**Lemma 1** *The borrowing limit for a generation-$(t-1)$ agent, $\bar{b}_{t-1}$, is increasing in $x_t$, his inheritance. That is, agents with bigger future inheritance are allowed to borrow more.*

**Proof.** Applying the implicit function theorem on equation (14), we get

$$\frac{\partial \bar{b}_{t-1}}{\partial x_t} = -\frac{\frac{\partial H}{\partial x_t}}{\frac{\partial H}{\partial b_{t-1}}\Big|_{b_{t-1}=\bar{b}_{t-1}}} = -\frac{\frac{1}{\omega_m + f\left(\bar{b}_{t-1}\right) + x_t - \bar{b}_{t-1}R - \widehat{s}_t} - \frac{1}{\omega_m + f\left(\bar{b}_{t-1}\right) + x_t}}{\frac{\partial H}{\partial b_{t-1}}\Big|_{b_{t-1}=\bar{b}_{t-1}}}.$$

It is obvious that the numerator of $\partial \bar{b}_{t-1}/\partial x_t$ is positive. Thus the sign of $\partial \bar{b}_{t-1}/\partial x_t$ is equivalent to the sign of $-\partial H/\partial b_{t-1}\Big|_{b_{t-1}=\bar{b}_{t-1}}$. Notice that $H$ is continuously differentiable with $b_{t-1}$. As stated earlier, we have $H(0) > 0$ and $H(\infty) < 0$. Hence $H$ must intersect the horizontal axis from above at least once. Moreover, if there are multiple intersections, it must be true that the curve $H$ will intersect the horizontal axis from above at the first and the last intersection points. Since $H \geq 0$ holds for all $b_{t-1} \in \left[0, \bar{b}_{t-1}\right]$, $\bar{b}_{t-1}$ corresponds to the first and the smallest intersection point. Since $H$ intersects the horizontal axis from above at $\bar{b}_{t-1}$, it must be that $\partial H/\partial b_{t-1}\Big|_{b_{t-1}=\bar{b}_{t-1}} < 0$. Hence, $\partial \bar{b}_{t-1}/\partial x_t > 0$. ■