Environmental *vs* Hedonic: Which Policy Can Help in Lowering Pollution Emissions?

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ONLINE APPENDIX

The supplementary material provided in this online appendix is organized as follows. Section 1 contains the more technical proofs of the results of our paper. Section 2 details the extension of our baseline model to the case of horizontal differentiation, that in the main text is only sketched in subsection 5.1. Finally, section 3 extends our baseline model to the case in which consumers are endowed with an initial level of green awareness which is independent from the policymaker's intervention.

1 Proofs of Proposition 1 and 2

1.1 Proof of Proposition 1

First of all, when the environmental campaign is adopted, social welfare is given by $SW^{*E}(\gamma)$, where:

$$CS_L^{*E} + CS_H^{*E} = \frac{1}{2\left(4q_H - q_L\right)^2 \left(q_H - q_L\right) q_L} \times$$

$$\left\{\gamma^2(q_H q_L + 4q_H^2 - q_L^2)\left(q_H - q_L\right)^2 - 2\gamma q_H q_L \left(q_H - q_L\right)\left[2c(2q_H - q_L) - \Theta q_L(q_H - q_L)\right]\right.$$
$$\left. + q_H q_L(6\Theta c q_H q_L + 4c^2 q_H - 3c^2 q_L - 8\Theta c q_H^2 + 2\Theta c q_L^2 + 4\Theta^2 q_H^3 - 5\Theta^2 q_H q_L^2 + \Theta^2 q_H^2 q_L)\right\}.$$

Lemma 2 and Remark 1 indicate that, by substituting into $SW^{*E}(\gamma)$ the corresponding optimal policy which depends on the emission level, we have:

$$SW|_{\gamma=0} = \frac{1}{2\left(4q_H - q_L\right)^2 \left(q_H - q_L\right)} \times$$

$$\left\{c^2\left(12q_H^2 - 9q_H q_L + 2q_L^2\right) - 2eq_H\left(q_H - 4q_L\right)\left[\Theta(q_H - q_L)\right]\right.$$
$$\left. + \Theta^2 q_H\left(q_H - q_L\right)\left(12q_H^2 - 2q_L^2 - q_H q_L\right) - 2c\Theta q_H\left(12q_H - 5q_L\right)\left(q_H - q_L\right)\right\},$$

$$SW|_{\gamma=\gamma^{*E}} = \frac{1}{2\left(2q_L^4 - 12q_H^4 + sq_L^3 - 15q_H q_L^3 - 7q_H^3 q_L - 8sq_H q_L^2 + 16sq_H^2 q_L + 32q_H^2 q_L^2\right)\left(q_H - q_L\right)} \times$$

$$\left\{2\Theta q_L q_H c\left(q_H - q_L\right)\left(9q_H^3 - 12sq_H q_L + 4q_L^3 + 5sq_L^2 - 9q_H q_L^2 - 4q_H^2 q_L\right)\right.$$
$$+ q_L c^2\left(2sq_L^3 - 9q_H^4 + 4q_H q_L^3 + 22q_H^3 q_L - 9sq_H q_L^2 + 12sq_H^2 q_L - 17q_H^2 q_L^2\right) - e^2 4q_H^4\left(q_H - q_L\right)$$
$$- q_L^2\Theta^2 q_H\left(q_H - q_L\right)\left(9q_H^4 + 4q_L^4 + 2sq_L^3 - 5q_H q_L^3 + 5q_H^3 q_L + sq_H q_L^2 - 12sq_H^2 q_L - 13q_H^2 q_L^2\right)$$
$$+ 2e[q_L q_H c\left(2q_L^3 - 7q_H^3 - 4sq_H q_L + sq_L^2 - 11q_H q_L^2 + 16q_H^2 q_L\right)]$$
$$\left. - \Theta\left(q_H - q_L\right)\left(7q_H^3 - 4sq_H q_L + 2q_L^3 + sq_L^2 - 7q_H q_L^2 - 2q_H^2 q_L\right)\right\}.$$

$$SW|_{\gamma=\overline{\gamma}^E} = \frac{1}{8\left(q_L - q_H\right)^2 q_H^2} \times$$

$$\left\{2\Theta c\left(q_L - q_H\right)\left(3q_H^3 + 2q_L^3 + sq_L^2 - 3q_H q_L^2 - 2q_H^2 q_L\right)\right.$$
$$+\Theta^2\left(q_L - q_H\right)^2\left(2q_L^3 - 3q_H^3 + sq_L^2 + q_H q_L^2 - 4q_H^2 q_L\right)$$
$$\left.-c^2\left(2q_L^3 - 3q_H^3 + sq_L^2 - 7q_H q_L^2 + 8q_H^2 q_L\right)\right\}.$$

Secondly, the expression for the social welfare function when the government decides to levy a tax on the polluting firm is $SW^{*E}(t)$, where:

$$CS_L^{**E} + CS_H^{**E} = \frac{q_H}{2\left(4q_H - q_L\right)^2\left(q_H - q_L\right)q_L} \times$$
$$\left\{t^2 q_H\left(4q_H - 3q_L\right) - 2tq_L(cq_L - 4\Theta q_H q_L + 4\Theta q_H^2) +\right.$$
$$\left. q_L(6\Theta c q_H q_L + 4c^2 q_H - 3c^2 q_L - 8\Theta c q_H^2 + 2\Theta c q_L^2 + 4\Theta^2 q_H^3 - 5\Theta^2 q_H q_L^2 + \Theta^2 q_H^2 q_L)\right\}.$$

Combining the results of Lemma 4 and Remark 2, by substituting into $SW^{**E}(t)$ the corresponding optimal taxation policy, we have:

$$SW|_{t=0} = SW|_{\gamma=0} \text{ by construction,}$$

$$SW|_{t=t^{*E}} = \frac{1}{2\left(q_H - q_L\right)\left(4q_H - 3q_L\right)q_L} \times$$

$$\left\{e^2\left(2q_H - q_L\right)^2 - 2eq_L\left(3cq_H - 2cq_L - 2\Theta q_H q_L + \Theta q_H^2 + \Theta q_L^2\right)\right.$$
$$\left.+q_L\left[c^2\left(3q_H - 2q_L\right) - 2\Theta c\left(3q_H - 2q_L\right)\left(q_H - q_L\right) + \Theta^2\left(3q_H^3 + q_L^3 - 4q_H^2 q_L\right)\right]\right\},$$

$$SW|_{t=\overline{t}^E} = \frac{\left(c - \Theta q_H\right)^2\left(3q_H - 2q_L\right)}{2\left(2q_H - q_L\right)^2}.$$

We can now formally prove the results of Proposition 1. Limiting our attention to internal solutions ($s > \underline{s}$), we first compare the different e-thresholds values and find four relevant cases. Then, for each case, we perform the welfare comparisons. However, we omit the precise expressions for the welfare differences for the sake of brevity.[1]

[1]They are available upon request, as are the analytical solutions and numerical simulations which confirm our results.

(i) When $\Theta \in \left(\underline{\Theta}^E, \Theta_1\right)$, with $\Theta_1 = \dfrac{2c(28q_H^4 - 53q_H^3 q_L + 36q_H^2 q_L^2 - 10q_H q_L^3 + q_L^4)}{q_H\left(4q_H - 3q_L\right)\left(q_H - q_L\right)\left(10q_H^2 - 6q_H q_L + q_L^2\right)}$ and $\underline{\Theta}^E$ defined in (2), the ranking is as follows:

$$\underline{e}^{tE} < \overline{e}^{tE} < \underline{e}^{\gamma E} < \overline{e}^{\gamma E}.$$

For each subinterval we compare the relevant social welfare, and demonstrate that the taxation instrument is always preferred.

(ii) When $\Theta \in (\Theta_1, \Theta_2)$, where $\Theta_2 = \dfrac{2c\left(2q_H - q_L\right)^2}{q_H\left(4q_H^2 - 7q_H q_L + 3q_L^2\right)}$, with $\Theta_2 > \Theta_1$, we have two subcases:

$$\begin{aligned}(a)\ \underline{e}^{tE} &< \underline{e}^{\gamma E} < \overline{e}^{\gamma E} < \overline{e}^{tE} \text{ when } s \in \left(\underline{s}, \widetilde{s}\right), \\ (b)\ \underline{e}^{tE} &< \underline{e}^{\gamma E} < \overline{e}^{tE} < \overline{e}^{\gamma E} \text{ when } s > \widetilde{s},\end{aligned}$$

$$\begin{aligned}\widetilde{s} &= \frac{\left(q_H - q_L\right)}{q_L\left(4q_H - q_L\right)\left(2q_H - q_L\right)^2\left[\Theta\left(2q_H - q_L\right)\left(q_H - q_L\right) - cq_H\right]} \times \\ &\quad [cq_H(97q_H^3 q_L - 72q_H^4 - 40q_H^2 q_L^2 + 2q_H q_L^3 + q_L^4) \\ &\quad +\Theta\left(q_H - q_L\right)\left(3q_H - q_L\right)\left(8q_H^4 + 12q_H^3 q_L - 25q_H^2 q_L^2 + 11q_H q_L^3 - 2q_L^4\right)].\end{aligned}$$

Subcase (a) is the most interesting, as we find that:

1. for $e \in (\underline{e}^{tE}, \underline{e}^{\gamma E})$, $SW|_{\gamma=0} - SW|_{t=t^{*E}} < 0$;
2. for $e \in (\underline{e}^{\gamma E}, \overline{e}^{\gamma E})$, $SW_{\gamma=\gamma^{*E}} - SW|_{t=t^{*E}} < 0$ when q_H/q_L is not excessive. For high values of q_H/q_L, there exists a threshold value of e above which the opposite holds: $SW|_{\gamma=\gamma^{*E}} - SW|_{t=t^{*E}} > 0$.
3. For $e \in (\overline{e}^{\gamma E}, \overline{e}^{tE})$, we compare $SW|_{\gamma=\overline{\gamma}^E}$ vs. $SW|_{t=t^{*E}}$, and find that the campaign is preferred only for very high levels of q_H/q_L. As the environmental damage is more severe in this region, the result holds for each level of $e \in (\overline{e}^{\gamma E}, \overline{e}^{tE})$. When the quality ratio is lower than a certain threshold value, then taxation prevails.
4. For $e > \overline{e}^{tE}$, we compare $SW|_{\gamma=\overline{\gamma}^E}$ with $SW|_{t=\overline{t}^E}$ and find similar results as in $e \in (\overline{e}^{\gamma E}, \overline{e}^{tE})$.

In subcase (b) the taxation instrument always prevails, as the cost for activating the campaign is now higher. We omit all the different subcases for brevity.

(iii) When $\Theta \in (\Theta_2, \widehat{\Theta})$, with $\widehat{\Theta} = \dfrac{2c\,(2q_H - q_L)}{(q_H - q_L)\,q_L}$, we have three subcases:

$$\begin{aligned}
(a)\ \underline{e}^{\gamma E} &< \overline{e}^{\gamma E} < \underline{e}^{tE} < \overline{e}^{tE} \text{ when } s \in (\underline{s}, s_1)\,, \\
(b)\ \underline{e}^{\gamma E} &< \underline{e}^{tE} < \overline{e}^{\gamma E} < \overline{e}^{tE} \text{ when } s \in (s_1, \widetilde{s})\,, \\
(c)\ \underline{e}^{\gamma E} &< \underline{e}^{tE} < \overline{e}^{tE} < \overline{e}^{\gamma E} \text{when } s > \widetilde{s},
\end{aligned}$$

where $\widetilde{s}$ is defined in the previous page, $\underline{s}$ can be found in (3), and

$$\begin{aligned}
s_1 &= \frac{(q_H - q_L)}{q_L\,(4q_H - q_L)^2\,(2q_H - q_L)\,[c + \Theta\,(q_H - q_L)]} \times \\
&\quad [8q_H^4(7\Theta q_H - 5c) - 2q_H^3 q_L(31\Theta q_H - 53c) - 11q_H^2 q_L^2(3\Theta q_H + 7c) \\
&\quad + 7q_H q_L^3(8\Theta q_H + 3c) - q_L^4(19\Theta q_H + 2c) + 2\Theta q_L^5].
\end{aligned}$$

The first two subcases can be explained together. They provide the result that the campaign is preferred, given that its cost is relatively low. Things change in subcase (c), where s is higher. By analyzing the different subintervals of e, we find that:

1. for $e \in (\underline{e}^{\gamma E}, \underline{e}^{tE})$, $SW|_{\gamma=\gamma^{*E}} > SW|_{t=0}$: the (relatively more costly) campaign still prevails when the environmental damage is perceived as limited.
2. for $e \in (\underline{e}^{tE}, \overline{e}^{tE})$, we evaluate $SW|_{\gamma=\gamma^{*E}}$ *vs* $SW|_{t=t^{*E}}$. We find that the campaign is more efficient only when its cost is not too excessive and when q_H/q_L is sufficiently high; on the contrary, when q_H/q_L decreases, there exists a threshold value of e above which taxation is to be preferred, when s is sufficiently high.
3. for $e \in (\overline{e}^{tE}, \overline{e}^{\gamma E})$, we compare $SW|_{\gamma=\gamma^{*E}}$ with $SW|_{t=\bar{t}^{E}}$, and find the same results as in the previous subinterval.
4. Finally, for $e > \overline{e}^{\gamma E}$, we find that $SW|_{\gamma=\overline{\gamma}^{E}} > SW|_{t=\bar{t}^{E}}$ when q_H/q_L is high and s does not overcome a certain limit. Otherwise taxation is more efficient from the welfare standpoint.

