Online Appendix. Elasticity of substitution and technical progress: Is there a misspecification problem?

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1 Data used in the estimation

The data used in this study are of the Italian economy, quarterly from 1980:Q2, to 2006:Q1. GDP and NDP, fixed Capital, and total remuneration are defined as € bn., employment in millions of employees, any parameters of variables such as interest rates, rate of time preference, rates of growth, etc. as rates per quarter in natural numbers. All real variables are defined with base year 2000. The stock of fixed capital is calculated from net capital formation divided by the GDP deflator and accumulated from a base stock of 3572.4 (€ bn.) in 2000:Q2. The ICT capital stock is calculated from annual data for gross real investment less depreciation for each of three sub-sectors (office machinery, communication devices and software), each separately interpolated to provide quarterly observations, and total net investment cumulated on a base figure of 80.717 (€ bn.) in 2000:Q4.

The main sources of the data are ISTAT, Bank of Italy, EU KLEMS, OECD, AMECO, European Commssion.

The ICT vs. non ICT sector in Italy and labor share dynamics in Italy (and selected countries) are described below.

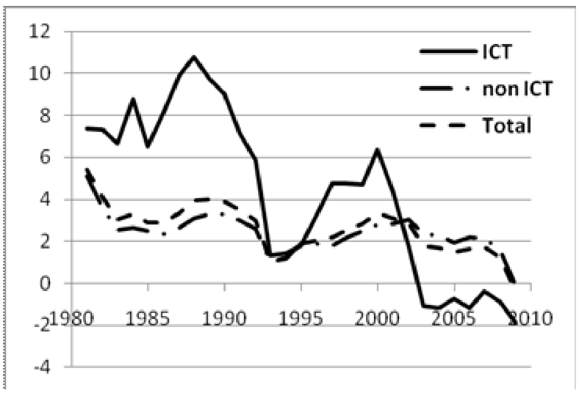

Figure 1: Capital accumulation in Italy (growth rates percent)

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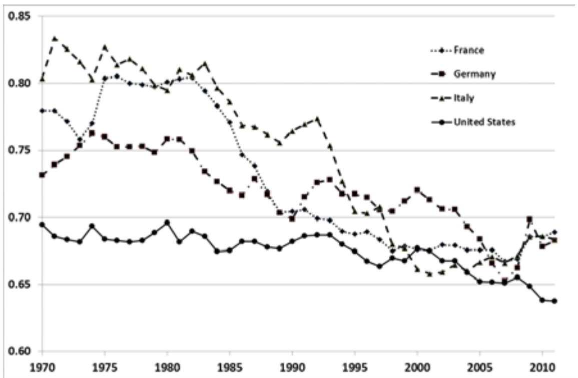

Figure 2: Labour share dynamics

2 The model

The core of the model is composed by the following equations, where for brevity we omitted the time index (for more details, see Saltari et al. 2012):

1. Production functions:

   (a) General production function

$$Y = f\,(C, K\,, Ls, Lu) = \beta_3 \left[\,(\,C^{\,\gamma_1} K\,)^{\,-\beta_1} \,+\, \left(\,\beta_2\, e^{\mu_K\, t}\, Ls^{\gamma_s}\, Lu^{\gamma_u}\right)^{\,-\beta_1} \right]^{-\frac{1}{\beta_1}} \quad (1)$$

   (b) Production function of ICT

$$I = f_I\,(C, L_{I\,s}) = \beta_7 \left[C^{-\beta_6} + (\beta_8 \exp(\lambda_c) L_{I\,s})^{-\beta_6}\right]^{-\frac{1}{\beta_6}} \quad (2)$$

   where $L_{I\,s}$ is the skilled labour employed in the ICT sector.

2. Investment functions:

   (a) Traditional capital

$$\dot{k} \,=\, \alpha_1 \left[\alpha_2 \left(\frac{\partial f}{\partial K} \,-\, (r \,-\, \beta_7\, D \ln\, p \,+\, \beta_8)\right) - (k - \mu_K)\right] \quad (3)$$

   where $k = D \ln{(K)}\,,\, \dot{k} = D^2 \ln{(K)}$

   (b) ICT capital

$$\dot{c} = \alpha_3 \left[\alpha_4 \left(\frac{\partial f_I}{\partial C} \,-\, (r \,-\, \beta_9\, D \ln\, p \,+\, \beta_{10})\right) - (c \,-\, \mu_C)\right] \quad (4)$$

   where in Equation (3) $\mu_K = \lambda_K + (\gamma_S - \gamma_1)\,\lambda_S + \gamma_u \lambda_u$ and in Equation (4) $\mu_C = \lambda_s + \lambda_c$, and $c = D \ln{(C)}\,,\, \dot{c} = D^2 \ln{(C)}$

