Online Appendix to: Environmental context and termination uncertainty in games with a dynamic public bad

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A Proofs of propositions

We note that some of the proofs below can be simplified using the techniques of dynamic programming. We use a more direct alternative method for consistency across all proofs.

A.1 Proof of Proposition 1

Let $X_t = \sum_j x_{jt}$ denote the total amount of emissions in period t. Further, let $X_{-i,t} = X_t - x_{it}$ denote the total amount of period t emissions of all players except player i. From Eq. (1), pollution level in period t can be written as

$$Y_t = \sum_{k=1}^{t} \gamma^{t-k} X_k.$$

The payoff of player i in period t is, then,

$$\pi_{it} = m + (a-1)x_{it} - b\gamma \sum_{k=1}^{t-1} \gamma^{t-1-k} X_k. \tag{A.1}$$

The payoff of player i accumulated by the end of period T is, therefore,

$$\Pi_{iT} = \sum_{\tau=1}^{T} \pi_{i\tau} = mT + (a-1)\sum_{\tau=1}^{T} x_{i\tau} - b\gamma \sum_{\tau=1}^{T}\sum_{k=1}^{\tau-1} \gamma^{\tau-1-k} X_k.$$

Note that

$$\sum_{\tau=1}^{T}\sum_{k=1}^{\tau-1} \gamma^{\tau-1-k} X_k = \sum_{k=1}^{T-1}\sum_{\tau=k+1}^{T} \gamma^{\tau-1-k} X_k = \sum_{\tau=1}^{T-1}\sum_{k=\tau+1}^{T} \gamma^{k-1-\tau} X_\tau = \sum_{\tau=1}^{T-1} \frac{1-\gamma^{T-\tau}}{1-\gamma} X_\tau,$$

which gives for the cumulative payoff

$$\Pi_{iT} = mt + \sum_{\tau=1}^{T} \left(a - 1 - b\gamma \frac{1-\gamma^{T-\tau}}{1-\gamma} \right) x_{i\tau} - b\gamma \sum_{\tau=1}^{T-1} \frac{1-\gamma^{T-\tau}}{1-\gamma} X_{-i,\tau}. \tag{A.2}$$

As seen from Eq. (A.2), the payoff of player i is an additively separable linear function of her own and other players' production inputs. Thus, all terms but the second in Eq. (A.2) can be ignored in the calculation of player i's best response. Moreover, only the terms with $\tau \geq t$ are relevant for decision making in period t. Therefore, player i's objective function for the best response calculation in period t is

$$V_{it} = \sum_{\tau=t}^{T} \left(a - 1 - b\gamma \frac{1-\gamma^{T-\tau}}{1-\gamma} \right) x_{i\tau}. \tag{A.3}$$

Using backward induction, start with period $t = T$ when the objective function is $V_{iT} = (a-1)x_{iT}$. For $a > 1$, the best response is $x^*_{iT} = m$. In period $t = T-1$, the objective function is $V_{i,T-1} = (a - 1 - b\gamma)x_{i,T-1} + (a-1)m$. There are two possibilities: (i) $a \geq 1 + b\gamma$, in which case $x^*_{i,T-1} = m$;[1] (ii) $a < 1 + b\gamma$, in which case $x^*_{i,T-1} = 0$.

Continuing backward induction, in period $t = T-2$ the objective function is

$$V_{i,T-2} = \left(a - 1 - b\gamma \frac{1-\gamma^2}{1-\gamma} \right) x_{i,T-2} + (a - 1 - b\gamma)x^*_{i,T-1} + (a-1)m.$$

Again, there are two possibilities: (i) $a \geq 1 + b\gamma(1-\gamma^2)/(1-\gamma)$, in which case $x^*_{i,T-1} = m$; (ii) $a < 1 + b\gamma(1-\gamma^2)/(1-\gamma)$, in which case $x^*_{i,T-1} = 0$.