(iii)bis When $\Theta > \widehat{\Theta}$, then $\underline{e}^{\gamma E} < 0$ and $\underline{e}^{tE} < 0$. Therefore:

$$\begin{aligned}(a)\ \max\{0, \overline{e}^{\gamma E}\} &< \overline{e}^{tE} \text{ when } s \in (\underline{s}, \widetilde{s}),\\ (b)\ \overline{e}^{tE} &< \overline{e}^{\gamma E} \text{ when } s > \widetilde{s}.\end{aligned}$$

In subcase (a) the campaign is preferred as the average evaluation of the environmental quality is relatively high, and the cost to activate the campaign is low.

In subcase (b) we find that:

1. for $e \in (0, \overline{e}^{tE})$, we compare $SW|_{\gamma=\gamma^{*E}}$ with $SW|_{t=t^{*E}}$ and discover that the campaign prevails when q_H/q_L is sufficiently high and s is not excessive. When q_H/q_L decreases, there exists a threshold value of e above which taxation is to be preferred, provided the cost for the campaign s is high enough.
2. For $e \in (\overline{e}^{tE}, \overline{e}^{\gamma E})$, we compare $SW|_{\gamma=\gamma^{*E}}$ with $SW|_{t=\bar{t}^E}$ and find the same results as in the previous subinterval.
3. Finally, for $e > \overline{e}^{\gamma E}$, we evaluate $SW|_{\gamma=\overline{\gamma}^E}$ vs. $SW|_{t=\bar{t}^E}$ and replicate the results found in $e \in (0, \overline{e}^{\gamma E})$ with the only difference that, when q_H/q_L is very low and s is sufficiently high, taxation prevails for each value of $e > \overline{e}^{\gamma E}$.

1.2 Proof of Proposition 2

First of all, when activating the campaign, the social welfare function as a function of γ is given by $SW^{*H}(\gamma)$, where:

$$CS_L^{*H} = \frac{\{q_H [(q_H - q_L)(2\gamma q_H - q_L) - c(2q_H - q_L)]\}^2}{2q_L(4q_H - q_L)(q_H - q_L)},$$

$$CS_H^{*H} = \frac{q_H [(2\Theta + \gamma)(q_H^2 - q_L^2) - c(3q_H + 2q_L)]\{cq_H + (q_H - q_L)[2\Theta q_H - \gamma(3q_H - q_L)]\}^2}{2(4q_H - q_L)(q_H - q_L)}.$$

By combining the results in Lemma 6 with the considerations reported in Remark 3, and by substituting into $SW^{*H}(\gamma)$, we obtain that:

$$SW|_{\gamma=0} = \frac{q_H}{2q_L(4q_H - q_L)^2(q_H - q_L)} \times$$

$$\{c^2(12q_H^2 - 9q_Hq_L + 2q_L^2) - 2cq_L[4q_H(e + \Theta q_H) - q_L(e + 6\Theta q_H - 2\Theta q_L)]$$
$$-\Theta(q_H - q_L)q_L[4e(4q_H - q_L) + \Theta(12q_H^2 - q_Hq_L - 2q_L^2)]\},$$

$$SW|_{\gamma=\gamma^{*H}} = \frac{1}{2(q_L-q_H)\left[(q_L-q_H)(12q_H^3+2q_L^3-13q_Hq_L^2+19q_H^2q_L)-q_L(4q_H-q_L)^2 s\right]}\times$$

$$\{2cq_H(q_H-q_L)^2[e(6q_H-q_L)]-2\Theta cq_H(9q_H-4q_L)(q_H+q_L)+8cq_H^2q_L(e+\Theta q_H)$$

$$-2cq_Hq_L(e+6\Theta q_H)q_L+2\Theta q_L^2)s+(q_H-q_L)[\Theta^2q_H(9q_H-4q_L)(q_H-q_L)(q_L+q_H)^2]$$

$$+c^2q_H\cdot\left[(9q_H-4q_L)(q_H+q_L)^2-s\left(12q_H^2-9q_Hq_L+2q_L^2\right)\right]+$$

$$+(q_H-q_L)[\Theta q_Hq_L(16eq_H-12\Theta q_H^2-4eq_L+\Theta q_Hq_L+2\Theta q_L^2)s]$$

$$-(q_H-q_L)e^2q_L(3q_H-q_L)^2+2\Theta eq_H(q_H-q_L)(6q_H-q_L)(q_H+q_L)\},$$

$$SW|_{\gamma=\overline{\gamma}^H} = \frac{1}{2q_L(5q_H^2-9q_Hq_L+2q_L^2)}\times$$

$$\left\{cq_H\left[27q_H^3-36q_H^2q_L-2q_L^3+q_Hq_L(11q_L-s)\right]\right.$$

$$+\Theta\cdot(q_H-q_L)^2(q_H+q_L)\left[\Theta(q_H+q_L)\left(3q_H^2+3q_Hq_L-2q_L^2\right)-2eq_L(5q_H-q_L)\right]$$

$$-\Theta q_L\left(2q_H^2-3q_Hq_L+q_L^2\right)^2 s-4cq_Hq_L(q_H-q_L)(5q_H-q_L)e$$

$$\left.-2\Theta cq_H(q_H-q_L)\left[9q_H^2-5q_H^2q_L+q_L^2(5q_L+s)-q_Hq_L(9q_L+2s)\right]\right\}.$$

For exposition purposes, we take $\overline{\gamma}^H = \frac{\Theta(2q_H-q_L)(q_H-q_L)-cq_H}{(5q_H-q_L)(q_H-q_L)}$. Tedious numerical calculations show that similar results also hold when $\overline{\gamma}^H = \frac{2cq_H+\Theta(q_H-q_L)q_L}{(q_L-2q_H)(q_L-q_H)}$.

Secondly, when taxing the polluting good, the social welfare is $SW^{**H}(t)$, where:

$$CS_L^{**H} = \frac{q_H^2q_L\left[q_L(t+\Theta q_H-\Theta q_L)-c(2q_H-q_L)\right]^2}{2(4q_H-q_L)(q_H-q_L)q_L},$$

$$CS_H^{**H} = \frac{\{q_H[2\Theta(q_H-q_L)+c]-t(2q_H-q_L)\}^2\left[2\Theta\left(q_H^2-q_L^2\right)-c(3q_H-2q_L)-t(2q_H-q_L)\right]}{(4q_H-q_L)(q_H-q_L)}.$$

From Lemma 8, we know that $t^{*H}\in[0,\overline{t}^H)$ *when* $e\in[\underline{e}^{tH},\overline{e}^{tH})$. Hence, as reported in Remark 4, the government sets: *(i)* $t=0$ when $e<\underline{e}^{tH}$; *(ii)* $t=t^{*H}$ when $e\in[\underline{e}^{tH},\overline{e}^{tH})$; *(iii)* $t=\overline{t}^H$ when $e\geqslant\overline{e}^{tH}$. By substituting into $SW^{**H}(t)$, we have:

$$SW|_{t=0} = SW|_{\gamma=0} \text{ by construction},$$

$$SW|_{t=t^{*H}} = \frac{1}{24}\left[4(\frac{e^2}{q_H}+3\Theta^2q_H-6\Theta e)+\frac{12(e-c)^2}{(q_H-q_L)}+\frac{9c^2}{q_L}-\frac{(3c-4e)^2}{4q_H-q_L}\right],$$

$$SW|_{t=\overline{t}^H} = \frac{q_H(3q_H-2q_L)(c-\Theta q_L)^2}{2q_L(2q_H-q_L)^2}.$$

We can now formally prove the results of Proposition 2. When $s>\underline{s}$, by comparing the different e-thresholds, one can find the following cases:

- When $\Theta \in (\underline{\Theta}^H, \Theta_3)$, with $\underline{\Theta}^H$ in (7) and $\Theta_3 = \dfrac{c\,(3q_H - q_L)\,(16q_H^3 - 24q_H^2 q_L + 11q_H q_L^2 - q_L^3)}{q_H\,(q_H - q_L)\,q_L\,(16q_H^2 - 20q_H q_L + 5q_L^2)}$, the ranking is as follows:

$$\underline{e}^{tH} < \overline{e}^{tH} < \underline{e}^{\gamma H} < \overline{e}^{\gamma H}.$$

- When $\Theta > \Theta_3$ and $s \in (\underline{s}, \widetilde{s})$, with $\underline{s}$ defined in (3) and $\widetilde{s}$ already appearing in the Proof of Proposition 1, the ranking is:

$$\underline{e}^{tH} < \underline{e}^{\gamma H} < \overline{e}^{\gamma H} < \overline{e}^{tH}.$$

- Finally, for $\Theta > \Theta_3$ and $s > \widetilde{s}$, we have that:

$$\underline{e}^{tH} < \underline{e}^{\gamma H} < \overline{e}^{tH} < \overline{e}^{\gamma H}.$$

Comparing the appropriate social welfare expressions, it is relatively easy to prove that, for any value of e, taxation always determines a higher welfare than the environmental campaign. Additional calculations are available upon request.

2 Relative Preferences in a Horizontally Differentiated Duopoly

The layout of the model is presented in subsection 5.1. The government has two alternative policy instruments at its disposal: either a campaign designed to raise environmental awareness, or a tax t proportional to the polluting emission. We build again the following two-stage game. First, the policymaker decides which policy instrument to use in order to reduce the environmental damage for any given level of the per-unit emissions. Then, firms compete in prices. Before considering the two different settings, we briefly focus on the unregulated market.

Lemma 9 *The unregulated market is characterized by a duopoly with full market coverage if* $c \leq 3r$.

Proof. In absence of policy instruments (i.e., $t = 0$ and $\gamma = 0$), the consumer type that is indifferent between buying good G or good B is $x_G = (r + p_B - p_G)/2r$. Profit functions are $\pi_B = x_B \cdot p_B$ and $\pi_G = (p_G - c)\,x_G$, where $x_B = 1 - x_G$. Equilibrium prices are $p_G^N = (2c + 3r)/3$, $p_B^N = (c + 3r)/3$, where N stands for Nash Equilibrium. It is immediate to verify that market coverage condition $x^N \in [0, 1]$ and non-negative profits condition $p_G^N - c > 0$ simultaneously hold when $c \leq 3r$. ■

Given that the following two cases are extension of the unregulated market, we assume that $c \leq 3r$. This will simplify the analysis without reducing the validity of our results.

2.1 Supporting the environmental campaign

When the government decides to support a campaign, the utility of a consumer indexed by $x \in [0, 1]$ is given by (11). Let x_G denote the consumer type that is indifferent between buying good G or good B:

$$x_G = \frac{2\gamma + r + p_B - p_G}{2r}. \tag{A1}$$

Consumer types $x \in [0, x_G)$ buy good G, whereas consumer types $x \in (x_G, 1]$ buy good B, such that the demand for good G is equal to x_G and the demand for good B is equal to $x_G = (1 - x_G)$ under the assumption of market coverage.

Before proceeding with the solution of the game, it is interesting to notice that, as in the baseline model, there exist two conflicting sources of product differentiation: parameter r measures the intensity of horizontal differentiation, while parameter γ indicates that also vertical differentiation plays a role. We need therefore to impose an additional condition in order to guarantee that horizontal differentiation prevails. Formally, in order for both demands to be positive when prices are equal, substituting $p_B = p_G$ in (A1), we find $x_G|_{p_G = p_B} = (2\gamma + r)/2r$. It is therefore immediate to prove that:

$$x_G|_{p_G = p_B} \in (0, 1) \Longleftrightarrow \gamma \leq \frac{r}{2}.$$

If instead $\gamma > r/2$, then the brown firm is stranded out of the market because the environmental concern is so high that no consumer is willing to buy the brown good. Hence,

Assumption 1 *We assume that $\gamma \leq r/2$ in order for the model to be characterized by horizontal differentiation.*

Now we proceed with the solution of the model. Demands are formally given by x_G in (A1) and $x_B = 1 - x_G$. Profit functions are again given by $\pi_B = x_B \cdot p_B$ and $\pi_G = (p_G - c)\, x_G$. Equilibrium prices can be easily obtained:

$$p_G^* = \frac{2(c+\gamma)+3r}{3},\ p_B^* = \frac{c-2\gamma+3r}{3}.$$

It is relatively easy to demonstrate that:

Lemma 10 *The market is covered and both firms are active under Assumption 1.*

Proof. To start with,

$$\begin{aligned} p_G^* - c &> 0 \iff c < 3r + 2\gamma, \\ p_B^* &> 0 \iff c > 2\gamma - 3r. \end{aligned}$$

Next, plugging p_G^* and p_B^* into in (A1), we need to verify that $x_G^* \in [0,1]$:

$$\begin{aligned} x_G^* &= \frac{3r+2\gamma-c}{6r} \geq 0 \iff c \leq 3r+2\gamma, \\ x_G^* &\leq 1 \iff c \geq 2\gamma - 3r. \end{aligned}$$

Notice that Lemma 9 guarantees that $c < 3r + 2\gamma$ for each $\gamma > 0$. Such a condition can therefore be discarded. Moreover, under Assumption 1, $2\gamma - 3r < 0$, and therefore $p_B^* > 0$ and $x_G^* < 1$ for every $c > 0$. It follows that the only condition that we need to impose is the one already highlighted in Assumption 1, i.e., $\gamma \leq r/2$, together with $c \leq 3r$ from Lemma 9. ■

Equilibrium profits are given by:

$$\pi_G^* = \frac{(3r+2\gamma-c)^2}{18r},\ \pi_B^* = \frac{(3r-2\gamma+c)^2}{18r}.$$

Consumer surplus accruing from consumption of the green good and the brown good is respectively given by:

$$CS_G^* = \int_0^{x_g} (v - r x_G^* - p_G^* + \gamma)dx = \frac{(3r+2\gamma-c)(12v-7c+2\gamma-15r)}{72r},$$

$$CS_B^* = \int_{x_g}^{1} (v - r x_B^* - p_B^* - \gamma)dx = \frac{(3r-2\gamma+c)(12v-5c-2\gamma-15r)}{72r}.$$

