot be under Case 1).

First, since $\theta_N^e(\delta)$ is increasing in δ, in this case, $\theta_N^e(\delta) > \underline{\theta}$. Second, by definition of $\theta_N^e(\delta)$ above, $\frac{\left(\frac{C_{SP}(\theta_N^e(\delta),0,0)}{L_S}\right)}{\left(\frac{C_N(\theta_N^e(\delta),0,0)}{L_N}\right)} = -\delta\frac{\frac{\partial C_{SP}(\theta_N^e(\delta),0,0)(0)}{\partial\theta_N}}{\left(\frac{\partial C_N(\theta_N^e(\delta),0,0)(0)}{\partial\theta_N}\right)}$, which is lower than δ because $\theta_N^e(\delta) > \underline{\theta}$. Finally, $\delta > (C_{SP}(\theta_N^e(\delta),0,0)/L_S)/(C_N(\theta_N^e(\delta),0,0)/L_N)$, i.e. we cannot be in Case 1.

- Assume $\delta < \frac{\left(\frac{C_{SP}(\underline{\theta},0,0)}{L_S}\right)}{\left(\frac{C_N(\underline{\theta},0,0)}{L_N}\right)} = \underline{\delta}$. Then $\theta_N^e = \underline{\theta}$ and $F^e(0) = \frac{\delta L_S C_N(\underline{\theta},0,0)(0) - L_N C_{SP}(\underline{\theta},0,0)(0)}{L_N+\delta L_S}$ is not a solution (because we cannot be in Case 2). Let us show that $\theta_N^e = \theta_N^e(\delta)$ and $F^e(0) = 0$ is a solution (i.e. we are in Case 1).

First, in this case, $\theta_N^e < \underline{\theta}$. Second, $\frac{\left(\frac{C_{SP}(\theta_N^e(\delta),0,0)}{L_S}\right)}{\left(\frac{C_N(\theta_N^e(\delta),0,0)}{L_N}\right)} = -\delta\frac{\frac{\partial C_{SP}(\theta_N^e(\delta),0,0)(0)}{\partial\theta_N}}{\left(\frac{\partial C_N(\theta_N^e(\delta),0,0)(0)}{\partial\theta_N}\right)}$ is greater than δ. Finally, $\delta < (C_{SP}(\theta_N^e(\delta),0,0)/L_S)/(C_N(\theta_N^e(\delta),0,0)/L_N)$, i.e. we are in Case 1.

- To sum up, the solution to the North's problem is $(\theta_N^e, F^e(0))$ such that

$\theta_N^e = \theta_N^e(\delta)$ and $F^e(0) = 0$, if $\delta \leq \frac{\left(\frac{C_{SP}(\underline{\theta},0,0)}{L_S}\right)}{\left(\frac{C_N(\underline{\theta},0,0)}{L_N}\right)} = \underline{\delta}$, where $\theta_N^e(\delta)$ is increasing from $\theta_0 = \theta_N^e(0)$ to $\underline{\theta} = \theta_N^e(\underline{\delta})$, and

$$\theta_N^e = \underline{\theta} \text{ and } F^e(0) = \frac{\delta L_S C_N(\underline{\theta},0,0)(0) - L_N C_{SP}(\underline{\theta},0,0)(0)}{L_N + \delta L_S} \geq 0, \text{ if } \delta \geq \frac{\left(\frac{C_{SP}(\underline{\theta},0,0)}{L_S}\right)}{\left(\frac{C_N(\underline{\theta},0,0)}{L_N}\right)} = \underline{\delta}.$$

Proof of Proposition 3

Whether the contract is accepted or not, the South's problem remains unchanged. In particular, if the contract is accepted, $I(t)$ is taken as given and the South seeks to maximize its consumption. Thus, $\theta_S^c = \theta_S^e = 0$.