Continuing similarly with backward induction, for $t = T - s$ we obtain $x^*_{i,T-s} = m$ for $a \geq 1 + b\gamma(1-\gamma^s)/(1-\gamma)$ and $x^*_{i,T-s} = 0$ otherwise. We conclude that in the subgame-perfect Nash equilibrium $x^*_{it} = m$ if $a \geq 1 + b\gamma(1-\gamma^{T-t})/(1-\gamma)$ and $x^*_{it} = 0$ otherwise.

Clearly, if $x^*_{it} = m$ for some t then $x^*_{is} = m$ for all $s > t$. Thus, there is a switching point, t^N_c, such that $x^*_{it} = 0$ for $t < t^N_c$ and $x^*_{it} = m$ for $t \geq t^N_c$. The switching period t^N_c is the lowest positive integer such that $t^N_c \geq \bar{t}_N$, where $\bar{t}_N$ is the real solution to the equation $a = 1 + b\gamma(1-\gamma^{T-t})/(1-\gamma)$, if it exists. For $a \geq 1 + b\gamma/(1-\gamma)$, the equation

[1] We assume, for concreteness, that if a player is indifferent among multiple production options, she chooses the maximal possible production input. Such constellations of parameters are of measure zero in the parameter space.

has no real solutions and part (i) of the proposition follows. For $a < 1 + b\gamma/(1-\gamma)$, the equation yields Eq. (2) and part (ii) of the proposition.

A.2 Proof of Proposition 2

To find the symmetric socially optimal (SO) outcome, we maximize total payoff of all players and assume symmetry. Let x_t^S denote the symmetric SO production input in period t. Then, using Eq. (A.2), the per capita cumulative payoff at the end of period T is

$$\bar{\Pi}_T = mT + \sum_{t=1}^{T} \left(a - 1 - b\gamma n \frac{1-\gamma^{T-t}}{1-\gamma} \right) x_t^S.$$

This is a linear function of x_t^S, therefore (ignoring the special cases with multiple solutions) the optimum is reached at one of the corners of the T-dimensional cube $S = \{(y_1, \ldots, y_T) \in \mathcal{R}_+^T : y_t \leq m\}$. Moreover, it is clear that $x_t^S = m$ whenever the coefficient at it is positive, and 0 whenever it is negative. Note that the coefficient increases monotonically in t, therefore, similarly to the symmetric SPNE, the SO profile of production inputs has the form $(0, \ldots, 0, m \ldots, m)$. The switching point is determined by the real solution of the equation $a = 1 + b\gamma n(1-\gamma^{T-t})/(1-\gamma)$, if it exists. For $a \geq 1 + nb\gamma/(1-\gamma)$, there is no real solution and switching occurs immediately, implying part (i) of the proposition. For $a < 1 + nb\gamma/(1-\gamma)$, the solution is given by Eq. (3) and thus follows part (ii) of the proposition.

A.3 Proof of Proposition 3

The expected payoff of player i in period t, conditional on period t occurring, is

$$\Pi_{it} = \tilde{\Pi}_{i,t-1} + \sum_{\tau=t}^{\infty} \beta^{\tau-t} \pi_{i\tau}.$$

Here, $\tilde{\Pi}_{i,t-1}$ is the payoff player i has accumulated by the end of period $t-1$; the second term is the expected payoff from current and future actions of player i and other players.

The latter term can be re-written using Eq. (A.1) as

$$\sum_{\tau=t}^{\infty}\beta^{\tau-t}\left[m+(a-1)x_{i\tau}-b\gamma\sum_{l=1}^{\tau-1}\gamma^{\tau-1-l}(x_{il}+X_{-i,l})-b\gamma\sum_{l=t}^{\tau-1}\gamma^{\tau-1-l}(x_{il}+X_{-i,l})\right]. \tag{A.4}$$

Here, we separated the sum containing emissions into the parts related to the periods before and after period t. We also separated the emissions created by player i from the emissions created by other players.