Remember that $x_B^* = 1 - x_G^*$. The social welfare function is written in a compact way as follows:

$$SW^*(\gamma) = \pi_G^* + \pi_B^* + CS_G^* + CS_B^* - e \cdot x_B^* - s\frac{\gamma^2}{2}.$$

The social welfare function is concave in γ if and only if $s > (10/9)r$, which is supposed to hold throughout the analysis. We compute the optimal γ level which maximizes social welfare:

$$\gamma^* = \frac{3e - 5c}{9rs - 10}.$$

We need now to impose conditions for γ^* to make economic sense:

Lemma 11 *$\gamma^* \in [0, r/2)$ when $e \in [\underline{e}^\gamma, \overline{e}^\gamma)$.*

Proof. It is immediate to prove that $\gamma^* \geq 0$ iff $e \geq \underline{e}^\gamma = (5/3)c$, and that $\gamma^* < r/2$ iff $e < \overline{e}^\gamma = [10c + r(9rs - 10)]/6$. Condition $s > (10/9)r$ guarantees that $\overline{e}^\gamma > \underline{e}^\gamma$. ■

It follows that:

Remark 5 *The government optimally sets: (i) $\gamma = 0$ when $e < \underline{e}^\gamma$; (ii) $\gamma = \gamma^*$ when $e \in [\underline{e}^\gamma, \overline{e}^\gamma)$; (iii) $\gamma = r/2$ when $e \geqslant \overline{e}^\gamma$.*

The complete expressions for the social welfare in the three cases reported in Remark 5 are as follows:

$$SW|_{\gamma=0} = \frac{5c^2 - 6c(e + 3r) + 9r(4v - 2e - r)}{36r}, \tag{A2}$$

$$SW|_{\gamma=\gamma^*} = \frac{2e^2 - 2er(9rs - 10 + 3cs) + r[5c^2 s - (4v - r - 2c)(9rs - 10)]}{4r(9rs - 10)}, \tag{A3}$$

$$SW|_{\gamma=r/2} = \frac{72vr + 10c^2 - 8r(3e + r) - 4c(3e + 14r) - 9r^3 s}{72r}. \tag{A4}$$

2.1.1 Taxing the polluting good

When the government opts for the tax, $\gamma = 0$ and the utility of a consumer indexed by $x \in [0, 1]$ is:

$$U(x) = \begin{cases} v - rx - p_G & \text{if buys } G \\ v - r(1 - x) - p_B & \text{if buys } B \end{cases}.$$

The consumer that is indifferent between buying good G or good B is obviously:

$$x_G = \frac{r + p_B - p_G}{2r}.$$

Profit functions in the presence of the taxation instrument are $\pi_G = (p_G - c)\, x_G$ and $\pi_B = (p_B - t)\, x_B$. Equilibrium prices are easily determined:

$$p_G^{**} = \frac{2c + t + 3r}{3},\ p_B^{**} = \frac{c + 2\gamma + 3r}{3}.$$

Lemma 12 *The market is covered and both firms are active iff $t < c + 3r$.*

Proof. $p_G^{**} - c > 0$ iff $c < 3r + t$ and $p_B^{**} - t > 0$ iff $c > t - 3r$. The same conditions also suffice to guarantee that $x_G^{**} \in [0, 1]$, as it can be easily verified. Following again Lemma 9, condition $c < 3r + t$ can be discarded, given that we assumed $c < 3r$. The second condition ($c > t - 3r$) can be turned into $t < c + 3r$ in order to have an explicit condition on t, as we did in the baseline model. ■

Equilibrium profits are given by:

$$\pi_G^{**} = \frac{(3r + t - c)^2}{18r},\ \pi_B^{**} = \frac{(3r - t + c)^2}{18r}.$$

Consumer surplus can be written as:

$$CS_G^{**} = \int_0^{x_g} (v - r x_G^{**} - p_G^{**}) dx = \frac{(3r + t - c)(12v - 7c - 5t - 15r)}{72r},$$

$$CS_B^{**} = \int_{x_g}^{1} (v - r x_B^{**} - p_B^{**}) dx = \frac{(3r - t + c)(12v - 5c - 7t - 15r)}{72r}.$$

The social welfare function is written in a compact way as follows:

$$SW^{**} = \pi_G^{**} + \pi_B^{**} + CS_G^{**} + CS_B^{**} - e \cdot x_B^{**} + t \cdot x_B^{**}.$$

Algebraic calculations confirm that SW^{**} is concave in t. The optimal tax rate is:

$$t^* = 3e - 2c.$$

Lemma 13 $t^* \in [0, c + 3r)$ *when* $e \in [\underline{e}^t, \overline{e}^t)$.

Proof. $t^* \geqslant 0$ iff $e \geqslant \underline{e}^t = (2/3)c$; $t^* < c + 3r$ iff $e < \overline{e}^t = r + c$. ■

Remark 6 *The government optimally sets: (i) $t = 0$ when $e < \underline{e}^t$; (ii) $t = t^*$ when $e \in [\underline{e}^t, \overline{e}^t)$; (iii) $t = c + 3r$ when $e \geqslant \overline{e}^t$.*

The expressions for the social welfare for the three different values of t which appear in Remark 6 are given by:

$$\begin{aligned} SW|_{t=0} &= SW|_{\gamma=0} \text{ by construction,} && \text{(A5)} \\ SW|_{t=t^*} &= \frac{c^2+e^2-2er-2c(e+r)+r(4v-r)}{4r}, && \text{(A6)} \\ SW|_{t=c+3r} &= \frac{2(v-c)-r}{2}. && \text{(A7)} \end{aligned}$$

2.1.2 Comparing the two instruments

In order to compare social welfare expressions (A2)-(A4) with (A5)-(A7), we first need to rank the threshold values of e which appear in Lemma 11 and Lemma 13 respectively. Provided $s > (10/9)r$ and $c \leq 3r$, three relevant rankings for the e-thresholds can be found, depending on the interplay between c and s:

(a) $\underline{e}^t < \underline{e}^\gamma < \overline{e}^\gamma < \overline{e}^t$ when $c \in (0,(3/2)r) \cup s \in ((10/9)r, 4(4r-c)/9r^2)$;

(b) $\underline{e}^t < \underline{e}^\gamma < \overline{e}^t < \overline{e}^\gamma$ when $c \in (0,(3/2)r) \cup s \geq 4(4r-c)/9r^2$;

(c) $\underline{e}^t < \overline{e}^t < \underline{e}^\gamma < \overline{e}^\gamma$ when $c \in [(3/2)r, 3r]$.

We demonstrate that:

Proposition 3 *The environmental campaign prevails only if both the cost for producing the green product and that for supporting the campaign itself are sufficiently low, provided some specific conditions for the emission level are satisfied. In all the other cases the taxation instrument is more efficient than the environmental campaign.*

Proof. In each of the three relevant rankings for the e-thresholds outlined above we perform the appropriate social welfare comparison.

(a) When $c \in (0,(3/2)r) \cup s \in ((10/9)r, 4(4r-c)/9r^2)$:

1. for $e \in (\underline{e}^t, \underline{e}^\gamma)$, $SW|_{\gamma=0} - SW|_{t=t^*} < 0$, hence taxation always prevails.
2. For $e \in (\underline{e}^\gamma, \overline{e}^\gamma)$, $SW|_{\gamma=\gamma^*} - SW|_{t=t^*} > 0$ when $s < \dfrac{2(5c^2-10ce+6e^2)}{r(2c-3e)^2} \equiv s_1$; however, $s_1 \in ((10/9)r, 4(4r-c)/9r^2)$ only when c is low, and e is sufficiently high, otherwise $s_1 < (10/9)r$ and $SW|_{\gamma=\gamma^*} - SW|_{t=t^*} < 0$ always.

3. For $e \in (\overline{e}^{\gamma}, \overline{e}^{t})$, $SW|_{\gamma=r/2} - SW|_{t=t^*} > 0$ when $s < \dfrac{10r^2 - 2(2c-3e)^2 + 4(3e-5c)}{9r^3} \equiv s_2$; however, $s_2 \in ((10/9)r, 4(4r-c)/9r^2)$ only when c is low, but this time combined with a value of e that must not be excessive, otherwise $s_2 < (10/9)r$ and $SW|_{\gamma=r/2} - SW|_{t=t^*} < 0$ always.

4. For $e > \overline{e}^{t}$, $SW|_{\gamma=r/2} - SW|_{t=c+3r} < 0$, and the taxation instrument is always more efficient than the environmental campaign.

(b) In the second interval, it is relatively easy to verify that taxation always prevails, given that we consider a higher cost for activating the campaign ($s \geq 4(4r-c)/9r^2)$).

(c) Also in the third interval the comparison is straightforward. The possibility for the campaign to prevail relies on the possibility for the green producer to bear a very low value of c, as we demonstrated above. As we are considering now relatively higher values of c, i.e., $c \in [(3/2)r, 3r]$, then taxation always proves to be more efficient than the campaign in terms of social welfare. ■

The results obtained from the above proposition reveal that also for the horizontal differentiation case there are circumstances in which the campaign can be preferred by the government. As in the baseline model, one of the preconditions for this to occur is that the cost of the campaign be not excessive. Here we also need a relatively low cost c of producing the green good. Notice that also here the role played by parameter e is ambiguous, as we need an intermediate level of emissions in order for the campaign to prevail. Finally, as already explained in the main text, consumers' heterogeneity r does not help in the social comparison between the two policy instruments.

3 The environmental campaign does not modify the structure of consumer preferences

As in the baseline model, we consider two different scenarios characterized by vertical differentiation (environmental quality *vs.* hedonic quality) and evaluate in each case whether a government committed to reducing polluting emissions would prefer either to tax the polluting firm or to support an environmental campaign. The structure of the game is the same as in the baseline model. The only difference is represented by the presence of an initial level of personal moral norms that favor pro-environmental behaviors. Parameter $\alpha > 0$ represents such an initial level, independent from the policymaker's intervention. The effect of the campaign is therefore not to *activate* pro-environmental behaviors, but rather to increase their weight in the utility function. As the analysis that will be carried out in this subsection is qualitatively similar to that of the baseline model, lemmas and propositions will be labeled accordingly.

3.1 Environmental qualities

Consider two goods of different *environmental* quality: good H produced by firm H is green and therefore of higher quality, while good L produced by firm L is brown and of lower quality. There is a continuum of consumers indexed by θ which is uniformly distributed in the interval $[0, \Theta]$. Each consumer can buy either one unit of good H or one unit of L or not buy anything at all. Pollution creates an environmental damage $D = e \cdot x_L$, where x_L is the quantity produced by firm L. The government can adopt two alternative policy instruments, either a campaign designed to *increase* environmental awareness, or a tax t proportional to the polluting emission.

As introduced above, the unique difference with respect to the baseline model consists of the presence of an initial level of *personal moral norms* captured by parameter $\alpha > 0$, which is independent from the policymaker's intervention. In order to reproduce interval regions which are comparable to those of the baseline model, the following holds:

Assumption 2 *We assume that the initial level of relative preferences is sufficiently low, i.e.,* $\alpha < \dfrac{cq_L}{(q_H - q_L)(q_H + q_L)} = \overline{\alpha}^E$.

If the results of the baseline model hold for low levels of α, then it can be easily shown what happens when $\alpha > \overline{\alpha}^E$. In particular, we will show that, when considering progressively higher values of such parameter: *(i)* first of all the threshold values of Θ decrease (and those of s increase), meaning that the interval regions where taxation prevails tend to reduce; *(ii)* then both instruments can not be adopted if the government wants to keep both firms in the market. More details will be provided in the following analysis, where we will also discuss the case $\alpha \geq \overline{\alpha}^E$.

3.1.1 Supporting the environmental campaign

Given the previous discussion, the utility of a consumer of type $\theta \in [0, \Theta]$ becomes:

$$U(\theta) = \begin{cases} \theta q_H - p_H + (\alpha + \gamma)(q_H - q_L) \text{ if she buys the green good,} \\ \theta q_L - p_L - (\alpha + \gamma)(q_H - q_L) \text{ if she buys the brown good,} \\ 0 \text{ if she refrains from buying,} \end{cases}$$

where $(\alpha + \gamma)$ now indicates relative preferences; $\alpha > 0$ is independent from the policy instrument, while $\gamma \geq 0$ measures the *increase* in the intensity of the pro-environmental component of consumer's utility induced by the campaign supported by the government.

The consumer indifferent between buying the low quality good and not buying at all is now given by:

$$\theta_L = \frac{p_L + (\alpha + \gamma)(q_H - q_L)}{q_L}, \tag{A8}$$

while the consumer indifferent between buying the low quality good and the high quality good is:

$$\theta_H = \frac{p_H - p_L - 2(\alpha + \gamma)(q_H - q_L)}{q_H - q_L}. \tag{A9}$$

The demands for the two goods are: $x_L = \theta_H - \theta_L$ and $x_H = \Theta - \theta_H$. The presence of $\alpha \geq 0$ obviously increases the *stealing effect* highlighted in the baseline model. Profits are again given by $\pi_H = x_H(p_H - c)$ and $\pi_L = p_L x_L$. Taking F.O.C.s, equilibrium prices are:

$$p_L^{*E}(\alpha) = \frac{q_L[c + \Theta(q_H - q_L)] - 2(\alpha + \gamma)q_H(q_H - q_L)}{4q_H - q_L}, \tag{A10}$$

$$p_H^{*E}(\alpha) = \frac{2cq_H + (q_H - q_L)[2\Theta q_H + (\alpha + \gamma)(3q_H - q_L)]}{4q_H - q_L}. \tag{A11}$$

Additional superscript E indicates the case of environmental qualities. Moreover, in order to differentiate the present case from the baseline one, we specify that both equilibrium variables and relevant threshold values of the main parameters of the model now depend on α. Notice that $p_L^{*E}(\alpha) < p_L^{*E}$ while $p_H^{*E}(\alpha) > p_H^{*E}$. We focus on the case in which both firms are active in the market, and replicate previous conditions. Define:

$$\underline{\Theta}^E(\alpha) \equiv \frac{c\,(2q_H - q_L) - a(3q_H^2 - 4q_H q_L + q_L^2)}{2q_H\,(q_H - q_L)}, \tag{A12}$$
$$\overline{\gamma}^E(\alpha) \equiv \frac{q_L\,[c + (\Theta - 2\alpha q_H)\,(q_H - q_L)]}{2q_H\,(q_H - q_L)}.$$

Lemma 1 bis *Provided Assumption 2 holds, the market is uncovered and both firms are active iff* $\Theta > \underline{\Theta}^E(\alpha)$ *and* $\gamma < \overline{\gamma}^E(\alpha)$.

