UNRAVELING THE SKILL PREMIUM:

ONLINE APPENDICES

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A FIRST ORDER CONDITIONS ASSOCIATED WITH THE FOUR-FACTOR THREE-LEVEL CASE

that the representative firm faces an isoelastic demand curve, $Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Y_t$. Profit maximizing under the specified CES technology, (4), implies the following four first order conditions:[17]

$$\ln w_1 = \mathcal{C}_1 + \left(\frac{\psi-1}{\psi}\right)\gamma_1 \tilde{t} + \frac{1}{\psi}\left[\ln \widetilde{Y} - \ln \widetilde{V}_1\right] \tag{A.1}$$

$$\begin{aligned} \ln w_2 = \mathcal{C}_2 &+ \left(\frac{\sigma-1}{\sigma}\right)\gamma_2 \tilde{t} + \frac{1}{\psi}\ln \widetilde{Y} - \frac{1}{\sigma}\ln \widetilde{V}_2 \\ &+ \frac{(\psi-\sigma)}{\psi(\sigma-1)} \ln\left\{(1-\beta_0)\left(A_2\widetilde{V}_2\right)^{\frac{\sigma-1}{\sigma}} + \beta_0\left[(1-\pi_0)\left(A_3\widetilde{V}_3\right)^{\frac{\eta-1}{\eta}} + \pi_0\left(A_4\widetilde{V}_4\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}\frac{\sigma-1}{\sigma}}\right\} \end{aligned} \tag{A.2}$$

$$\begin{aligned} \ln w_3 = \mathcal{C}_3 &+ \left(\frac{\eta-1}{\eta}\right)\gamma_3 \tilde{t} + \frac{1}{\psi}\ln \widetilde{Y} - \frac{1}{\eta}\ln \widetilde{V}_3 \\ &+ \frac{(\psi-\sigma)}{\psi(\sigma-1)} \ln\left\{(1-\beta_0)\left(A_2\widetilde{V}_2\right)^{\frac{\sigma-1}{\sigma}} + \beta_0\left[(1-\pi_0)\left(A_3\widetilde{V}_3\right)^{\frac{\eta-1}{\eta}} + \pi_0\left(A_4\widetilde{V}_4\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}\frac{\sigma-1}{\sigma}}\right\} \\ &+ \frac{(\sigma-\eta)}{\sigma(\eta-1)} \ln\left\{(1-\pi_0)\left(A_3\widetilde{V}_3\right)^{\frac{\eta-1}{\eta}} + \pi_0\left(A_4\widetilde{V}_4\right)^{\frac{\eta-1}{\eta}}\right\} \end{aligned} \tag{A.3}$$

$$\begin{aligned} \ln w_4 = \mathcal{C}_4 &+ \left(\frac{\eta-1}{\eta}\right)\gamma_4 \tilde{t} + \frac{1}{\psi}\ln \widetilde{Y} - \frac{1}{\eta}\ln \widetilde{V}_4 \\ &+ \frac{(\psi-\sigma)}{\psi(\sigma-1)} \ln\left\{(1-\beta_0)\left(A_2\widetilde{V}_2\right)^{\frac{\sigma-1}{\sigma}} + \beta_0\left[(1-\pi_0)\left(A_3\widetilde{V}_3\right)^{\frac{\eta-1}{\eta}} + \pi_0\left(A_4\widetilde{V}_4\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}\frac{\sigma-1}{\sigma}}\right\} \\ &+ \frac{(\sigma-\eta)}{\sigma(\eta-1)} \ln\left\{(1-\pi_0)\left(A_3\widetilde{V}_3\right)^{\frac{\eta-1}{\eta}} + \pi_0\left(A_4\widetilde{V}_4\right)^{\frac{\eta-1}{\eta}}\right\} \end{aligned} \tag{A.4}$$