As seen from Eq. (A.4), the objective function of player i is additive separable in her and other players' decisions and across time. Therefore, to find the best response of player i in period t, only the term containing x_{it} should be considered. From Eq. (A.4), this term is

$$\left(a-1-b\gamma\sum_{\tau=t+1}^{\infty}\beta^{\tau-t}\gamma^{\tau-1-t}\right)x_{it}=\left(a-1-\frac{b\beta\gamma}{1-\beta\gamma}\right)x_{it}.$$

When the coefficient at x_{it} is nonnegative (negative), player i's best response is $x_{it}=m$ ($x_{it}=0$), which gives parts (i) and (ii) of the proposition.

A.4 Proof of Proposition 4

To find the socially optimal profile of production inputs, we maximize the expected sum of all players' payoffs. In the symmetric case under consideration, this is equivalent to setting $x_{it}=x_t^S$ for all i in Eq. (A.4). and maximizing the resulting value function over the stream of inputs x_s^S, $s\geq t$. Equation (A.4) becomes

$$\sum_{\tau=t}^{\infty}\beta^{\tau-t}\left[m+(a-1)x_\tau^S-bn\gamma\sum_{l=1}^{\tau-1}\gamma^{\tau-1-l}x_l^S-bn\gamma\sum_{l=t}^{\tau-1}\gamma^{\tau-1-l}x_l^S\right].$$

As in the proof of Proposition 3, the objective function is additive separable across time, therefore the optimal value of x_t^S can be obtained by only considering the term proportional to it, which is

$$\left(a - 1 - bn\gamma \sum_{\tau=t+1}^{\infty} \beta^{\tau-t}\gamma^{\tau-1-t}\right) x_t^S = \left(a - 1 - \frac{bn\beta\gamma}{1-\beta\gamma}\right) x_t^S.$$

When the coefficient at x_t^S is nonnegative (negative), the optimal production input is $x_t^S = m$ ($x_t^S = 0$), which gives parts (i) and (ii) of the proposition.

B Instructions

Instructions – experimenter[2]

Thank you for participating in today's experiment. During the experiment you will make decisions and may earn money. Your earnings may depend on your own decisions and the decisions of other participants.

All amounts are expressed in tokens. The exchange rate is 1 token = 1 cent. At the end of the experiment your total earnings in tokens will be exchanged into dollars and cents and added to your $10 show-up fee. You will be given a check for the total amount in private. No other participant will be informed about your payment.

Please do not communicate with other participants or look at their monitors during the experiment. If you violate the rules we may ask you to leave the experiment. Raise your hand if you have a question at any point.

At the beginning of the experiment all participants will be randomly divided into groups and stay in the same group for the entire sequence of decisions. You will be given an initial balance of 250 tokens.

Decision

At the beginning of each round you will be endowed with 10 tokens. You can allocate these 10 tokens between two options: keep or use as production input. Therefore possible

[2]We present the instructions for the treatment with environmental context. In the neutral context treatments, "pollution" is called "common stock," and "the cost of environmental damage" is called "common stock maintenance cost."

allocations for tokens (kept, production input) are (0,10), (1,9), (8,2), (4,6) and so on.

Production

Each token you put into production yields you 5 units of output. Each unit of output is sold at a price of 1 token. Therefore each token you use in production input brings you production revenue of 5 tokens.

Production process generates pollution proportional to the scale of production. The level of pollution generated from production is exactly equal to the number of tokens you allocate for production. So if you put 1 token as input into production you generate 1 unit of pollution and so on. Pollution generated by production activity of all members of your group is accumulated together and leads to environmental damage. The total pollution amount and environmental damage impose a cost on all members of the group. Specifically each group member pays environmental damage cost proportional to pollution level. The cost of each unit of pollution is 1 token. This cost each round is based on the pollution level at the beginning of this round. Pollution level at the beginning of the first round is zero. At the end of each round the level of pollution grows by the number of tokens all group members put into production. Environment partially recovers from pollution but part of pollution is transferred to the next rounds. Specifically, pollution retention rate is .75 or 75% meaning that $\frac{3}{4}$ of pollution level in the current round will be the level of pollution at the beginning of the next round.