The same way as in Proposition 2, one can check that the North's problem reduces to the maximization of date 0 utility:

$$\max_{\theta_N, I(0), F(0)} L_N \ln\left(\frac{C_N(\theta_N,\theta_S,0)(0) - F(0)}{L_N}\right) + \delta L_S \ln\left(\frac{C_{SP}(\theta_N,\theta_S,0)(0) + F(0) + I(0)}{L_S}\right),$$ subject to $F(0) \geq 0$, $I(0) \geq 0$ and $C_{SR}(\theta_N^e, 0)(0) \leq C_{SR}(\theta_N, 0)(0) - I(0)$. Since the North's objective function is increasing with $I(0)$, the South's participation constraint is binding: $I(0) = C_{SR}(\theta_N, 0)(0) - C_{SR}(\theta_N^e, 0)(0)$. Here, $C_{SR}(\theta_N, 0)$ can be shown to decrease with θ_N: From Proof of Proposition 2 b, one can see that $C_{SR}(\theta_N, 0)(0) = p(0)R(0) = \frac{\alpha}{\theta_N + 1} Y_N(0) + \alpha Y_S(0)$, where $p(0)R(0)$ appears to be decreasing in θ_N. Hence, $I(0) \geq 0$ is equivalent to $\theta_N \leq \theta_N^e$. Taking as given the South's dominant strategy $\theta_S^c = 0$, the North's problem becomes

$$\max_{\theta_N, F(0)} L_N \ln\left(\frac{C_N(\theta_N,0,0)(0) - F(0)}{L_N}\right) + \delta L_S \ln\left(\frac{C_{SP}(\theta_N,0,0)(0) + F(0) + C_{SR}(\theta_N,0) - C_{SR}(\theta_N^e,0)}{L_S}\right),$$ subject to $F(0) \geq 0$ and $\theta_N \leq \theta_N^e$. The associated Lagrangian function writes $\mathcal{L} = L_N \ln\left(\frac{C_N(\theta_N,0,0)(0) - F(0)}{L_N}\right) + \delta L_S \ln\left(\frac{C_{SP}(\theta_N,0,0)(0) + F(0) + C_{SR}(\theta_N,0) - C_{SR}(\theta_N^e,0)}{L_S}\right) + \mu F(0) +$

$\vartheta({\theta_N}^e - \theta_N)$, where μ and ϑ are respectively the multipliers associated to the positivity constraint on $F(0)$ and to the constraint on the tax rate. The first-order conditions are the following:

(C6) $\partial\mathcal{L}/\partial\theta_N = 0,$

(C7) $\partial\mathcal{L}/\partial F(0) = 0,$

(C8) $\mu F(0) = 0,$

(C9) $F(0) \geq 0,$

(C10) $\mu \geq 0,$

(C11) $\vartheta(\theta_N^e - \theta_N) = 0,$

(C12) $\vartheta \geq 0,$

(C13) $\theta_N \leq \theta_N^e.$

- **Case 1:** $\theta_N = \theta_N^e, F(0) = 0.$

Then, (C6) is equivalent to

$$\frac{L_N}{C_N(\theta_N,0,0)(0)}\frac{\partial C_N(\theta_N,0,0)(0)}{\partial\theta_N} + \frac{\delta L_S}{C_{SP}(\theta_N,0,0)(0)}\left[\frac{\partial C_{SP}(\theta_N,0,0)(0)}{\partial\theta_N} + \frac{\partial C_{SR}(\theta_N,0)(0)}{\partial\theta_N}\right]\Bigg|_{\theta_N=\theta_N^e} = \vartheta \geq 0.$$