Proof. First of all:

$$\begin{aligned}
p_L^{*E}(\alpha) &> 0 \iff \gamma < \frac{q_L\,[c + (\Theta - 2aq_H)\,(q_H - q_L)]}{2q_H\,(q_H - q_L)} \equiv \overline{\gamma}^E(\alpha), \\
p_H^{*E}(\alpha) - c &> 0 \iff \gamma > \frac{c\,(2q_H - q_L) - (q_H - q_L)\,[\alpha\,(3q_H - q_L) + 2q_H\Theta]}{(3q_H - q_L)\,(q_H - q_L)} \equiv \underline{\gamma}^E(\alpha), \\
p_H^{*E}(\alpha) - p_L^{*E}(\alpha) &= \frac{(2q_H - q_L)\,(\Theta q_H - \Theta q_L + c) + (\alpha + \gamma)\,(5q_H - q_L)\,(q_H - q_L)}{(4q_H - q_L)} > 0.
\end{aligned}$$

The last inequality is straightforward in the present scenario. It will, however, play a relevant role when consumers are characterized by hedonic preferences. Next, we insert $p_L^{*E}(\alpha)$ and $p_H^{*E}(\alpha)$, (defined in (A10) and (A11), respectively) into θ_L and θ_H (expressions (A8) and (A9), respectively) and verify that $0 < \theta_L^{*E}(\alpha) < \theta_H^{*E}(\alpha) < \Theta$. This gives the following conditions:

$$\begin{aligned}
\theta_L^{*E}(\alpha) &= \frac{q_L\,(c + \Theta q_H - \Theta q_L) + (\alpha + \gamma)\,(2q_H - q_L)\,(q_H - q_L)}{(4q_H - q_L)\,q_L} > 0, \text{ always;} \\
\theta_H^{*E}(\alpha) - \theta_L^{*E}(\alpha) &= \frac{q_H[c + (2\alpha + \Theta + 2\gamma)q_H - \Theta q_H q_L^2] - 2q_H^3(\alpha + \gamma)}{(4q_H - q_L)\,(q_H - q_L)\,q_L} > 0 \iff \gamma < \overline{\gamma}^E(\alpha); \\
\Theta - \theta_H^{*E}(\alpha) &> 0 \iff \gamma > \underline{\gamma}^E(\alpha),
\end{aligned}$$

with $\underline{\gamma}^E(\alpha) \leq 0 \iff \Theta \geq \underline{\Theta}^E(\alpha)$. Therefore, similarly to the baseline model, we assume that $\Theta > \underline{\Theta}^E(\alpha)$. This implies that $\underline{\gamma}^E(\alpha)$ is negative, and then $\theta_H^{*E} < \Theta$ always holds. Notice that $\underline{\Theta}^E(\alpha)$ decreases in α. However, Assumption 2 guarantees that $\underline{\Theta}^E(\alpha) > 0$. If follows that the conditions that have to be simultaneously satisfied are $\Theta > \underline{\Theta}^E(\alpha)$ and $\gamma < \underline{\gamma}^E(\alpha)$. ■

We assume that the conditions reported in Lemma 1 *bis* hold throughout the following analysis. However, notice that, when α is relatively high, then *(i)* condition $\Theta > \underline{\Theta}^E(\alpha)$

can be discarded, as $\underline{\Theta}^{E}(\alpha) < 0$, and *(ii)* $\overline{\gamma}^{E}(\alpha) < 0$, meaning that the brown firm is forced to exit the market.

Equilibrium demands/outputs $x_L^{*E}(\alpha)$ and $x_H^{*E}(\alpha)$ are not reported but they are similar to equivalent expressions in the baseline case. Total output decreases both with α and γ, as expected, given that the higher the intensity of relative preferences, the lower the willingness to pay for the brown product. Equilibrium profits and social welfare can still be expressed in the compact form:

$$\pi_L^{*E}(\alpha) = \frac{q_L}{q_H}(q_H - q_L)\left[x_L^{*E}(\alpha)\right]^2, \tag{A13}$$

$$\pi_H^{*E}(\alpha) = (q_H - q_L)\left[x_H^{*E}(\alpha)\right]^2, \tag{A14}$$

$$SW^{*E}(\alpha,\gamma) = \pi_L^{*E}(\alpha) + \pi_H^{*E}(\alpha) + CS_L^{*E}(\alpha) + CS_H^{*E}(\alpha) - e \cdot x_L^{*E}(\alpha) - s\frac{\gamma^2}{2}.$$

The precise expressions of (A13)-(A14), together with that of $SW^{*E}(\alpha,\gamma)$, are omitted for brevity. The social welfare is concave in γ if $s > \underline{s}$, where $\underline{s}$ is defined as in (3) in the baseline model.[2] We assume that $s > \underline{s}$ and compute the optimal level of γ that maximizes social welfare:

$$\gamma^{*E}(\alpha) = \frac{2eq_H^2(4q_H - q_L) + q_L[2c(6q_Hq_L - 10q_H^2 - q_L^2) + \Theta q_H(8q_H^2 - 11q_Hq_L + 3q_L^2)] + \alpha\Phi}{sq_L(4q_H - q_L)^2 - (q_H - q_L)(12q_H^3 + 2q_L^3 - 13q_Hq_L^2 + 19q_H^2q_L)},$$

$$\Phi = (q_H - q_L)\left(12q_H^3 + 2q_L^3 - 13q_Hq_L^2 + 19q_H^2q_L\right).$$

Notice that $\gamma^{*E}(\alpha)$ is increasing in α. Given a cost of the campaign $s > \underline{s}$, the investment effort is increasing in the initial level of relative preferences. The higher α is, the more resources can be devoted to additionally reinforce such a *social component* of consumption. It is relatively easy to formally show that α and γ are complementary by computing cross-partial derivatives:[3]

$$\frac{\partial^2(SW^{*E}(\alpha,\gamma))}{\partial\alpha\partial\gamma} = \frac{\partial^2(SW^{*E}(\alpha,\gamma))}{\partial\gamma\partial\alpha} = \frac{(12q_H^3 + 19q_H^2q_L - 13q_Hq_L^2 + 2q_L^3)}{q_L(4q_H - q_L)^2} > 0. \tag{A15}$$

Moreover, we need to verify that $\gamma^{*E}(\alpha)$ is compatible with the assumptions of our model (see in particular Lemma 1 *bis*):

[2] The second order derivative of $SW^{*E}(\alpha,\gamma)$ is independent of α, as it can be easily demonstrated.

[3] Following Topkis (1978), a function is supermodular when cross-partial derivatives between each pair of variables are positive. This formalizes the notion of complementarity.

Lemma 2 bis *Under Assumption 2,* $\gamma^{*E}(\alpha) \in [0, \overline{\gamma}^{E}(\alpha))$ *iff* $e \in [\max\{0, \underline{e}^{\gamma E}(\alpha)\}, \overline{e}^{\gamma E}(\alpha))$.

Proof. First of all, notice that $\gamma^{*E}(\alpha) \geq 0 \iff e \geq \underline{e}^{\gamma E}(\alpha)$, where $\underline{e}^{\gamma E}(\alpha) = \frac{q_L(12cq_Hq_L + 8\Theta q_H^3 - 20cq_H^2 - 2cq_L^2 + 3\Theta q_H q_L^2 - 11\Theta q_H^2 q_L) + \alpha\Phi}{2q_H^2(4q_H - q_L)}$. However, $\underline{e}^{\gamma E}(\alpha) \geq 0 \iff \Theta \leq \widetilde{\Theta}(\alpha) = \frac{2c(10q_H^2 - 6q_Hq_L + q_L^2) - \alpha \cdot \Phi}{q_H(q_H - q_L)(8q_H - 3q_L)}$. Similarly to the baseline model, under Assumption 2 we find that $\widetilde{\Theta}(\alpha) > \underline{\Theta}^E(\alpha)$. Hence, $\gamma^{*E}(\alpha) \geq 0$ always for $\Theta \geq \widetilde{\Theta}(\alpha)$, while in $\Theta \in (\underline{\Theta}^E(\alpha), \widetilde{\Theta}(\alpha))$ we have that $\gamma^{*E}(\alpha) \geq 0$ only when $e \geq \underline{e}^{\gamma E}(\alpha)(> 0)$. In short, $\gamma^{*E}(\alpha) \geq 0$ when $e \geq \max\{0, \underline{e}^{\gamma E}(\alpha)\}$. Notice that also $\widetilde{\Theta}(\alpha)$ decreases in α. If Assumption 2 ceases to hold, then $\widetilde{\Theta}(\alpha) < \underline{\Theta}^E(\alpha)$, and $\widetilde{\Theta}(\alpha)$ becomes irrelevant. The second condition requires that $\gamma^{*E}(\alpha) < \overline{\gamma}^E(\alpha)$. This holds when $e < \overline{e}^{\gamma E}(\alpha) \equiv \frac{q_L\{(7q_H - 2q_L)(q_H - q_L)^2[c - \Theta(q_H - q_L)] + [c + \Theta s(q_H - q_L) - 2\alpha s q_H](4q_H - q_L)q_L\}}{4q_H^3(q_H - q_L)}$.

■

In order to complete this section, Remark 1 *bis* parallels Remark 1 from the baseline model:

Remark 1 bis *The government optimally sets: (i)* $\gamma = 0$ *when* $e < \max\{0, \underline{e}^{\gamma E}(\alpha)\}$; *(ii)* $\gamma = \gamma^{*E}(\alpha)$ *when* $e \in [\max\{0, \underline{e}^{\gamma E}(\alpha)\}, \overline{e}^{\gamma E}(\alpha))$; *(iii)* $\gamma = \overline{\gamma}^E$ *when* $e \geq \overline{e}^{\gamma E}(\alpha)$.

The complete expressions for the social welfare in the three cases reported in Remark 1 are extremely long and therefore we decided to omit them from the main text. They are, however, available upon request.

3.1.2 Taxing the polluting good

Next, we analyze the traditional taxation instrument, assuming that consumers are endowed with an initial level of pro-environmental behaviors represented by $\alpha > 0$. Based on the previous discussion, a consumer of type $\theta \in [0, \Theta]$ has thus the following utility:

$$U(\theta) = \begin{cases} \theta q_H - p_H + \alpha(q_H - q_L), \text{ if she buys the high (green) quality good,} \\ \theta q_L - p_L - \alpha(q_H - q_L), \text{ if she buys the low (brown) quality good,} \\ 0, \text{ if she refrains from buying.} \end{cases}$$

The consumer indifferent between buying the brown quality good and not buying at all is:

$$\theta_L = \frac{p_L + \alpha(q_H - q_L)}{q_L}.$$

The consumer indifferent between buying the brown quality good and the green quality good is:

$$\theta_H = \frac{p_H - p_L - 2\alpha\,(q_H - q_L)}{q_H - q_L}.$$

Demands are given by $x_L = \theta_H - \theta_L$ and $x_H = \Theta - \theta_H$. Again, we assume that producing the green good implies a higher cost than the brown good ($c > 0$). Nonetheless, the polluting good is now subject to a per-unit tax t. Profit functions are therefore $\pi_L = (p_L - t)\,x_L$ and $\pi_H = (p_H - c)\,x_H$. Price competition yields:

$$\begin{aligned} p_L^{**E}(\alpha) &= \frac{cq_L + 2tq_H + q_L\Theta\,(q_H - q_L) - 2\alpha q_H\,(q_H - q_L)}{4q_H - q_L}, \\ p_H^{**E}(\alpha) &= \frac{q_H\,[2c + t + 2\Theta\,(q_H - q_L)] + \alpha\,(3q_H - q_L)\,(q_H - q_L)}{4q_H - q_L}. \end{aligned}$$

Let us now define:

$$\bar{t}^E(\alpha) = \overline{\gamma}^E(\alpha) \equiv \frac{q_L\,[c + (\Theta - 2\alpha q_H)\,(q_H - q_L)]}{2q_H\,(q_H - q_L)}.$$

Lemma 3 bis *Provided Assumption 2 holds, both firms stay in the market iff* $\Theta > \underline{\Theta}^E(\alpha)$ *and* $t < \bar{t}^E(\alpha)$.