Payoffs

At the end of each round your revenue is obtained by adding the number of tokens you decided to keep and production revenue to your balance while subtracting the cost of environmental damage from your balance. This part of the experiment will consist of several rounds and your balance will be updated after every round as described above. At the end of the experiment your balance will be paid to you at the exchange rate of one cent per one token.

Are there any questions?

Practice

We will now illustrate the interface of the program and show you the decision screens.

All subjects will be randomly divided into groups of 2 and stay in the same group for the sequence of decision rounds.

Please do not press any buttons until you are instructed to, you are not paid for practice. Every round you will decide how to allocate 10 tokens as shown in the box in the center of the screen. In the upper left part of the screen you are reminded about the return per token you allocate as production input and cost of environmental damage per unit of pollution. Recall that every token you put into production gives 5 tokens of production revenue. Production inputs of all members of your group are added to the pollution level. Each unit of pollution has environmental damage cost of 1 token therefore the cost paid by each group member is equal to the level of pollution at the beginning of the round. For example if pollution is 0 each group member pays a cost of 0, if pollution level is 10, cost is 10, if pollution level is 50, cost paid by each group member is 50 and so forth.

In the upper right part of the screen you see your current balance and pollution level. Since this is the first round, there is no pollution. Your initial balance is 250 tokens. Your earnings after each round will be added to your balance. Note that if your earnings in a round are negative, your balance will decrease.

You will indicate your allocation decision in the fields provided in the box in the center of the screen. You see that the total number of tokens to allocate is 10. In the corresponding fields please enter the number of tokens you wish to keep and use for production. Note that the numbers should be integers and sum up to 10. Please make your decision, click on the Submit button, confirm and wait. Next screen reports your decision. You can change your decision by clicking on the grey Back button. Please click on the grey Back button now. You see that you are back at the decision screen. Please make a decision and click Submit. To submit your final decision click on the red Confirm button. When all members of your group make their decision you will see the results screen. Please click Confirm.

You now see the results screen. Your allocation decision is reported back to you in a box. To the right of the box you will see allocation decision of the other member(s) of your group. In the left column below the box you see revenue from tokens you decided to keep, revenue from production, cost of environmental damage and earnings this round. Your earnings in a round are obtained by summing these top three numbers. The cost of environmental damage is 0 since there was no pollution at the beginning of this round. Cost of environmental damage is determined by the level of pollution at the beginning of current round. Some numbers are replaced by xx in the practice round since they may depend on the decisions of other members of your group. In the actual rounds all these numbers will be shown to you. Note that your earnings have been added to your balance and your updated balance is shown in the upper right part of the screen. The right column provides information on pollution. You see the pollution level at the beginning of this round, which was used to determine cost of environmental damage this round. Next line reports increase in pollution level this round which is equal to production input by all members of your group. Next you see pollution level at the end of current round, which is the sum of the first two values. Pollution retention rate is shown next. Retention rate of 75% will not change during the experiment. The last line reports the size of pollution at the beginning of next round. It is equal to $\frac{3}{4}$ of current pollution level and will be used to determine environmental damage cost to each group member next round. When you review the results please click on the Continue button. In the actual rounds please remember to click on the Continue button to go to the next round. We will proceed to next round only after everyone has completed the previous round.

Are there any questions?

All subjects are randomly divided into groups of 2 and stay in the same group for the sequence of decision rounds. There will be 20 decision rounds.

We will now ask you to answer a short questionnaire to make sure our explanation was clear. As soon as you are done raise completed questionnaire and one of us will come and check your answers.