Using the expressions of the consumption levels and after some computations, one can check that $\frac{-\frac{\partial C_{SR}(\theta_N,0)(0)}{\partial\theta_N}}{\frac{\partial C_{SP}(\theta_N,0,0)(0)}{\partial\theta_N}} = 1 + \frac{L_N}{\varphi L_S}(\theta_N+1)^{-1/(1-\alpha)}$. This implies that $\frac{-\frac{\partial C_{SR}(\theta_N,0)(0)}{\partial\theta_N}}{\frac{\partial C_{SP}(\theta_N,0,0)(0)}{\partial\theta_N}} > 1$ for all θ_N, which is equivalent to $\frac{\partial C_{SP}(\theta_N,0,0)(0)}{\partial\theta_N} + \frac{\partial C_{SR}(\theta_N,0)(0)}{\partial\theta_N} < 0$, since $\frac{\partial C_{SP}(\theta_N,0,0)(0)}{\partial\theta_N} = (1-\alpha)\frac{\partial Y_S(0)}{\partial\theta_N} > 0$. Hence, from the expression of (C6) above, θ_N^e is such that $\frac{\partial C_N(\theta_N,0,0)(0)}{\partial\theta_N}\Big|_{\theta_N=\theta_N^e} > 0$ which is contradicted by $\frac{\partial C_N(\theta_N,0,0)(0)}{\partial\theta_N}\Big|_{\theta_N=\theta_N^e} < 0$ from Proof of Proposition 2. Thus, $\theta_N = \theta_N^e$, $F(0) = 0$ cannot be a solution.

- **Case 2:** $\theta_N = \theta_N^e$, $F(0) > 0$.

  Then, (C6) is equivalent to

$$\frac{L_N}{C_N(\theta_N,0,0)(0) - F(0)} \frac{\partial C_N(\theta_N,0,0)(0)}{\partial \theta_N} + \frac{\delta L_S}{C_{SP}(\theta_N,0,0)(0) + F(0)} \left[\frac{\partial C_{SP}(\theta_N,0,0)(0)}{\partial \theta_N} + \frac{\partial C_{SR}(\theta_N,0)(0)}{\partial \theta_N}\right]\Bigg|_{\theta_N = \theta_N^e} = \vartheta.$$

For the same reason as in Case 1, this cannot be a solution.

- **Case 3:** $\theta_N < \theta_N^e$, $F(0) = 0$.

  Then, (C6) is equivalent to

$$\frac{L_N}{C_N(\theta_N,0,0)(0)} \frac{\partial C_N(\theta_N,0,0)(0)}{\partial \theta_N} + \frac{\delta L_S}{C_{SP}(\theta_N,0,0)(0) + C_{SR}(\theta_N,0) - C_{SR}(\theta_N^e,0)} \left[\frac{\partial C_{SP}(\theta_N,0,0)(0)}{\partial \theta_N} + \frac{\partial C_{SR}(\theta_N,0)(0)}{\partial \theta_N}\right] =$$

0. Let us denote any solution to this equation by $\theta_N^c(\delta)$. As $\frac{\partial C_{SP}(\theta_N,0,0)(0)}{\partial \theta_N} + \frac{\partial C_{SR}(\theta_N,0)(0)}{\partial \theta_N} < 0$ from above, $\theta_N^c(\delta)$ is such that $\frac{\partial C_N(\theta_N^c(\delta),0,0)(0)}{\partial \theta_N} > 0$. Note that, from Proof of Proposition 2, $\frac{\partial C_N(\theta_N,0,0)(0)}{\partial \theta_N} > 0$ is equivalent to $\theta_N < \theta_0$, thus implying that any solution $\theta_N^c(\delta)$ is such that $\theta_N^c(\delta) < \theta_0$ for all $\delta > 0$. One can check that $\theta_N^c(0) = \theta_0$.

  Moreover, (C7) implies $\mu = \frac{L_N}{C_N(\theta_N^c(\delta),0,0)(0)} - \frac{\delta L_S}{C_{SP}(\theta_N^c(\delta),0,0)(0) + C_{SR}(\theta_N^c(\delta),0)(0) - C_{SR}(\theta_N^e,0)(0)}$.

Hence, (C10) requires $\delta \leq \frac{\left(C_{SP}(\theta_N^c(\delta),0,0)(0) + C_{SR}(\theta_N^c(\delta),0)(0) - C_{SR}(\theta_N^e,0)(0)\right)/L_S}{C_N(\theta_N^c(\delta),0,0)(0)/L_N}$.