Proof. First of all:

$$\begin{aligned} p_L^{**E}(\alpha) - t &\geq 0 \iff \frac{q_L\,[c + \Theta(q_H - q_L)] - 2\alpha q_H(q_H - q_L)}{(2q_H - q_L)} \equiv \bar{t}^E(\alpha), \\ p_H^{**E}(\alpha) - c &\geq 0 \iff t > \frac{c(2q_H - q_L) - (q_H - q_L)[\alpha(3q_H - q_L) + 2\Theta q_H]}{q_H} \equiv \underline{t}^E(\alpha) \\ p_H^{**E}(\alpha) - p_L^{**E}(\alpha) &> 0 \iff t < \frac{c\,(2q_H - q_L) + (q_H - q_L)[q_H(5\alpha + 2\Theta) - q_L(\alpha + \Theta)]}{q_H} \equiv \widehat{t}(\alpha). \end{aligned}$$

Moreover, we have to guarantee that at equilibrium prices $0 < \theta_L^{**E}(\alpha) < \theta_H^{**E}(\alpha) < \Theta$:

$$\begin{aligned} \theta_L^{**E}(\alpha) &= \frac{cq_L + (q_H - q_L)\,[\alpha(2q_H - q_L) + \Theta q_L)] + 2tq_H}{q_L\,(4q_H - q_L)} > 0, \\ \theta_H^{**E}(\alpha) - \theta_L^{**E}(\alpha) &= \frac{q_H\{q_L[c + \Theta\,(q_H - q_L)] - 2\alpha q_H\,(q_H - q_L) - t\,(2q_H - q_L)\}}{(4q_H - q_L)\,(q_H - q_L)\,q_L} > 0 \\ &\iff t < \bar{t}^E(\alpha), \\ \theta_H^{**E}(\alpha) - \Theta &= \frac{c(2q_H - q_L) - (q_H - q_L)\,[\alpha(3q_H - q_L) + 2\Theta q_H] - tq_H}{(4q_H - q_L)\,(q_H - q_L)} < 0 \\ &\iff t > \underline{t}^E(\alpha). \end{aligned}$$

However, it is easy to show that $\underline{t}^E(\alpha) < 0 \iff \Theta > \underline{\Theta}^E(\alpha)$. Moreover, in the relevant interval region it also holds that $\underline{t}^E(\alpha) > \widehat{t}(\alpha)$. In addition, Assumption 2 guarantees that $\underline{\Theta}^E(\alpha) > 0$ and that $\bar{t}^E(\alpha) > 0$. It follows that the conditions that must simultaneously hold in order to guarantee that both firms are active in the market are $t < \bar{t}^E(\alpha)$ and $\Theta > \underline{\Theta}^E(\alpha)$. ■

We assume that the conditions reported in Lemma 3 *bis* hold throughout the paper. Remember that the precise value of $\underline{\Theta}^E(\alpha)$ is reported in (A12). Equilibrium demands/outputs $x_L^{**E}(\alpha)$ and $x_H^{**E}(\alpha)$ are not reported but they are similar to equivalent expressions in the baseline case. We also confirm that $\partial x_L^{**E}(\alpha)/\partial t < 0$, $\partial x_H^{**E}(\alpha)/\partial t > 0$, and $\partial(x_H^{**E}(\alpha) + x_L^{**E}(\alpha))/\partial t < 0$. Equilibrium profits and social welfare can still be expressed in the compact form:

$$\pi_L^{**E}(\alpha) = \frac{q_L}{q_H}(q_H - q_L)\,[x_L^{**E}(\alpha)]^2, \tag{A16}$$

$$\pi_H^{**E}(\alpha) = (q_H - q_L)\,[x_H^{**E}(\alpha)]^2, \tag{A17}$$

$$SW^{*E}(\alpha, t) = \pi_L^{**E}(\alpha) + \pi_H^{**E}(\alpha) + CS_L^{**E}(\alpha) + CS_H^{**E}(\alpha) - e \cdot x_L^{**E}(\alpha) + t \cdot x_L^{**E}(\alpha),$$

where the precise expressions of (A16)-(A17) as well as that of $SW^{*E}(\alpha, t)$ are omitted for brevity. Algebraic calculations show that the social welfare function is still concave in t. The optimal tax rate can therefore be computed:

$$\begin{aligned} t^{*E}(\alpha) &= \frac{e\,(4q_H - q_L)\,(2q_H - q_L) + q_L[\Theta q_L(q_H - q_L) - 2c\,(2q_H - q_L)] + \alpha \cdot \Psi}{q_H\,(4q_H - 3q_L)}, \\ \Psi &= (q_H - q_L)(4q_H^2 + q_H q_L - q_L^2). \end{aligned}$$

Notice that also $t^{*E}(\alpha)$ is increasing in α. Also in this case it is immediate to demonstrate that t is complementary with α:

$$\frac{\partial^2(SW^{**E}(\alpha, t))}{\partial\alpha\partial t} = \frac{\partial^2(SW^{**E}(\alpha, t))}{\partial t\partial\alpha} = \frac{(4q_H^2 + q_H q_L - q_L^2)}{q_L\,(4q_H - q_L)^2} > 0. \tag{A18}$$

However, the intensity of the complementarity effect is different. By comparing (A18) with (A15), one can easily find that

$$\frac{\partial^2(SW^{*E}(\alpha, \gamma))}{\partial\gamma\partial\alpha} > \frac{\partial^2(SW^{**E}(\alpha, t))}{\partial t\partial\alpha}. \tag{A19}$$

The intuition is simple. The presence of an initial level of *personal norms* that favor the green good at the expense of the brown one obviously goes in the same direction as the two policy instruments, as it restrains the consumption of the polluting product. However, α is more directly connected to γ, given that these variables reinforce each other in shaping relative preferences. The complementarity effect is therefore stronger when the government adopts the campaign instead of the tax.

Taking into account that $t^{*E}(\alpha)$ has to be non-negative, and that the conditions from Lemma 3 *bis* have to be satisfied:

Lemma 4 bis $t^{*E}(\alpha) \in [0, \bar{t}^E(\alpha))$ *when* $e \in [\max\{0, \underline{e}^{tE}(\alpha)\}, \bar{e}^{tE}(\alpha))$.

Proof. Firstly, $t^{*E} \geq 0$ iff $e \geq \underline{e}^{tE}(\alpha) = \dfrac{q_L[2c(2q_H - q_L) - \Theta q_L(q_H - q_L)] - \alpha \cdot \Psi}{(4q_H - q_L)(2q_H - q_L)}$. However, $\underline{e}^{tE}(\alpha) \geq 0 \iff \Theta \leq \dfrac{2c(2q_H - q_L) - \alpha \cdot \Psi}{q_L(q_H - q_L)} \equiv \widehat{\Theta}(\alpha)$. Similarly to what we did in the Proof of Lemma 2 *bis*, in order to avoid unnecessary notational complications, we simply write that $t^{*E}(\alpha) \geq 0$ when $e \geq \max\{0, \underline{e}^{tE}(\alpha)\}$. Secondly, notice that $t^{*E}(\alpha) < \bar{t}^E(\alpha)$ iff $e < \bar{e}^{tE}(\alpha) = \dfrac{c(2q_H - q_L) + 2\Theta(q_H - q_L)^2 - \alpha \cdot \Psi}{(2q_H - q_L)}$, with $\bar{e}^{tE}(\alpha) > \underline{e}^{tE}(\alpha)$, as expected. Finally, notice that $\widehat{\Theta}(\alpha)$ is decreasing in α, as well as $\bar{t}^E(\alpha)$ and threshold values $\underline{e}^{tE}(\alpha)$ and $\bar{e}^{tE}(\alpha)$. However, Assumption 2 guarantees that $\widehat{\Theta}(\alpha) > \underline{\Theta}^E(\alpha) > 0$. ■

We summarize the optimal tax policy in the following remark:

Remark 2 bis *The government optimally sets: (i)* $t = 0$ *when* $e < \max\{0, \underline{e}^{tE}(\alpha)\}$; *(ii)* $t = t^{*E}(\alpha)$ *when* $e \in [\max\{0, \underline{e}^{tE}(\alpha)\}, \bar{e}^{tE}(\alpha))$; *(iii)* $t = \bar{t}^E(\alpha)$ *when* $e \geq \bar{e}^{tE}(\alpha)$.

Depending on the emission level, we find therefore three different expressions for the social welfare function. They are omitted for brevity, but they are available upon request.

3.1.3 Comparing the two instruments under environmental qualities

Following (A19), we can already anticipate that, compared to the baseline model, the campaign will perform better than the taxation instrument. However, in order to have a precise comparison of the social efficiency of both instruments in presence of $\alpha > 0$, we replicate the analysis carried out in the main text. Let

$$\Theta_1(\alpha) = \frac{2cq_L(28q_H^4 - 53q_H^3 q_L + 36q_H^2 q_L^2 - 10q_H q_L^3 + q_L^4) - \alpha\Upsilon}{q_H\,(4q_H - 3q_L)\,(q_H - q_L)\,(10q_H^2 - 6q_H q_L + q_L^2)},$$
$$\Upsilon = (q_H - q_L)\,(16q_H^5 + 44q_H^4 q_L - 110q_H^3 q_L^2 + 77q_H^2 q_L^3 - 21q_H q_L^4 + 2q_L^5);$$
$$\Theta_2(\alpha) = \frac{2c\,(2q_H - q_L)^2 - \alpha\,(q_H - q_L)\,(4q_H^3 + 7q_H^2 q_L - 9q_H q_L^2 + 2q_L^3)}{q_H\,(4q_H - 3q_L)\,(8q_H - 3q_L)},$$

where $\underline{\Theta}^E(\alpha) < \Theta_1(\alpha) < \Theta_2(\alpha)$ under Assumption 2. Considering values of Θ such that $\Theta > \underline{\Theta}^E(\alpha)$, the following proposition replicates and extends the results that we outlined in the baseline model for environmental qualities.

Proposition 1 bis *Assume that consumers are environmentally concerned. The social welfare preferences are such that:*

(i) *when $\Theta \leq \Theta_1(\alpha)$, the tax instrument is socially more efficient than the environmental campaign.*

(ii) *When $\Theta \in (\Theta_1(\alpha), \Theta_2(\alpha))$, the tax instrument prevails as long as the cost of the campaign is sufficiently high. For a relatively low cost of the campaign, taxation dominates the campaign only when both the quality ratio q_H/q_L and the pollution level are not excessive. For each level of the quality ratio, there exists now a threshold level for the polluting emission e above which the environmental campaign is preferred to the taxation instrument. The higher the quality ratio, the lower the level of such threshold level of e.*

(iii) *When $\Theta \geq \Theta_2(\alpha)$, the environmental campaign is socially more efficient than the tax instrument, unless both the cost of the campaign and the emission levels are sufficiently high. However, the impact of α is to increase the interval region where the campaign is selected.*

Proof. We limit our attention to internal solutions ($s > \underline{s}$) and compare the different e-thresholds values. For each case, we perform welfare comparisons.[4]

[4] As in the baseline model, we omit the precise expressions for the welfare differences for the sake of brevity. They can be obtained upon request, as can the analytical solutions and many numerical simulations which confirm our results.

(i) When $\Theta \in \left(\underline{\Theta}^{E}(\alpha), \Theta_1(\alpha)\right)$, the ranking is as follows:

$$\underline{e}^{tE}(\alpha) < \overline{e}^{tE}(\alpha) < \underline{e}^{\gamma E}(\alpha) < \overline{e}^{\gamma E}(\alpha).$$

For each subinterval we compare the relevant social welfare:

1. for $e \in \left(\underline{e}^{tE}(\alpha), \overline{e}^{tE}(\alpha)\right)$, $SW(\alpha)|_{\gamma=0} - SW(\alpha)|_{t=t^{*E}} < 0$;

2. for $e \in \left(\overline{e}^{tE}(\alpha), \underline{e}^{\gamma E}(\alpha)\right)$, $SW(\alpha)|_{\gamma=0} - SW(\alpha)|_{t=\bar{t}^{E}} < 0$;

3. for $e \in \left(\underline{e}^{\gamma E}(\alpha), \overline{e}^{\gamma E}(\alpha)\right)$, $SW(\alpha)|_{\gamma=\gamma^{*E}} - SW(\alpha)|_{t=\bar{t}^{E}} < 0$;

4. for $e > \overline{e}^{\gamma E}(\alpha)$, $SW(\alpha)|_{\gamma=\overline{\gamma}^{E}} - SW(\alpha)|_{t=\bar{t}^{E}} < 0$.

We then confirm that for relatively low values of Θ, the taxation instrument is always preferred.

(ii) When $\Theta \in (\Theta_1(\alpha), \Theta_2(\alpha))$, we have two subcases:

$$\begin{aligned}
(a)\ \underline{e}^{tE}(\alpha) &< \underline{e}^{\gamma E}(\alpha) < \overline{e}^{\gamma E}(\alpha) < \overline{e}^{tE}(\alpha) \text{ when } s \in (\underline{s}, \widetilde{s}(\alpha)), \\
(b)\ \underline{e}^{tE}(\alpha) &< \underline{e}^{\gamma E}(\alpha) < \overline{e}^{tE}(\alpha) < \overline{e}^{\gamma E}(\alpha) \text{ when } s > \widetilde{s}(\alpha),
\end{aligned}$$

where

$$\begin{aligned}
\widetilde{s}(\alpha) &= \frac{(q_H - q_L)\left[cq_H\Omega + \Theta(q_H - q_L)(3q_H - q_L)\digamma + \alpha(16q_H^6 - 20q_H^5 q_L + 4q_H^3 q_L^3)\right]}{q_L(4q_H - q_L)(2q_H - q_L)^2\left[\Theta(q_L - 2q_H)(q_L - q_H) - cq_H\right]}, \\
\Omega &= 97q_H^3 q_L - 72q_H^4 - 40q_H^2 q_L^2 + 2q_H q_L^3 + q_L^4; \\
\digamma &= 8q_H^4 + 12q_H^3 q_L - 25q_H^2 q_L^2 + 11q_H q_L^3 - 2q_L^4.
\end{aligned}$$

As in the baseline model, the subcase $s \in (\underline{s}, \widetilde{s}(\alpha))$ is the most interesting, as we find that:

1. for $e \in (\underline{e}^{tE}(\alpha), \underline{e}^{\gamma E}(\alpha))$, $SW(\alpha)|_{\gamma=0} - SW(\alpha)|_{t=t^{*E}} < 0$;

2. for $e \in (\underline{e}^{\gamma E}(\alpha), \overline{e}^{\gamma E}(\alpha))$, $SW(\alpha)|_{\gamma=\gamma^{*E}} - SW(\alpha)|_{t=t^{*E}} < 0$ only when the q_H/q_L is not excessive and e is sufficiently low. For *every value of* q_H/q_L, there now exists a threshold value of e above which the environmental campaign is more efficient than

the taxation instrument $\left(SW(\alpha)|_{\gamma=\gamma^{*E}} - SW(\alpha)|_{t=t^{*E}} > 0\right)$.[5] This represents one of the most important differences with respect to the baseline model. The higher the quality ratio and the higher the initial value of relative preference, the lower the level of such threshold value, meaning that the area where the campaign prevails is increasing in both q_H/q_L and α.