where the individual constants are given by $\mathcal{C}_1 = \ln\left[\frac{\alpha_0}{(1+\mu)}\frac{Y_0}{V_{1,0}}\right]$, $\mathcal{C}_2 = \ln\left[\frac{(1-\alpha_0)(1-\beta_0)}{(1+\mu)}\frac{Y_0}{V_{2,0}}\right]$, $\mathcal{C}_3 = \ln\left[\frac{(1-\alpha_0)\beta_0(1-\pi_0)}{(1+\mu)}\frac{Y_0}{V_{3,0}}\right]$, $\mathcal{C}_4 = \ln\left[\frac{(1-\alpha_0)\beta_0\pi_0}{(1+\mu)}\frac{Y_0}{V_{4,0}}\right]$ and where $\mu = \varepsilon/(\varepsilon-1)$.

Denoting factor prices by w_i the distribution parameters α_0, β_0 and π_0 in equations (1)-(3) are defined by factor incomes at the normalization point,

$$\alpha_0 = (w_{1,0} \cdot V_{1,0}) \ / \ (w_{1,0} \cdot V_{1,0} + w_{2,0} \cdot V_{2,0} + w_{3,0} \cdot V_{3,0} + w_{4,0} \cdot V_{4,0}) \tag{A.5}$$

$$\beta_0 = (w_{3,0} \cdot V_{3,0} + w_{4,0} \cdot V_{4,0}) \ / \ (w_{2,0} \cdot V_{2,0} + w_{3,0} \cdot V_{3,0} + w_{4,0} \cdot V_{4,0}) \tag{A.6}$$

$$\pi_0 = (w_{4,0} \cdot V_{4,0}) \ / \ (w_{3,0} \cdot V_{3,0} + w_{4,0} \cdot V_{4,0}) \tag{A.7}$$

Equations (4)-(A.4) define a 5-equation system with manifest cross-equation parameter constraints. This encompasses the 3-equation system estimated by Krusell et al. (2000) who constrained the elasticity of substitution, ψ, between variable V_1 (structures capital) and the compound factor Z (capturing unskilled labor V_2, equipment capital V_3 and skilled labor V_4) to unity, i.e. Cobb Douglas.[18]

B FIRST ORDER CONDITIONS ASSOCIATED WITH $\psi = 1$ THREE-LEVEL CASE

The implied first order maximization conditions with respect to inputs corresponding (A.1)-(A.4) equations are:

$$\ln w_{1,t} = \mathcal{C}_1 + \ln\left[\frac{\alpha_0}{(1+\mu)}\frac{\widetilde{Y}_t}{\widetilde{V}_{1,t}}\right] \tag{B.1}$$

$$\begin{aligned}\ln w_{2,t} &= \mathcal{C}_2 + \left(\frac{\sigma-1}{\sigma}\right)\gamma_{24}t + \ln\left(\widetilde{Y}_t\right) - \frac{1}{\sigma}\ln\left(\widetilde{V}_{2,t}\right) \\ &- \ln\left\{\begin{array}{c}(1-\beta_0)\left(e^{\gamma_{24}t}\widetilde{V}_{2,t}\right)^{\frac{\sigma-1}{\sigma}} \\ +\beta_0\left[(1-\pi_0)\left(e^{\gamma_{34}t}\widetilde{V}_{3,t}\right)^{\frac{\eta-1}{\eta}} + \pi_0\left(\widetilde{V}_{4,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}\frac{\sigma-1}{\sigma}}\end{array}\right\}\end{aligned} \tag{B.2}$$