C Detailed results

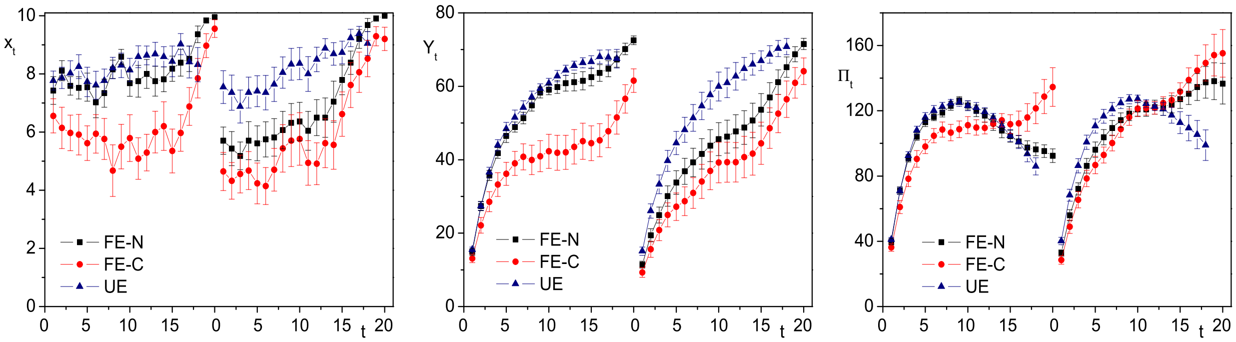

Figure C.1: Mean per capita inputs (left), mean pollution (center), mean per capita cumulative payoffs (right) in treatments FE-N, FE-C and UE, by period, with group-level error bars.

Period	$\bar{x}_t^{FE-N} - \bar{x}_t^{FE-C}$	Std. Err.	$\bar{x}_t^{FE-N} - \bar{x}_t^{UE}$	Std. Err.
1	0.87	0.76	-0.34	0.58
2	1.97**	0.79	0.25	0.51
3	1.62**	0.69	-0.43	0.53
4	1.61**	0.79	-0.73	0.56
5	1.93**	0.77	-0.18	0.70
6	1.08	0.85	-0.59	0.72
7	1.58*	0.85	-0.43	0.62
8	3.48***	0.94	-0.05	0.49
9	3.09***	0.80	0.30	0.49
10	1.89**	0.92	-0.45	0.61
11	2.66***	0.90	-0.84	0.66
12	2.71***	0.80	-0.64	0.59
13	1.75**	0.88	-0.93	0.66
14	1.61*	0.94	-0.77	0.55
15	2.83***	0.87	-0.25	0.68
16	2.42***	0.78	-0.64	0.59
17	1.64**	0.78	0.11	0.70
18	1.51**	0.74	1.05	0.66
19	0.87**	0.42		
20	0.40	0.31		
1	1.06	0.81	-1.84**	0.73
2	1.11	0.86	-1.93**	0.84
3	0.62	0.81	-1.70**	0.74
4	1.03	0.92	-1.65**	0.84
5	1.38	0.93	-1.79**	0.80
6	1.60*	0.88	-1.61*	0.85
7	1.11	0.98	-1.82**	0.88
8	0.63	1.04	-1.98**	0.84
9	0.61	1.11	-2.02**	0.86
10	0.60	1.12	-1.99**	0.86
11	1.10	1.02	-1.95**	0.90
12	1.59	0.95	-2.00***	0.75
13	0.88	1.08	-2.38***	0.75
14	1.49	1.09	-1.65**	0.80
15	1.18	0.92	-0.94	0.64
16	0.77	0.87	-0.85*	0.50
17	1.15	0.77	-0.18	0.41
18	1.15*	0.67	0.63	0.47
19	0.61*	0.34		
20	0.79*	0.40		

Table C.1: Mean differences in per capita inputs between treatments, by period, with group-level standard errors. Significance levels: *** - 1%, ** - 5%, * - 10%.