3. For $e \in (\overline{e}^{\gamma E}(\alpha), \overline{e}^{tE}(\alpha))$, we compare $SW(\alpha)|_{\gamma=\overline{\gamma}^E}$ vs. $SW(\alpha)|_{t=t^{*E}}$, and find similar results as in the previous interval region. However, for sufficiently high levels of q_H/q_L, the campaign always prevails.

4. For $e > \overline{e}^{tE}(\alpha)$, we compare $SW(\alpha)|_{\gamma=\overline{\gamma}^E}$ with $SW(\alpha)|_{t=\overline{t}^E}$ and find that the campaign always prevails.

Conversely, in the second subcase ($s > \widetilde{s}(\alpha)$) the taxation instrument always prevails, as the cost for activating the campaign is now higher. We omit all the different subcases for brevity. However, notice that $\widetilde{s}(\alpha)$ is increasing in α. This implies that the presence of such an initial degree of relative preferences reduces the area where taxation is preferred.

(iii) When $\Theta \in (\Theta_2(\alpha), \widehat{\Theta}(\alpha))$, with $\widehat{\Theta}(\alpha) = \dfrac{2c\,(2q_H - q_L) - \alpha\,(q_H - q_L)\,(4q_H^2 + q_H q_L - q_L^2))}{(q_H - q_L)\,q_L}$, we have three subcases:

$$
\begin{aligned}
(a)\ \underline{e}^{\gamma E}(\alpha) &< \overline{e}^{\gamma E}(\alpha) < \underline{e}^{tE}(\alpha) < \overline{e}^{tE}(\alpha) \text{ when } s \in (\underline{s}, s_1(\alpha)),\\
(b)\ \underline{e}^{\gamma E}(\alpha) &< \underline{e}^{tE}(\alpha) < \overline{e}^{\gamma E}(\alpha) < \overline{e}^{tE}(\alpha) \text{ when } s \in (s_1(\alpha), \widetilde{s}(\alpha)),\\
(c)\ \underline{e}^{\gamma E}(\alpha) &< \underline{e}^{tE}(\alpha) < \overline{e}^{tE}(\alpha) < \overline{e}^{\gamma E}(\alpha) \text{ when } s > \widetilde{s}.
\end{aligned}
$$

The first two subcases can be explained together, as they provide the same result. For this reason, the precise value of $s_1(\alpha)$ is not reported. Consider for example subcase (a), where:

1. for $e \in (\underline{e}^{\gamma E}(\alpha), \overline{e}^{\gamma E}(\alpha))$, $SW(\alpha)|_{\gamma=\gamma^{*E}} > SW(\alpha)|_{t=0}$, as it can be immediately ascertained;

2. for $e \in (\overline{e}^{\gamma E}(\alpha), \underline{e}^{tE}(\alpha))$, $SW(\alpha)|_{\gamma=\overline{\gamma}^E} > SW(\alpha)|_{t=0}$, and also this result is obvious;

[5] The precise expression of such e-thresold value is extremely long and it is therefore omitted for brevity.

3. for $e \in (\underline{e}^{tE}(\alpha), \overline{e}^{tE}(\alpha))$, we evaluate $SW(\alpha)|_{\gamma=\overline{\gamma}^E}$ *vis à vis* $SW(\alpha)|_{t=t^{*E}}$. Algebraic calculations confirm that $SW(\alpha)|_{\gamma=\overline{\gamma}^E} > SW(\alpha)|_{t=t^{*E}}$, and therefore the campaign is preferred.

4. Finally, for $e > \overline{e}^{tE}(\alpha)$, we find that $SW(\alpha)|_{\gamma=\overline{\gamma}^E} > SW(\alpha)|_{t=\overline{t}^E}$.

In subcase (b), the only difference is that the taxation instrument can be adopted for a higher interval region of parameter e, given that $\underline{e}^{tE}(\alpha) < \overline{e}^{\gamma E}(\alpha)$. However, similar results as those for subcase (a) can be obtained, given that the cost for the campaign is still relatively low.

As in the baseline model, things change in subcase (c). We find that:

1. for $e \in (\underline{e}^{\gamma E}(\alpha), \underline{e}^{tE}(\alpha))$, $SW(\alpha)|_{\gamma=\gamma^{*E}} > SW(\alpha)|_{t=0}$, hence the (relatively more costly) campaign still prevails when the environmental damage is perceived as limited.

2. For $e \in (\underline{e}^{tE}(\alpha), \overline{e}^{tE}(\alpha))$, we evaluate $SW(\alpha)|_{\gamma=\gamma^{*E}}$ *vs* $SW(\alpha)|_{t=t^{*E}}$. We find that the campaign is more efficient only when the quality ratio q_H/q_L is sufficiently high and its cost is not too excessive; on the contrary, when q_H/q_L decreases (meaning that the environmental quality of the brown good is not very different from that of the green good), there exists a threshold value of e above which taxation is to be preferred, when s is sufficiently high. Such a value is, however, smaller than in the baseline model, thus confirming that the presence of α reduces the interval region where taxation prevails.

3. For $e \in (\overline{e}^{tE}(\alpha), \overline{e}^{\gamma E}(\alpha))$, we compare $SW(\alpha)|_{\gamma=\gamma^{*E}}$ with $SW(\alpha)|_{t=\overline{t}^E}$, and find the same results as in the previous subinterval.

4. Finally, for $e(\alpha) > \overline{e}^{\gamma E}(\alpha)$, we find that $SW(\alpha)|_{\gamma=\overline{\gamma}^E} > SW(\alpha)|_{t=\overline{t}^E}$ when the quality ratio is high and the cost of the campaign does not overcome a certain limit. Otherwise taxation is more efficient from the welfare standpoint.

(*iii bis*) When $\Theta > \widehat{\Theta}(\alpha)$, then $\underline{e}^{\gamma E}(\alpha) < 0$ and $\underline{e}^{tE}(\alpha) < 0$. Therefore:

$$(a)\ \max\{0, \overline{e}^{\gamma E}(\alpha)\} < \overline{e}^{tE} \text{ when } s \in (\underline{s}, \widetilde{s}(\alpha)),$$
$$(b)\ \overline{e}^{tE}(\alpha) < \overline{e}^{\gamma E}(\alpha) \text{ when } s > \widetilde{s}(\alpha).$$

In the first subcase, the campaign is always more efficient than taxation. We find that:

1. for $e \in (\max\{0, \overline{e}^{\gamma E}(\alpha)\}, \overline{e}^{\gamma E}(\alpha))$, algebraic calculations show that $SW(\alpha)|_{\gamma=\overline{\gamma}^E} > SW(\alpha)|_{t=t^{*E}}$;

2. for $e > \overline{e}^{\gamma E}(\alpha)$, $SW(\alpha)|_{\gamma=\overline{\gamma}^E} > SW(\alpha)|_{t=\overline{t}^E}$.

In the second subcase, we find that:

1. for $e \in (0, \overline{e}^{tE})$, we compare $SW(\alpha)|_{\gamma=\gamma^{*E}}$ with $SW(\alpha)|_{t=t^{*E}}$. The campaign prevails when q_H/q_L is sufficiently high and s is not too high. When q_H/q_L decreases, there exists a threshold value of e above which taxation is preferred, provided s is high enough. Such a threshold value of e increases in α, thus confirming the main intuition behind this robustness check.

2. For $e \in (\overline{e}^{tE}, \overline{e}^{\gamma E})$, we compare $SW(\alpha)|_{\gamma=\gamma^{*E}}$ with $SW(\alpha)|_{t=\overline{t}^E}$ and find the same results as in the previous subinterval.

3. Finally, for $e > \overline{e}^{\gamma E}$, we evaluate $SW(\alpha)|_{\gamma=\overline{\gamma}^E}$ *vs.* $SW(\alpha)|_{t=\overline{t}^E}$ and replicate the results of $e \in (0, \overline{e}^{\gamma E})$ with the only difference that, when q_H/q_L is very low and s is high, then taxation prevails for each value of $e > \overline{e}^{\gamma E}$.

■

The results outlined above confirm the validity of Proposition 1 in the baseline model. The preconditions for the campaign to perform better than taxation in terms of total welfare are a moderate cost s and a sufficiently high Θ, which measures the average evaluation for the environmental quality. The higher the value of Θ, the higher the *green expansion effect* induced by the campaign. For this reason, when $\Theta \leq \Theta_1(\alpha)$, taxation always prevails, while when $\Theta \geq \Theta_2(\alpha)$, it is the campaign that performs better in terms of total welfare, provided its cost is not excessive. When $\Theta \in (\Theta_1(\alpha), \Theta_2(\alpha))$, the campaign is more efficient than the tax instrument for sufficiently high levels of e. However, differently from the baseline model, this is valid for *each* level of q_H/q_L when $\alpha > 0$. Of course, the higher the quality ratio q_H/q_L, the lower the level of such threshold level of e, and in turn the larger the interval region in which the campaign prevails. We confirm therefore that α acts in reinforcing the efficiency of the environmental campaign as compared to the pollution tax.

Finally, notice that all threshold values are affected by the presence of an initial level α of relative preferences. As we showed above, the higher α is, the lower are the *threshold values* of Θ, and the higher are those of s. This implies that, *ceteris paribus*, an increase in α reduces the interval regions where both Θ is so low that the impact of the campaign is limited and s is so high that the campaign is too costly. Hence, when α progressively increases and Assumption 1 ceases to hold, we progressively eliminate cases *(i)* and *(ii)* of Proposition 1 *bis*. We are left with case *(iii)*, in which the campaign prevails for each value of Θ (given that $\Theta_2(\alpha) < 0$) if its cost is not prohibitive. However, given that $\widetilde{s}(\alpha)$ is also increasing in α, the interval region where the campaign is relatively affordable tend to enlarge. This holds as long as

$$\alpha < \frac{c + bq_L\left(q_H - q_L\right)}{2q_H\left(q_H + q_L\right)} \equiv \widehat{\alpha},$$

where $\widehat{\alpha} > \overline{\alpha}^E$. Condition $\alpha \leq \widehat{\alpha}$ guarantees in fact that both $\overline{\gamma}^E(\alpha)$ and $\overline{t}^E(\alpha)$ are non-negative. When $\alpha > \widehat{\alpha}$, our analysis becomes immaterial as both policy instruments cannot be adopted at the duopoly market at equilibrium.

3.2 Hedonic qualities

In the second scenario consumers value the hedonic quality of the goods above all else. Hence, good H produced by firm H is the high quality but brown good, while good L produced by firm L is the low quality but green good. As before, there is a continuum of consumers indexed by θ which is uniformly distributed in the interval $[0, \Theta]$, where θ now measures consumers' valuation for the hedonic quality. Pollution generates an environmental damage $D = e \cdot x_H$. We solve a two-stage game in which the policymaker decides whether to tax polluting firms or to support an environmental campaign, having anticipated the price game played by firms in the second stage.

Again, the unique difference with respect to the baseline model consists in the presence of $\alpha > 0$, which does not depend on the policy instrument adopted by the government. Similarly to the previous section, the following holds:

Assumption 3 *The initial level of relative preferences is sufficiently low,* i.e., $\alpha < \dfrac{c}{(q_H + q_L)} = \overline{\alpha}^H$.

When $\alpha \geq \overline{\alpha}^H$, then two situations may occur. For intermediate values of α, the interval regions where the analysis can be carried out tend to reduce, but the main results still hold. For high values of α, we also confirm that both policy instruments cannot be implemented if the government wants to preserve a market duopoly at equilibrium.

3.2.1 Supporting the environmental campaign

Introducing $\alpha > 0$ in the baseline model implies that the utility for a consumer of type $\theta \in [0, \Theta]$ is as follows:

$$U(\theta) = \begin{cases} \theta q_H - p_H - (\alpha + \gamma)(q_H - q_L), \text{ if she buys the high quality good,} \\ \theta q_L - p_L + (\alpha + \gamma)(q_H - q_L), \text{ if she buys the low quality good,} \\ 0, \text{ if she refrains from buying.} \end{cases}$$

The hedonic qualities of the two goods are indicated by $q_H > q_L$. The consumer indifferent between buying the low quality good and not buying at all is:

$$\theta_L = \frac{p_L - (\alpha + \gamma)(q_H - q_L)}{q_L}. \tag{A20}$$

The consumer indifferent between buying the low and the high quality good is:

$$\theta_H = \frac{p_H - p_L + 2(\alpha + \gamma)(q_H - q_L)}{q_H - q_L}. \tag{A21}$$

Demands are formally written as: $x_L = \theta_H - \theta_L$ and $x_H = \Theta - \theta_H$. Obviously, the market share of the green firm L increases in α, which amplifies the *green expansion effect* already outlined in the baseline model. Under Assumption 3 the market remains uncovered. Profit functions are again given by $\pi_H = x_H \cdot p_H$ and $\pi_L = (p_L - c)\, x_L$.