- **Case 4:** $\theta_N < \theta_N^e$, $F(0) > 0$.

  Then, (C7) implies $F(0) = \frac{\delta L_S C_N(\theta_N,0,0)(0) - L_N[C_{SP}(\theta_N,0,0)(0) + C_{SR}(\theta_N,0)(0) - C_{SR}(\theta_N^e,0)(0)]}{L_N + \delta L_S}$.

Replacing $F(0)$ and simplifying, (C6) becomes $\frac{\partial C_N(\theta_N,0,0)(0)}{\partial \theta_N} + \frac{\partial C_{SP}(\theta_N,0,0)(0)}{\partial \theta_N} + \frac{\partial C_{SR}(\theta_N,0)(0)}{\partial \theta_N} = \frac{\partial Y_N(0)}{\partial \theta_N} + \frac{\partial Y_S(0)}{\partial \theta_N} = \frac{\partial Y(0)}{\partial \theta_N} = 0$. One can check that the unique solution to this equation is $\theta_N = 0$. Moreover $F(0) > 0$ requires $\delta > \frac{\left(C_{SP}(0,0,0)(0) + C_{SR}(0,0)(0) - C_{SR}(\theta_N^e,0)(0)\right)/L_S}{C_N(0,0,0)(0)/L_N} \equiv \underline{\underline{\delta}}$.

- Let us now review the solutions to the North's problem for all δ.

First, one can check that for $\delta = \frac{\left(C_{SP}(\theta_N^c(\delta),0,0)(0)+C_{SR}(\theta_N^c(\delta),0)(0)-C_{SR}(\theta_N^e,0)(0)\right)/L_S}{C_N(\theta_N^c(\delta),0,0)(0)/L_N}$, $\theta_N^c(\delta) = 0$.

Second, as $\theta_N \geq 0$ is equivalent to $\frac{\partial Y(0)}{\partial \theta_N} \leq 0$, it is also equivalent to $-\left[\frac{\partial C_{SP}(\theta_N,0,0)(0)}{\partial \theta_N} + \frac{\partial C_{SR}(\theta_N,0)(0)}{\partial \theta_N}\right] \geq \frac{\partial C_N(\theta_N,0,0)(0)}{\partial \theta_N}$.

- Assume $\delta > \underline{\underline{\delta}}$.