3. Skilled labour:

(a) Demand for skilled labour

$$\dot{\ell}_s = \alpha_5 \left[\alpha_6 \ln \left(\frac{\partial f}{\partial L_s} \Big/ \frac{w_s}{p} \right) + \alpha'_6 \ln \left(\frac{\partial f_I}{\partial L_{I\,s}} \Big/ \frac{w_s}{p} \right) - (\ell_s - \lambda_s) \right] \quad (5)$$

where w_s is the wage of the skilled labour, and $\ell_s = D \ln (Ls)$, $\dot{\ell}_s = D^2 \ln (Ls)$

(b) Skilled wages (w_s)

$$D^2 \ln w_s = \alpha_7 \left[\alpha_8 \ln \left(\frac{\partial f}{\partial L_s} \frac{w_s}{p} \right) + \alpha'_8 \ln \left(\frac{\partial f_I}{\partial L_{I\,s}} \frac{w_s}{p} \right) - \right.$$
$$\left. (\alpha_7 + \alpha_8 + \alpha'_8) \; (D \ln w_s - \beta_{11} D \ln p - \lambda_K - \gamma_1 \lambda_C) \right] \quad (6)$$

where β_{11} measures money illusion.[1]

4. Unskilled labour:

(a) Employment

$$\dot{\ell}_u = \alpha_9 \alpha_{10} \ln \left(L_u^d / L_u \right) - (\alpha_9 + \alpha_{10}) (\ell_u - \lambda_u) \quad (7)$$

where $\ell_u = D \ln (Lu)$, $\dot{\ell}_u = D^2 \ln (Lu)$, and L_u^d is the demand for unskilled labour.

(b) Unskilled wages (w_u)

$$D^2 \ln w_u = \alpha_{11} \left[\alpha_{12} \ln \left(\frac{L_u^d}{L_u^s} \right) - (D \ln \frac{w_u}{p} - \lambda_K - \gamma_1 \lambda_C) \right] \quad (8)$$

where the labour supply is $L_u^s = L_{u0} \left(\frac{w_u}{p} \right)^{\beta_{12}} e^{\lambda_u t}$. In the model, changes in the unskilled labor supply depends on the real wage, with elasticity β_{12}.

5. Price determination:

The marginal cost of labour is obtained in the usual way as a ratio between the mean wage and the marginal product of labour, where labour is defined as a Cobb-Douglas function of the two labor components, $L = L_s^{\gamma_u} L_u^{\gamma_u}$. The short term marginal cost is a weighted average of skilled and unskilled wage rates

$$mc \left(\frac{\partial L}{\partial Y} \right) = \left(\frac{w_s L_s}{\gamma_s} + \frac{w_u L_u}{\gamma_u} \right) L_s^{-\gamma_u} L_u^{-\gamma_u} (\beta_2 \beta_3)^{-1} e^{-(\lambda_K + \gamma_1 \lambda_C)\, t} \left[1 + \left(\beta_2 e^{(\lambda_K + \gamma_1 \lambda_C)\, t} \psi \right)^{\beta_1} \right]^{\frac{1+\beta_1}{\beta_1}}$$

where $\psi = \frac{L}{K C^{\gamma_1}}$.

The dynamics of price determination are described by a second-order process:

$$D^2 \ln(p) = \alpha_{15} \ln \left(\frac{\beta_{13} \tau mc \left(\frac{\partial L}{\partial Y} \right)}{p} \right) + \alpha_{13} \left(D \ln \left(\frac{w_s}{p} \right) - \lambda_c \right) +$$
$$+ \alpha_{14} \left\{ D \ln \left(\frac{w_u}{p} \right) - (\lambda_k + \gamma_1 \lambda_c) \right\} + \alpha_{16} \ln \left\{ \frac{vM}{pY} \left(1 + \lambda_m - \lambda_m e^{-\lambda_m t} \right) \right\} \quad (9)$$

where β_{13} is the mark-up and τ is the indirect tax rate. M is the volume of money (M_2), ν the velocity of money increasing by a factor $(1 + \lambda_m - \lambda_m e^{-\lambda_m t})$. This factor is one when $t = 0$ and increases at a reducing rate to an asymptote $(1 + \lambda_m)$. Hence it is assumed that ν increases over time from a base level at $t = 0$ owing to more efficient banking services.

3 Normalization used in Section 5

The normalization procedure identifies a family of CES production functions that are distinguished only by the elasticity parameter. Normalization is a way to represent the production function so that the variables are independent of the unit of measure, i.e. they are in index number form. This makes the parameter estimation easier.[2]

To begin with, we set the base period used for the normalization at the middle of the sample, $t = 48$, corresponding to 1993:Q3. To simplify notation, we denote this period by the index 0. Normalization implies that all the variables are expressed in terms of their baseline values.

To normalize the production function, we start from equation (3) of the main text:

$$Y_t = \beta_3 \left[\left(KIT_t \right)^{-\beta_1} + \left(\beta_2 e^{\mu_K (t-t_0)} L_t \right)^{-\beta_1} \right]^{-\frac{1}{\beta_1}}, \tag{10}$$

where t_0 is the base period and, to simplify notation, we set $KIT_t = K_t C_t^{\gamma_1}$.