Equilibrium prices can be easily obtained:

$$p_L^{*H}(\alpha) = \frac{2cq_H + (q_H - q_L)\left[2(\alpha+\gamma)q_H + \Theta q_L\right]}{4q_H - q_L},$$
$$p_H^{*H}(\alpha) = \frac{cq_H + (q_H - q_L)\left[2\Theta q_H - (\alpha+\gamma)(3q_H - q_L)\right]}{4q_H - q_L},$$

where additional superscript H indicates hedonic preferences. It is immediate to notice that $p_L^{*H}(\alpha) > p_L^{*H}$ and $p_H^{*H}(\alpha) < p_H^{*H}$, given that the green good is now the low quality one. Let us define:

$$\underline{\Theta}^H(\alpha) \equiv \frac{c\left(2q_H - q_L\right) - 2\alpha q_H\left(q_H - q_L\right)}{q_L\left(q_H - q_L\right)},$$
$$\overline{\gamma}^H(\alpha) \equiv \min\left\{\frac{\Theta\left(2q_H - q_L\right)\left(q_H - q_L\right) - cq_H}{\left(5q_H - q_L\right)\left(q_H - q_L\right)}, \frac{2cq_H + \Theta\left(q_H - q_L\right)q_L}{\left(2q_H - q_L\right)\left(q_H - q_L\right)}\right\} - \alpha.$$

Lemma 5 bis *The market is uncovered and both firms are active in the market iff* $\Theta > \underline{\Theta}^H(\alpha)$ *and* $\gamma < \overline{\gamma}^H(\alpha)$.

Proof. Notice that $p_L^{*H}(\alpha) > 0$, while $p_H^{*H}(\alpha) > 0$ only when γ is sufficiently low. However, if $p_H^{*H}(\alpha) > p_L^{*H}(\alpha)$, then *a fortiori* also $p_H^{*H}(\alpha) > 0$. The first condition that we need to impose is then:

$$p_H^{*H}(\alpha) - p_L^{*H}(\alpha) > 0 \iff \gamma < \frac{\Theta\left(2q_H - q_L\right)\left(q_H - q_L\right) - cq_H - \alpha(5q_H^2 - 6q_Hq_L + q_L^2)}{\left(5q_H - q_L\right)\left(q_H - q_L\right)}.$$

This threshold value of γ decreases in α, as expected. However, Assumption 3 guarantees that it is positive, hence $\gamma < \overline{\gamma}^H(\alpha)$ can be met. Next, we verify that π_L is positive:

$$p_L^{*H}(\alpha) - c > 0 \iff \gamma > \frac{c\left(2q_H - q_L\right) - \left(q_H - q_L\right)\left(2\alpha q_H + \Theta q_L\right)}{2q_H\left(q_H - q_L\right)} \equiv \underline{\gamma}^H(\alpha),$$

and that $0 < \theta_L^*(\alpha) < \theta_H^*(\alpha) < \Theta$:[6]

$$\begin{aligned}
\theta_L^*(\alpha) &= \frac{2cq_H + (q_H - q_L)\left[\Theta q_L - \gamma(2q_H + q_L)\right] - \alpha(2q_H^2 - 3q_Hq_L + q_L^2)}{\left(4q_H - q_L\right)q_L} > 0 \\
&\iff \gamma < \frac{2cq_H + \Theta\left(q_H - q_L\right)q_L}{\left(2q_H - 2q_L\right)\left(q_H - q_L\right)} - \alpha, \\
\theta_H^*(\alpha) - \theta_L^*(\alpha) &= \frac{q_H\left\{c\left(q_L - 2q_H\right) + \left(q_H - q_L\right)\left[\Theta q_L + 2(\alpha+\gamma)q_H\right]\right\}}{\left(q_H - q_L\right)\left(4q_H - q_L\right)q_L} > 0 \\
&\iff \gamma > \underline{\gamma}^H(\alpha). \\
\theta_H^*(\alpha) - \Theta &= \frac{\left(q_H - q_L\right)\left[(\alpha+\gamma)(3q_H - q_L) - 2\Theta q_H\right] - cq_H}{\left(q_H - q_L\right)\left(4q_H - q_L\right)} < 0 \\
&\iff \gamma < \frac{cq_H - \left(q_H - q_L\right)\left[\alpha(3q_H - q_L) - 2\Theta q_H\right]}{\left(3q_H - q_L\right)\left(q_H - q_L\right)} \equiv \widehat{\gamma}(\alpha).
\end{aligned}$$

[6] As usual, in order to obtain $\theta_L^*(\alpha)$ and $\theta_H^*(\alpha)$, we plug $p_L^{*H}(\alpha)$ and $p_H^{*H}(\alpha)$ into (A20) and (A21).

Assuming $\Theta > \underline{\Theta}^H(\alpha)$: *(i)* $\underline{\gamma}^H(\alpha) < 0$, and therefore we can discard such condition; *(ii)* $\widehat{\gamma}(\alpha) > \max\left\{\frac{\Theta(2q_H - q_L)(q_H - q_L) - cq_H}{(5q_H - q_L)(q_H - q_L)} - \alpha, \frac{2cq_H + \Theta(q_H - q_L)q_L}{(2q_H - q_L)(q_H - q_L)}\right\}$. Moreover, as $\underline{\Theta}^H(\alpha) > 0$ under Assumption 2, the requirements that have to be satisfied for the market to be uncovered and for both firms to be active boil down to $\Theta > \underline{\Theta}^H(\alpha)$ and $\overline{\gamma}^H = \min\left\{\frac{\Theta(2q_H - q_L)(q_H - q_L) - cq_H}{(5q_H - q_L)(q_H - q_L)} - \alpha, \frac{2cq_H + \Theta(q_H - q_L)q_L}{(2q_H - q_L)(q_H - q_L)}\right\}$. The comparison between these two threshold values of γ depends on the quality ratio. However, we will demonstrate that the same qualitative results holds under both restrictions. ■

The conditions reported in Lemma 5 *bis* hold throughout the paper. Equilibrium demands $x_L^{*H}(\alpha)$ and $x_H^{*H}(\alpha)$ are not reported for the sake of brevity, and profits can be written again in the compact form:

$$\pi_L^{*H}(\alpha) = \frac{q_L}{q_H}(q_H - q_L)\left[x_L^{*H}(\alpha)\right]^2,\ \pi_H^{*H}(\alpha) = \frac{\left[x_H^{*H}(\alpha)\right]^2}{(q_H - q_L)}.$$

We also verify that $\partial(x_L^{*H}(\alpha) + x_H^{*H}(\alpha))/\partial\alpha > 0$, meaning that total output increases not only in γ, but also in α. This is due to analogous reasons as those reported in the baseline model. The social welfare function is written in a compact way as follows:

$$SW^{*H}(\alpha,\gamma) = \pi_L^{*H}(\alpha) + \pi_H^{*H}(\alpha) + CS_L^{*H}(\alpha) + CS_H^{*H}(\alpha) - e\cdot x_H^{*H}(\alpha) - s\frac{\gamma^2}{2},$$

where the precise expression for $CS_L^{*H}(\alpha)$ and $CS_H^{*H}(\alpha)$ are omitted for brevity. The social welfare function is concave in γ if and only if $s > \underline{s}$, as in the previous scenario. We compute the optimal γ level which maximizes social welfare:

$$\begin{aligned}
\gamma^{*H}(\alpha) &= \frac{q_H\left(cq_Hq_L - 12cq_H^2 - 3\Theta q_L^3 + cq_L^2 + 11\Theta q_Hq_L^2 - 8\Theta q_H^2q_L\right) + \Xi}{s\,q_L\left(q_L - 4q_H\right)^2 - \left(q_H - q_L\right)\left(12q_H^3 + 2q_L^3 - 13q_Hq_L^2 + 19q_H^2q_L\right)}, \\
\Xi &= e\,q_L\left(q_L - 4q_H\right)\left(q_L - 3q_H\right) + \alpha\left(q_H - q_L\right)\left(12q_H^3 + 19q_H^2q_L - 13q_Hq_L^2 + 2q_L^3q\right).
\end{aligned}$$

Notice that $\gamma^{*H}(\alpha)$ is increasing in α, as in the case of environmental qualities. In particular, taking cross-partial derivatives, we replicate the same result as in (A15). Hence,

$$\frac{\partial^2(SW^{*E}(\alpha,\gamma))}{\partial\alpha\partial\gamma} = \frac{\partial^2(SW^{*E}(\alpha,\gamma))}{\partial\gamma\partial\alpha} = \frac{\left(q_H - q_L\right)\left(12q_H^3 + 19q_H^2q_L - 13q_Hq_L^2 + 2q_L^3\right)}{q_L\left(4q_H - q_L\right)^2} > 0. \tag{A22}$$

We need now to impose conditions for $\gamma^{*H}(\alpha)$ to make economic sense. The following lemma replicates the conditions outlined in the baseline model:

Lemma 6 bis *Assume* $\overline{\gamma}^H(\alpha) = \dfrac{\Theta(2q_H - q_L)(q_H - q_L) - cq_H - \alpha(5q_H^2 - 6q_H q_L + q_L^2)}{(5q_H - q_L)(q_H - q_L)}$.

Then $\gamma^{*H}(\alpha) \in [0, \overline{\gamma}^H(\alpha))$ *when* $e \in [\underline{e}^{\gamma H}(\alpha), \overline{e}^{\gamma H}(\alpha))$.

Proof. First, $\gamma^{*H}(\alpha) \geq 0$ iff $e \geq \underline{e}^{\gamma H} = \dfrac{q_H [q_L \Theta(8q_H - 3q_L)(q_H - q_L)] - \Xi}{q_L(4q_H - q_L)(3q_H - q_L)}$.
Next, we have to find the condition for $\gamma^{*H}(\alpha) < \overline{\gamma}^H(\alpha)$. This holds when $e < \overline{e}^{\gamma H}(\alpha) =$
$\dfrac{(q_H - q_L)[cq_H(18q_H^2 + 5q_H q_L - 3q_L^2) - 2\Theta(q_H - q_L)^2(3q_H - q_L)(q_H + q_L)] - sq_L(4q_H - q_L)\cdot\Lambda}{q_L(3q_H - q_L)(q_H - q_L)(5q_H - q_L)}$,
where $\Lambda = cq_H + (q_H - q_L)[\alpha(5q_H - q_L) - \Theta(2q_H - q_L)]$. Similar conditions would have been obtained by using $\overline{\gamma}^H = \dfrac{2cq_H + \Theta(q_H - q_L)q_L}{(2q_H - q_L)(q_H - q_L)} - \alpha$, without changing the qualitative results of our paper. ■

It follows that:

Remark 3 bis *The government optimally sets: (i)* $\gamma = 0$ *when* $e < \underline{e}^{\gamma H}(\alpha)$*; (ii)* $\gamma = \gamma^{*H}(\alpha)$ *when* $e \in [\underline{e}^{\gamma H}(\alpha), \overline{e}^{\gamma H}(\alpha))$*; (iii)* $\gamma = \overline{\gamma}^H(\alpha)$ *when* $e \geqslant \overline{e}^{\gamma H}(\alpha)$.

The complete expressions for the social welfare in the three cases reported in Remark 3 bis are available upon request.

3.2.2 Taxing the polluting good

A consumer of type $\theta \in [0, \Theta]$ has the following utility:

$$U(\theta) = \begin{cases} \theta q_H - p_H - \alpha(q_H - q_L), \text{ if she buys the high quality good,} \\ \theta q_L - p_L + \alpha(q_H - q_L), \text{ if she buys the low quality good,} \\ 0, \text{ if she refrains from buying.} \end{cases}$$

The consumer who is indifferent between buying the low quality good and not buying at all is:

$$\theta_L = \frac{p_L - \alpha(q_H - q_L)}{q_L},$$

while the consumer indifferent between buying the low quality good and the high quality good is now given by:

$$\theta_H = \frac{p_H - p_L + 2\alpha(q_H - q_L)}{q_H - q_L}.$$

Profit functions in the presence of the taxation instrument are $\pi_L = (p_L - c)x_L$ and $\pi_H = (p_H - t)x_H$.

Equilibrium prices as a function of qualities are:

$$\begin{aligned}
p_L^{**H}(\alpha) &= \frac{(2\alpha q_H + \Theta q_L)(q_H - q_L) + 2cq_H + tq_L}{4q_H - q_L}, \\
p_H^{**H}(\alpha) &= \frac{q_H(c+2t) - 2(q_H - q_L)[\alpha(3q_H - q_L) - 2\Theta q_H]}{4q_H - q_L}.
\end{aligned}$$

Let us now define:

$$\bar{t}^H(\alpha) \equiv \frac{cq_H - (q_H - q_L)[\alpha(3q_H - q_L) - 2\Theta q_H]}{2q_H - q_L}.$$

Lemma 7 bis *Both firms are active in the market iff* $\Theta > \underline{\Theta}^H(\alpha)$ *and* $t < \bar{t}^H(\alpha)$.