Then, $\theta_N^c = 0$ and $F^c(0) = \frac{\delta L_S C_N(0,0,0)(0) - L_N[C_{SP}(0,0,0)(0)+C_{SR}(0,0)(0)-C_{SR}(\theta_N^e,0)(0)]}{L_N+\delta L_S}$ is a solution. Let us show that $\theta_N^c = \theta_N^c(\delta)$ and $F^c(0) = 0$ cannot be solution. From Case 3, any $\theta_N^c(\delta)$ must satisfy $\delta \leq \frac{\left(C_{SP}(\theta_N^c(\delta),0,0)(0)+C_{SR}(\theta_N^c(\delta),0)(0)-C_{SR}(\theta_N^e,0)(0)\right)/L_S}{C_N(\theta_N^c(\delta),0,0)(0)/L_N} \equiv \delta(\theta_N^c(\delta))$, where $\delta(\theta_N^c(\delta))$ is strictly decreasing in $\theta_N^c(\delta)$ since $\theta_N^c(\delta)$ is such that $\frac{\partial C_N(\theta_N^c(\delta),0,0)(0)}{\partial \theta_N} > 0$ and $\frac{\partial C_{SP}(\theta_N,0,0)(0)}{\partial \theta_N} + \frac{\partial C_{SR}(\theta_N,0)(0)}{\partial \theta_N} < 0$ for all θ_N. Moreover, $\delta(0) = \underline{\underline{\delta}}$. Hence, if $\delta(\theta_N^c(\delta)) \geq \delta > \underline{\underline{\delta}}$, then any $\theta_N^c(\delta)$ is strictly negative. Then, by definition of $\theta_N^c(\delta)$, $\delta(\theta_N^c(\delta)) = \frac{\left(C_{SP}(\theta_N^c(\delta),0,0)(0)+C_{SR}(\theta_N^c(\delta),0)(0)-C_{SR}(\theta_N^c(\delta),0)(0)\right)/L_S}{C_N(\theta_N^c(\delta),0,0)(0)/L_N} = -\frac{\delta\left[\frac{\partial C_{SP}(\theta_N^c(\delta),0,0)(0)}{\partial \theta_N}+\frac{\partial C_{SR}(\theta_N^c(\delta),0)(0)}{\partial \theta_N}\right]}{\frac{\partial C_N(\theta_N^c(\delta),0,0)(0)}{\partial \theta_N}}$, which is strictly lower than δ since $\theta_N^c(\delta) < 0$ is equivalent to $-\left[\frac{\partial C_{SP}(\theta_N,0,0)(0)}{\partial \theta_N} + \frac{\partial C_{SR}(\theta_N,0)(0)}{\partial \theta_N}\right] < \frac{\partial C_N(\theta_N,0,0)(0)}{\partial \theta_N}$. Thus, $\delta(\theta_N^c(\delta)) = \frac{\left(C_{SP}(\theta_N^c(\delta)\,,0,0)(0)+C_{SR}(\theta_N^c(\delta)\,,0)(0)-C_{SR}(\theta_N^c(\delta)\,,0)(0)\right)/L_S}{C_N(\theta_N^c(\delta)\,,0,0)(0)/L_N} < \delta$, i.e. we cannot be in Case 3.

- Assume $\delta < \underline{\underline{\delta}}$.

Then $\theta_N^c = 0$ and $F^c(0) = \frac{\delta L_S C_N(0,0,0)(0) - L_N[C_{SP}(0,0,0)(0)+C_{SR}(0,0)(0)-C_{SR}(\theta_N^e,0)(0)]}{L_N+\delta L_S}$ is not a solution. Let us show that any $\theta_N^c(\delta)$ together with $F^c(0) = 0$ is solution. From above, $\theta_N^c(\delta)$

cannot be negative. Then, $\theta_N^c(\delta) \geq 0$, thus implying

$$\delta(\theta_N^c(\delta)) = \frac{\left(C_{SP}(\theta_N^c(\delta),0,0)(0) + C_{SR}(\theta_N^c(\delta),0)(0) - C_{SR}(\theta_N^c(\delta),0)(0)\right)/L_S}{C_N(\theta_N^c(\delta),0,0)(0)/L_N} =$$

$$\frac{\delta\left[\frac{\partial C_{SP}(\theta_N^c(\delta),0,0)(0)}{\partial \theta_N} + \frac{\partial C_{SR}(\theta_N^c(\delta),0)(0)}{\partial \theta_N}\right]}{\frac{\partial C_N(\theta_N^c(\delta),0,0)(0)}{\partial \theta_N}} \geq \delta,$$ which is consistent with Case 3.

- Since we are restricting our attention to $\delta \geq \underline{\delta}$, $\theta_N^e = \underline{\theta}$ from Proposition 2. Hence,

$$\underline{\underline{\delta}} \equiv \frac{\left(C_{SP}(0,0,0)(0) + C_{SR}(0,0)(0) - C_{SR}(\underline{\theta},0)(0)\right)/L_S}{C_N(0,0,0)(0)/L_N}.$$

Then, to sum up, the solution to the North's problem is

$I^c(0) = C_{SR}(\theta_N^c, 0)(0) - C_{SR}\left(\underline{\theta}, 0\right)(0)$ and

$\theta_N^c = 0$, $F^c(0) = \frac{\delta L_S C_N(0,0,0)(0) - L_N\left[C_{SP}(0,0,0)(0) + C_{SR}(0,0)(0) - C_{SR}(\underline{\theta},0)(0)\right]}{L_N + \delta L_S}$, if $\delta \geq \underline{\underline{\delta}}$, and

any $\theta_N^c(\delta)$, such as defined above, and $F^c(0) = 0$, if $\delta < \underline{\underline{\delta}}$.