$$\begin{aligned}\ln w_{3,t} &= \mathcal{C}_3 + \left(\frac{\eta-1}{\eta}\right)\gamma_{34}t + \ln\left(\widetilde{Y}_t\right) - \frac{1}{\eta}\ln\left(\widetilde{V}_{3,t}\right) \\ &- \ln\left\{\begin{array}{c}(1-\beta_0)\left(e^{\gamma_{24}t}\widetilde{V}_{2,t}\right)^{\frac{\sigma-1}{\sigma}} \\ +\beta_0\left[(1-\pi_0)\left(e^{\gamma_{34}t}\widetilde{V}_{3,t}\right)^{\frac{\eta-1}{\eta}} + \pi_0\left(\widetilde{V}_{4,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}\frac{\sigma-1}{\sigma}}\end{array}\right\} \\ &+ \frac{(\sigma-\eta)}{\sigma(\eta-1)}\ln\left[(1-\pi_0)\left(e^{\gamma_{34}t}\widetilde{V}_{3,t}\right)^{\frac{\eta-1}{\eta}} + \pi_0\left(\widetilde{V}_{4,t}\right)^{\frac{\eta-1}{\eta}}\right]\end{aligned} \tag{B.3}$$

$$\begin{aligned}\ln w_{4,t} &= \mathcal{C}_4 + \ln\left(\widetilde{Y}_t\right) - \frac{1}{\eta}\ln\left(\widetilde{V}_{4,t}\right) \\ &- \ln\left\{\begin{array}{c}(1-\beta_0)\left(e^{\gamma_{24}t}\widetilde{V}_{2,t}\right)^{\frac{\sigma-1}{\sigma}} \\ +\beta_0\left[(1-\pi_0)\left(e^{\gamma_{34}t}\widetilde{V}_{3,t}\right)^{\frac{\eta-1}{\eta}} + \pi_0\left(\widetilde{V}_{4,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}\frac{\sigma-1}{\sigma}}\end{array}\right\} \\ &+ \frac{(\sigma-\eta)}{\sigma(\eta-1)}\ln\left[(1-\pi_0)\left(e^{\gamma_{34}t}\widetilde{V}_{3,t}\right)^{\frac{\eta-1}{\eta}} + \pi_0\left(\widetilde{V}_{4,t}\right)^{\frac{\eta-1}{\eta}}\right]\end{aligned} \tag{B.4}$$

where $\mathcal{C}_1 = \ln\frac{\alpha_0}{(1+\mu)}$, $\mathcal{C}_2 = \ln\left[\frac{\alpha_0(1-\beta_0)}{(1+\mu)}\frac{Y_0}{V_{2,0}}\right]$, $\mathcal{C}_3 = \ln\left[\frac{(1-\alpha_0)\beta_0(1-\pi_0)}{(1+\mu)}\frac{Y_0}{V_{3,0}}\right]$, $\mathcal{C}_4 = \ln\left[\frac{(1-\alpha_0)\beta_0\pi_0}{(1+\mu)}\frac{Y_0}{V_{4,0}}\right]$.

C FIRST ORDER CONDITIONS ASSOCIATED WITH THE TWO-LEVEL CASE

Isoelastic demand, and profit maximization, implies the first order conditions:

$$\begin{aligned}\ln w_{1,t} = {} & \mathcal{C}_1 + \frac{(\zeta-1)\,\gamma_1}{\zeta} t + \frac{1}{\sigma}\ln\left(\widetilde{Y}_t\right) - \frac{1}{\zeta}\ln\left(\widetilde{V}_{1,t}\right) \\ & + \frac{\sigma-\zeta}{\sigma(\zeta-1)}\ln\left[(1-\beta_0)\left(e^{\gamma_1 t}\widetilde{V}_{1,t}\right)^{\frac{\zeta-1}{\zeta}} + \beta_0\left(e^{\gamma_2 t}\widetilde{V}_{2,t}\right)^{\frac{\zeta-1}{\zeta}}\right]\end{aligned} \tag{C.1}$$