Proof. We focus on the case in which the price of the high quality good is higher than that of the low quality good:

$$p_H^{**H}(\alpha) - p_L^{**H}(\alpha) \geqslant 0 \iff t > \frac{cq_H + (q_H - q_L)[\alpha(5q_H - q_L) - \Theta(2q_H - q_L)]}{2q_H - q_L} \equiv \widetilde{t}(\alpha).$$

This also guarantees that $p_H^{**H}(\alpha) > 0$, given that $p_L^{**H}(\alpha)$ is always positive. Moreover:

$$\begin{aligned}
p_L^{**H}(\alpha) - c &> 0 \iff t > \frac{c(2q_H - q_L) - (q_H - q_L)(2\alpha q_H + \Theta q_L)}{q_L} \equiv \underline{t}^H(\alpha) \ ; \\
p_H^{**H}(\alpha) - t &> 0 \iff t < \bar{t}^H(\alpha).
\end{aligned}$$

At equilibrium it must also hold that $0 < \theta_L^*(\alpha) < \theta_H^*(\alpha) < \Theta$:

$$\theta_L^{**}(\alpha) = \frac{2cq_H + tq_L + (q_H - q_L)[\Theta q_L - \alpha(2q_H - q_L)]}{(4q_H - q_L)\, q_L} > 0 \text{ under Assumption 2;}$$

$$\begin{aligned}
\theta_H^{**}(\alpha) - \theta_L^*(\alpha) &> 0 \iff t > \underline{t}^H(\alpha) \ ; \\
\theta_H^{**}(\alpha) - \Theta &< 0 \iff t < \bar{t}^H(\alpha).
\end{aligned}$$

It is easy again to demonstrate that $\underline{t}^H(\alpha) > \widetilde{t}(\alpha)$. Moreover, $\underline{t}^H(\alpha) < 0 \iff \Theta > \underline{\Theta}^H(\alpha)$, as in baseline model. Hence, we assume that $\Theta > \underline{\Theta}^H$ in order to eliminate the condition $t > \underline{t}^H(\alpha)$. It follows that conditions $\Theta > \underline{\Theta}^H(\alpha)$ and $t < \bar{t}^H(\alpha)$ are required to have both players active in an uncovered market, provided Assumption 3 holds. ■

Equilibrium demands/outputs $x_L^{**H}(\alpha)$ and $x_H^{**H}(\alpha)$ are omitted for brevity. We confirm that taxation always shrinks total output. Equilibrium profits are given by:

$$\begin{aligned}
\pi_L^{**H}(\alpha) &= \frac{q_L}{q_H}(q_H - q_L)[x_L^{**H}(\alpha)]^2, \\
\pi_H^{**H}(\alpha) &= (q_H - q_L)[x_H^{**H}(\alpha)]^2.
\end{aligned}$$

Social welfare writes in a compact form as:

$$SW^{**H}(\alpha,t)=\pi_L^{**H}(\alpha)+\pi_H^{**H}(\alpha)+CS_L^{**H}(\alpha)+CS_H^{**H}(\alpha)-e\cdot x_H^{**H}(\alpha)+t\cdot x_H^{**H}(\alpha).$$

The precise expression of $CS_L^{**H}(\alpha)+CS_H^{**H}(\alpha)$ is available upon request. Algebraic calculations confirm that $SW^{**H}(\alpha,t)$ is concave in t. The optimal tax rate is:

$$t^{*H}(\alpha)=\frac{1}{q_H\left(4q_H-3q_L\right)}\times$$

$$e(4q_H-q_L)\left(2q_H-q_L\right)-q_H\left[\Theta\left(4q_H-3q_L\right)\left(q_H-q_L\right)+2c\left(2q_H-q_L\right)\right]+\alpha(8q_H^2-5q_Hq_L+q_L^2).$$

Also in this case it is immediate to demonstrate that t is complementary with α:

$$\frac{\partial^2(SW^{**H}(\alpha,t))}{\partial\alpha\partial t}=\frac{\partial^2(SW^{**H}(\alpha,t))}{\partial t\partial\alpha}=\frac{(8q_H^2-q_Hq_L+q_L^2)}{q_L\left(4q_H-q_L\right)^2}>0. \tag{A23}$$

As before, we compare (A22) with (A23) and obtain that:

$$\frac{\partial^2(SW^{*H}(\alpha,\gamma))}{\partial\gamma\partial\alpha}>\frac{\partial^2(SW^{**H}(\alpha,t))}{\partial t\partial\alpha}. \tag{A24}$$

This confirms also in the hedonic case that the presence of α, although benefitting both policy instruments, contributes to a higher social efficiency of the campaign with respect to the tax. The following lemma states the conditions for t^{*H} to have economic meaning:

Lemma 8 bis $t^{*H}\in[0,\bar{t}^H(\alpha))$ *when* $e\in[\underline{e}^{tH}(\alpha),\bar{e}^{tH}(\alpha))$.

Proof. Two conditions have to simultaneously hold: *(i)* $t^{*H}(\alpha)\geqslant 0$ iff $e\geqslant\underline{e}^{tH}(\alpha)\equiv\frac{q_H\left[\Theta\left(4q_H-3q_L\right)\left(q_H-q_L\right)+2c\left(2q_H-q_L\right)\right]-\alpha(8q_H^3-13q_H^2q_L+6q_Hq_L^2-q_L^3)}{\left(4q_H-q_L\right)\left(2q_H-q_L\right)}$; *(ii)* $t^{*H}<\bar{t}^H$ iff $e<\bar{e}^{tH}(\alpha)\equiv\frac{q_H\left[\Theta\left(4q_H-3q_L\right)\left(q_H-q_L\right)+c\left(3q_H-2q_L\right)\right]-\alpha(7q_H^3-13q_H^2q_L+7q_Hq_L^2-q_L^3)}{\left(2q_H-q_L\right)^2}$.
■

Remark 4 bis *The government optimally sets: (i)* $t=0$ *when* $e<\underline{e}^{tH}(\alpha)$; *(ii)* $t=t^{*H}(\alpha)$ *when* $e\in[\underline{e}^{tH}(\alpha),\bar{e}^{tH}(\alpha))$; *(iii)* $t=\bar{t}^H(\alpha)$ *when* $e\geqslant\bar{e}^{tH}(\alpha)$.

The expressions for the social welfare for the three different values of t which appear in Remark 4 *bis* are available upon request.

3.2.3 Comparing the two instruments under hedonic qualities

As in the previous scenario, (A24) clearly indicates that the campaign benefits more than the taxation instrument from the presence of the initial level of pro-environmental behavior α. However, from the baseline model we know that taxation always prevails over the campaign when consumers are characterized by hedonic preferences. Factoring out the effects of the different parameters at stake, is it now possible to find interval regions in which the government would opt for the campaign? The following proposition answers to this question:

Proposition 2 bis *Assume that consumers are not environmentally concerned (hedonic qualities). Even in the presence of an initial level of pro-environmental behaviors $\alpha > 0$, the social welfare preferences are such that, if the cost of the campaign is sufficiently high ($s > \underline{s}$), then the tax instrument is again always preferred to the environmental campaign. However, the performance gap between the two instruments reduces as α increases.*

Proof. We consider $\overline{\gamma}^H(\alpha) = \dfrac{\Theta(2q_H - q_L)(q_H - q_L) - cq_H - \alpha(5q_H^2 - 6q_Hq_L + q_L^2)}{(5q_H - q_L)(q_H - q_L)}$. However, tedious numerical calculations show that similar results would hold even with $\overline{\gamma}^H(\alpha) = \dfrac{2cq_H + \Theta(q_H - q_L)q_L}{(2q_H - q_L)(q_H - q_L)} - \alpha$. When $s > \underline{s}$, by comparing the different e-thresholds, one can find the following cases:

(i) $\Theta \in (\underline{\Theta}^H(\alpha), \Theta_3(\alpha))$, $\Theta_3(\alpha) = \dfrac{c(3q_H - q_L)(16q_H^3 - 24q_H^2q_L + 11q_Hq_L^2 - q_L^3) - \alpha\Gamma}{q_H(q_H - q_L)q_L(16q_H^2 - 20q_Hq_L + 5q_L^2)}$, $\Gamma = (q_H - q_L)(48q_H^5 + 16q_H^4q_L - 97q_H^3q_L^2 + 71q_H^2q_L^3 - 20q_Hq_L^4 + 2q_L^5)$, the ranking is:

$$\underline{e}^{tH}(\alpha) < \overline{e}^{tH}(\alpha) < \underline{e}^{\gamma H}(\alpha) < \overline{e}^{\gamma H}(\alpha).$$

(ii) $\Theta > \Theta_3$ and $s \in (\underline{s}, \widetilde{s}(\alpha))$, with $\underline{s}$ defined in the baseline model (see expression 13) and $\widetilde{s}(\alpha)$ appearing in the Proof of Proposition 1 *bis*, the ranking is:

$$\underline{e}^{tH}(\alpha) < \underline{e}^{\gamma H}(\alpha) < \overline{e}^{\gamma H}(\alpha) < \overline{e}^{tH}(\alpha).$$

(iii) $\Theta > \Theta_3(\alpha)$ and $s > \widetilde{s}(\alpha)$, we have that: $\underline{e}^{tH}(\alpha) < \underline{e}^{\gamma H}(\alpha) < \overline{e}^{tH}(\alpha) < \overline{e}^{\gamma H}(\alpha)$.

Comparing the appropriate social welfare expressions, we can prove that, for any e, taxation always determines a higher welfare than the environmental campaign. For the sake of brevity, we do not replicate all the welfare comparisons, whose precise expressions are very long and are not reported in the text. They are available upon request along with several numerical simulations that confirm our results. We confirm that taxation always provides a higher level of social welfare than the campaign. However, we also demonstrate that, the higher the value of α, the lower the difference between the social welfare computed under taxation and that obtained with the campaign. ■

When the social welfare associated with the environmental campaign is concave ($s > \underline{s}$), the government always prefers the taxation instrument. The higher increase in performance of the campaign with respect to the taxation instrument is not sufficient to counterbalance the forces highlighted in the baseline model in support of the polluting tax when consumers show hedonic preferences. Notice, however, that we carried out our analysis under the condition represented in Assumption 3. Given that all threshold values of γ still decrease in α, we are therefore assuming that the impact of the campaign is even more limited than in the baseline model. If we consider progressively higher levels of α, then it becomes more likely to alter the equilibrium market structure when adopting the campaign instead of the tax. In fact, we obtain that:

$$\overline{\gamma}^{H}(\alpha) \geq 0 \Leftrightarrow \alpha \leq \min\left\{ \frac{\Theta(2q_H^2 - 3q_Hq_L + q_L^2) - cq_H}{(5q_H - q_L)(q_H - q_L)}, \frac{2cq_H + \Theta(q_H - q_L)q_L}{2q_H^2 - 3q_Hq_L + q_L^2} \right\},$$

$$\overline{t}^{H}(\alpha) \geq 0 \Leftrightarrow a \leq \frac{q_H[c + 2\Theta(q_H - q_L)]}{3q_H^2 - 4q_Hq_L + q_L^2},$$

where

$$\frac{q_H[c + 2\Theta(q_H - q_L)]}{3q_H^2 - 4q_Hq_L + q_L^2} \geq \max\left\{ \frac{\Theta(2q_H^2 - 3q_Hq_L + q_L^2) - cq_H}{(5q_H - q_L)(q_H - q_L)}, \frac{2cq_H + \Theta(q_H - q_L)q_L}{2q_H^2 - 3q_Hq_L + q_L^2} \right\}.$$

This means that, *ceteris paribus*, assuming that the government aims at the complete elimination of the pollution activity, then the campaign induces the brown firm to exit the market for relatively low levels of α with respect to the taxation instrument.[7]

[7] More precisely, considering progressively higher levels of α, when supporting the campaign instead of the tax it is more likely to reach a situation where the only producer active in the market is the green one.

3.3 Discussion

In this last section we extended the basic model by considering a campaign that does not activate pro-environmental behaviors, but it rather increases their weight in the utility function. Although algebraically more challenging, we demonstrated that the main results of our paper are still valid. In particular, when consumers value the environmental quality, then a campaign proved to be socially more efficient than the pollution tax for similar parameter values than those outlined in Proposition 1 in the baseline model. The only relevant difference consists in the fact that the interval region in which the campaign prevails is larger than in the baseline model, and it is increasing in α. As a consequence, for intermediate levels of Θ, we demonstrated that the campaign prevails for *each* quality ratio. Second, when consumers favor the hedonic quality, taxation still prevails as the best policy instrument in the entire relevant interval region. However, the efficiency gap between the two instruments is not as significant as in the baseline model. This confirms that adding an initial value of *moral consumption* that rewards the green producer to the detriment of the brown one increases the efficiency of the environmental campaign as compared to the taxation instrument.

It was also relatively easy to demonstrate that the degree of complementarity between α and γ is higher than that between α and t. Indeed, the presence of an initial level of *personal norms* that favors the green good at the expense of the brown one goes in the same direction as the two policy instruments, as it reduces the consumption of the polluting product. However, α is more directly connected to γ, given that these variables reinforce each other in shaping relative preferences. The complementarity effect is then stronger when the government adopts the campaign instead of the tax.

The main message of our contribution is therefore not only confirmed but also reinforced when we account for a campaign that does not modify the structure of consumer preferences, but it simply amplifies their pro-environmental component. In fact, as we pointed out above, compared to the baseline model, we can now prove that our main results are independent from the quality specification, although we confirm that an increase in the quality ratio tends to favor the campaign. As a consequence, our assumption of keeping qualities as exogenous does not represent a limitation for the modeling strategy adopted in this paper. Our model provides therefore valuable environmental policy indications, depending on consumer preferences towards the environment.

One final remark. In order to make our demonstration as simple as possible, we initially considered relatively low levels of α. Then, we discussed the implications of extending our analysis to progressively higher values of α. The reason is the following. One of the crucial points of our paper consists in the possibility for the government to adopt consumer-based policy instruments as an alternative to taxing polluting firms. In particular, consumers can be persuaded to switch from the brown to the green good by raising their awareness about the negative impact of pollution on the ecosystem in which they live. Environmental campaigns designed for this purpose are often costly. However, if consumers demonstrate an initial sensibility for such issues, already captured in our baseline model by considering an initial scenario in which they value the environmental quality of the products, then the campaign may turn out to be socially preferable to traditional tax instruments. If we start from relatively high values of α, then a comparison between two different policy instruments may become immaterial, given that the brown firm would be since the beginning extremely penalized at no cost to society. Taxing such a firm would result therefore in the creation of a "green" monopoly, which is not the aim of the present paper.

References

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