In this latter case, one can hardly tell something precise about $\theta_N^c(\delta)$. However, we know that $\theta_N^c(0) = \theta_0$, $\theta_N^c(\underline{\underline{\delta}}) = 0$ and that any solution $\theta_N^c(\delta)$ for all δ in $(0, \underline{\underline{\delta}})$ is such that $0 < \theta_N^c(\delta) < \theta_0$.

In this case, the objective function is continuous in $\delta \geq 0$ and in $\theta_N > -1$, except at point $\theta_N = \alpha - 1$. Moreover, the objective is not maximized for $\theta_N \leq \alpha - 1$ and as θ_N tends to $\alpha - 1$ or to $+\infty$, because, then, the objective would tend to $-\infty$. Finally, it is bounded from above because $C_N(\theta_N, 0,0)(0) + C_{SP}(\theta_N, \theta_S, 0)(0) + I(0)$ is lower than $Y(0)$, finite. Thus, for any $\delta \geq 0$, the existence of a global maximum is ensured. Hence, for any $\delta \in \left[0, \underline{\delta}\right]$, there exists at least one $\theta_N^c(\delta) \in [0, \theta_0]$.

Proof of Proposition 4

The utility of northern households is obviously increased with an additional instrument. The southern rich are indifferent as their participation constraint is binding. We thus only have to show that the southern poor are better-off when the contract is used by the North. We will consider two cases: $\underline{\delta} \leq \underline{\underline{\delta}}$ and $\underline{\delta} > \underline{\underline{\delta}}$.

- **Case 1:** $\underline{\delta} \leq \underline{\underline{\delta}}$.

  - If $\delta \leq \underline{\delta}$, aid is nil whether the contract is used or not. Let us then show $C_{SP}(\theta_N^c(\delta),0,0) + I^c \geq C_{SP}(\theta_N^e, 0,0)$. Replacing I^c, this is equivalent to $C_{SP}(\theta_N^c(\delta),0,0) + C_{SR}(\theta_N^c(\delta),0) \geq C_{SR}(\theta_N^e, 0) + C_{SP}(\theta_N^e, 0,0)$, which is satisfied because $C_{SP}(\theta_N, 0,0) + C_{SR}(\theta_N, 0)$ is decreasing in θ_N and $\theta_N^c(\delta) \leq \theta_N^e$.

If $\underline{\delta} < \delta < \underline{\underline{\delta}}$, aid is nil under the contract and is positive without it. Let us then show $C_{SP}(\theta_N^c(\delta),0,0) + I^c \geq C_{SP}\big(\underline{\theta}, 0,0\big) + F^e$. This is equivalent to $C_{SP}(\theta_N^c(\delta),0,0) + C_{SR}(\theta_N^c(\delta),0) - C_{SR}\big(\underline{\theta}, 0\big) \geq C_{SP}\big(\underline{\theta}, 0,0\big) + \frac{\delta L_S C_N(\underline{\theta},0,0) - L_N C_{SP}(\underline{\theta},0,0)}{L_N + \delta L_S}$ which can be rearranged to give the necessary and sufficient condition $\frac{L_N}{\delta L_S}\big[C_{SP}(\theta_N^c(\delta), 0,0) + C_{SR}(\theta_N^c(\delta), 0) - C_{SR}\big(\underline{\theta}, 0\big)\big] \geq C_{SR}\big(\underline{\theta}, 0\big) + C_{SP}\big(\underline{\theta}, 0,0\big) + C_N\big(\underline{\theta}, 0,0\big) - C_{SP}(\theta_N^c(\delta), 0,0) - C_{SR}(\theta_N^c(\delta), 0)$. Here, left-hand side is positive. Then, if right-hand side is negative, the condition is satisfied. Let us assume this term is positive to check the condition is also satisfied in this case. Then, the inequality is equivalent to $\delta \leq \frac{\left(C_{SP}(\theta_N^c(\delta),0,0) + C_{SR}(\theta_N^c(\delta),0) - C_{SR}(\underline{\theta},0)\right)/L_S}{\left[C_{SR}(\underline{\theta},0) + C_{SP}(\underline{\theta},0,0) + C_N(\underline{\theta},0,0) - C_{SP}(\theta_N^c(\delta),0,0) - C_{SR}(\theta_N^c(\delta),0)\right]/L_N}$. We have shown that the case where aid is nil under the contract requires (from Case 3 of Proof of Proposition 3) $\delta \leq \frac{\left(C_{SP}(\theta_N^c(\delta),0,0) + C_{SR}(\theta_N^c(\delta),0) - C_{SR}(\underline{\theta},0)\right)/L_S}{C_N(\theta_N^c(\delta),0,0)/L_N}$. It is thus sufficient to have $C_N(\theta_N^c(\delta), 0,0) \geq C_{SR}\big(\underline{\theta}, 0\big) +$