Under imperfect competition, factor compensation is subject to a mark-up, by hypothesis constant and denoted by β_{13},[3] so that in any period t the following relation holds

$$Y_t = (i_t KIT_t + w_t L_t)\, \beta_{13},$$

where i_t is the real interest rate and w_t is the wage rate.[4]

In the reference period capital compensation is

$$i_0 = \frac{1}{\beta_{13}} \frac{\partial Y_0}{\partial KIT_0} = \frac{(\beta_3)^{-\beta_1}}{\beta_{13}} \left(\frac{Y_0}{KIT_0} \right)^{1+\beta_1},$$

so that total capital compensation over total factor income, or the capital share, in the base period is

$$\pi_0 = \frac{i_0 KIT_0}{Y_0} \beta_{13} = (\beta_3)^{-\beta_1} \left(\frac{Y_0}{KIT_0} \right)^{\beta_1}. \tag{11}$$

Likewise, the labor compensation in the base period is

$$w_0 = \frac{1}{\beta_{13}} \frac{\partial Y_0}{\partial L_0} = \frac{(\beta_3 \beta_2)^{-\beta_1}}{\beta_{13}} \left(\frac{Y_0}{L_0} \right)^{1+\beta_1},$$

so the labour share is

$$1 - \pi_0 = \frac{w_0 L_0}{Y_0} \beta_{13} = (\beta_3)^{-\beta_1} \left(\frac{Y_0}{\beta_2 L_0} \right)^{\beta_1}. \tag{12}$$

Notice that labour share in the base period expressed in efficiency units is simply $\beta_2 L$ since in the base period the time-dependent efficiency factor disappears.

Substituting into the production function (10) the capital share evaluated in the base period, we get

$$Y_t = \left[\pi_0 \left(\frac{Y_0}{KIT_0} \right)^{-\beta_1} (KIT_t)^{-\beta_1} + \left(\beta_3 \beta_2 e^{\mu_K (t-t_0)} L_t \right)^{-\beta_1} \right]^{-\frac{1}{\beta_1}}.$$

Following an analogous procedure for the labor share (12), we have

$$Y_t = Y_0 \left[\pi_0 \left(\frac{KIT_t}{KIT_0} \right)^{-\beta_1} + (1 - \pi_0) \left(e^{\mu_K (t-t_0)} \frac{L_t}{L_0} \right)^{-\beta_1} \right]^{-\frac{1}{\beta_1}}. \tag{13}$$

In index number form, the production function becomes

$$\frac{Y_t}{Y_0} = \left[\pi_0 \left(\frac{KIT_t}{KIT_0} \right)^{-\beta_1} + (1-\pi_0) \left(e^{\mu_K \,(t-t_0)} \frac{L_t}{L_0} \right)^{-\beta_1} \right]^{-\frac{1}{\beta_1}} . \tag{14}$$

For simplicity, the last equation is rewritten as

$$y_t = \left[\pi_0 \, (kit_t)^{-\beta_1} + (1-\pi_0) \left(e^{\mu_K \,(t-t_0)} l_t \right)^{-\beta_1} \right]^{-\frac{1}{\beta_1}} , \tag{15}$$

where small capital letters indicate index numbers. In the capital intensive form with inputs expressed in efficiency units, the last equation becomes

$$\frac{y_t}{e^{\mu_K \,(t-t_0)} l_t} = \left[\pi_0 \left(\frac{kit_t}{e^{\mu_K \,(t-t_0)} l_t} \right)^{-\beta_1} + (1-\pi_0) \right]^{-\frac{1}{\beta_1}} .$$

There are two points worth making about equation 15. First, under imperfect competition with a non-zero mark-up, the distribution parameter π_0 equals the share of capital income over total factor income, the sum of labour and capital income. Second, in the normalized production function there are two parameters to be estimated: β_1 which is related to the elasticity of substitution, σ_1, and γ_1 (which is behind the definitions of $kit_t = \frac{K_t \, C_t{}^{\gamma_1}}{K_0 \, C_0{}^{\gamma_1}}$ and $\mu_k = \lambda_K + \gamma_1 \, \lambda_C$).

Before performing any estimation exercise using normalization, we need to fix income shares in the benchmark period. Employing observed data for capital, labour and output and our parameters estimates in Table 1 in the main text, the capital share for the Italian economy, see equation (11), is

$$\pi_0 = (\beta_3)^{-\beta_1} \left(\frac{Y_0}{KIT_0} \right)^{\beta_1} = 0.24,$$

so that labour income share is

$$1 - \pi_0 = 0.76.$$

Since these estimates are quite close to those present in different databanks (such as OECD, EU KLEMS, AMECO), we adopt these shares for the reference period.