Period	$\bar{Y}_t^{FE-N} - \bar{Y}_t^{FE-C}$	Std. Err.	$\bar{Y}_t^{FE-N} - \bar{Y}_t^{UE}$	Std. Err.
1	1.75	1.51	-0.68	1.15
2	5.24**	2.50	-0.01	1.71
3	7.17**	3.04	-0.87	1.94
4	8.60**	3.54	-2.11	2.11
5	10.31***	3.62	-1.95	2.52
6	9.89***	3.56	-2.64	2.42
7	10.57***	3.88	-2.84	2.41
8	14.89***	4.30	-2.22	2.28
9	17.35***	4.50	-1.08	1.97
10	16.79***	4.85	-1.72	1.95
11	17.92***	5.06	-2.97	2.35
12	18.85***	5.13	-3.50	2.57
13	17.64***	5.26	-4.49 *	2.67
14	16.45***	5.50	-4.91 *	2.83
15	18.00***	5.30	-4.18	2.88
16	18.33***	5.04	-4.41	2.91
17	17.03***	4.96	-3.08	2.99
18	15.79***	5.01	-0.22	3.28
19	13.58***	4.18		
20	10.98***	3.41		
1	2.11	1.61	-3.69**	1.46
2	3.80	2.81	-6.62**	2.64
3	4.10	3.61	-8.36**	3.35
4	5.13	4.29	-9.57**	3.99
5	6.60	4.95	-10.76**	4.50
6	8.16	5.29	-11.29**	4.93
7	8.34	5.69	-12.11**	5.32
8	7.51	6.24	-13.04**	5.57
9	6.86	6.75	-13.81**	5.80
10	6.34	7.12	-14.35**	5.96
11	6.96	7.07	-14.67**	6.04
12	8.40	7.03	-15.00**	5.87
13	8.06	7.28	-16.01***	5.52
14	9.02	7.51	-15.30***	5.56
15	9.12	7.27	-13.36***	5.18
16	8.38	6.83	-11.72**	4.67
17	8.58	6.18	-9.15**	4.00
18	8.74	5.69	-5.59	3.55
19	7.78*	4.64		
20	7.43*	3.93		

Table C.2: Mean differences in the amount of pollution between treatments, by period, with group-level standard errors. Significance levels: *** - 1%, ** - 5%, * - 10%.

Period	$\bar{\Pi}_t^{FE-N} - \bar{\Pi}_t^{FE-C}$	Std. Err.	$\bar{\Pi}_t^{FE-N} - \bar{\Pi}_t^{UE}$	Std. Err.
1	3.49	3.02	-1.36	2.31
2	10.05**	4.67	0.15	3.17
3	12.60**	5.19	-1.57	3.31
4	13.66**	5.58	-3.83	3.40
5	14.92***	5.02	-2.97	3.93
6	11.52**	4.90	-3.88	3.81
7	10.40*	5.39	-3.62	3.77
8	16.41***	5.73	-1.67	3.54
9	17.60***	5.65	1.18	3.31
10	12.13**	5.93	0.17	4.11
11	10.19*	5.89	-1.91	4.77
12	7.58	5.52	-2.23	4.85
13	0.44	5.23	-3.33	5.15
14	-6.34	5.99	-3.05	5.21
15	-7.36	6.85	-0.37	5.20
16	-11.20	8.30	0.22	5.48
17	-18.38**	9.36	3.99	6.25
18	-25.11**	10.22	10.48	6.57
19	-33.47***	11.13		
20	-42.08***	12.56		
1	4.23	3.23	-7.37**	2.93
2	7.08	5.25	-12.31**	4.95
3	6.72	6.16	-14.14**	5.74
4	7.76	6.76	-14.48**	6.25
5	9.42	7.16	-14.47**	6.41
6	10.88	6.78	-12.82**	6.32
7	9.21	6.72	-11.66*	6.14
8	5.46	6.75	-10.49*	5.58
9	2.28	6.60	-8.77 *	5.14
10	-0.47	6.45	-6.38	4.81
11	-0.81	6.09	-3.44	4.90
12	0.32	5.81	-0.44	4.99
13	-2.45	6.40	1.29	6.46
14	-2.55	6.91	6.72	7.16
15	-4.61	7.78	14.43*	8.74
16	-8.38	9.91	21.04**	10.26
17	-10.08	12.53	29.13**	11.92
18	-11.90	14.71	38.52***	13.49
19	-15.99	17.07		
20	-18.65	19.19		

Table C.3: Mean differences in per capita payoffs between treatments, by period, with group-level standard errors. Significance levels: *** - 1%, ** - 5%, * - 10%.