$C_{SP}(\underline{\theta}, 0{,}0) + C_N(\underline{\theta}, 0{,}0) - C_{SP}(\theta_N^c(\delta), 0{,}0) - C_{SR}(\theta_N^c(\delta), 0)$. This is satisfied because $C_N(\theta_N^c(\delta), 0{,}0) + C_{SR}(\theta_N^c(\delta), 0) + C_{SP}(\theta_N^c(\delta), 0{,}0) \geq C_N(\underline{\theta}, 0{,}0) + C_{SR}(\underline{\theta}, 0) + C_{SP}(\underline{\theta}, 0{,}0)$, where both sides equal total output, decreasing in θ_N for all $\theta_N \geq 0$, and where $\theta_N^c(\delta) < \underline{\theta}$.

- If $\delta \geq \underline{\underline{\delta}}$, aid is positive with or without the contract. Let us show $C_{SP}(0{,}0{,}0) + F^c + I^c \geq C_{SP}(\underline{\theta}, 0{,}0) + F^e$. Developing F^c, I^c and F^e, one gets the equivalent condition $C_N(0{,}0{,}0) + C_{SP}(0{,}0{,}0) + C_{SR}(0{,}0) \geq C_N(\underline{\theta}, 0{,}0) + C_{SP}(\underline{\theta}, 0{,}0) + C_{SR}(\underline{\theta}, 0)$, which is satisfied since total output is maximized for $\theta_N = 0$.

- **Case 2:** $\underline{\delta} > \underline{\underline{\delta}}$.
  - If $\delta \leq \underline{\underline{\delta}}$, aid is nil with and without the contract. The proof is the same as in Case 1 ($\delta \leq \underline{\delta}$).
  - If $\delta \geq \underline{\delta}$, aid is positive with and without the contract. The proof is the same as in Case 1 ($\delta \geq \underline{\underline{\delta}}$).

If $\underline{\delta} > \delta > \underline{\underline{\delta}}$, aid is nil without the contract and positive under it. Let us then show $C_{SP}(0, 0{,}0) + F^c + I^c \geq C_{SP}(\theta_N^e, 0{,}0)$. This is equivalent to $C_{SP}(0{,}0{,}0) + C_{SR}(0, 0) - C_{SR}(\theta_N^e, 0) + F^c \geq C_{SP}(\theta_N^e, 0{,}0)$. Since $F^c > 0$, a sufficient condition is $C_{SP}(0{,}0{,}0) + C_{SR}(0, 0) \geq C_{SP}(\theta_N^e, 0{,}0) + C_{SR}(\theta_N^e, 0)$, which is satisfied because $C_{SP}(\theta_N, 0{,}0) + C_{SR}(\theta_N, 0)$ is decreasing in θ_N and $\theta_N^e > 0$.