$$\begin{aligned}\ln w_{2,t} = {} & \mathcal{C}_2 + \frac{(\zeta-1)\,\gamma_2}{\zeta} t + \frac{1}{\sigma}\ln\left(\widetilde{Y}_t\right) - \frac{1}{\zeta}\ln\left(\widetilde{V}_{2,t}\right) \\ & + \frac{\sigma-\zeta}{\sigma(\zeta-1)}\ln\left[(1-\beta_0)\left(e^{\gamma_1 t}\widetilde{V}_{1,t}\right)^{\frac{\zeta-1}{\zeta}} + \beta_0\left(e^{\gamma_2 t}\widetilde{V}_{2,t}\right)^{\frac{\zeta-1}{\zeta}}\right]\end{aligned} \tag{C.2}$$

$$\begin{aligned}\ln w_{3,t} = {} & \mathcal{C}_3 + \frac{(\eta-1)\,\gamma_3}{\eta} t + \frac{1}{\sigma}\ln\left(\widetilde{Y}_t\right) - \frac{1}{\eta}\ln\left(\widetilde{V}_{3,t}\right) \\ & + \frac{\sigma-\eta}{\sigma(\eta-1)}\ln\left[(1-\pi_0)\left(e^{\gamma_3 t}\widetilde{V}_{3,t}\right)^{\frac{\eta-1}{\eta}} + \pi_0\left(e^{\gamma_4 t}\widetilde{V}_{4,t}\right)^{\frac{\eta-1}{\eta}}\right]\end{aligned} \tag{C.3}$$

$$\begin{aligned}\ln w_{4,t} = {} & \mathcal{C}_4 + \frac{(\eta-1)\,\gamma_4}{\eta} t + \frac{1}{\sigma}\ln\left(\widetilde{Y}_t\right) - \frac{1}{\eta}\ln\left(\widetilde{V}_{4,t}\right) \\ & + \frac{\sigma-\eta}{\sigma(\eta-1)}\ln\left[(1-\pi_0)\left(e^{\gamma_3 t}\widetilde{V}_{3,t}\right)^{\frac{\eta-1}{\eta}} + \pi_0\left(e^{\gamma_4 t}\widetilde{V}_{4,t}\right)^{\frac{\eta-1}{\eta}}\right]\end{aligned} \tag{C.4}$$

where the individual constants are given by $\mathcal{C}_1 = \ln\left[\frac{\alpha_0(1-\beta_0)}{(1+\mu)}\frac{Y_0}{V_{1,0}}\right]$, $\mathcal{C}_2 = \ln\left[\frac{\alpha_0\beta_0}{(1+\mu)}\frac{Y_0}{V_{2,0}}\right]$, $\mathcal{C}_3 = \ln\left[\frac{(1-\alpha_0)(1-\pi_0)}{(1+\mu)}\frac{Y_0}{V_{3,0}}\right]$, and $\mathcal{C}_4 = \ln\left[\frac{(1-\alpha_0)\pi_0}{(1+\mu)}\frac{Y_0}{V_{4,0}}\right]$.

Denoting factor prices by w_i $(i = 1, 2, 3)$ normalization implies that the distribution parameters α_0, β_0 and π_0 in (12)-(14) are defined by,

$$\alpha_0 = (w_{1,0}\cdot V_{1,0} + w_{2,0}\cdot V_{2,0}) \,/\, (w_{1,0}\cdot V_{1,0} + w_{2,0}\cdot V_{2,0} + w_{3,0}\cdot V_{3,0} + w_{4,0}\cdot V_{4,0}) \tag{C.5}$$

$$\beta_0 = (w_{2,0}\cdot V_{2,0}) \,/\, (w_{1,0}\cdot V_{1,0} + w_{2,0}\cdot V_{2,0}) \tag{C.6}$$

$$\pi_0 = (w_{4,0}\cdot V_{4,0}) \,/\, (w_{3,0}\cdot V_{3,0} + w_{4,0}\cdot V_{4,0}) \tag{C.7}$$

Following Krusell et al. (2000) and others, we that $\widetilde{V}_1$ is structures capital, $\widetilde{V}_2$ unskilled labor, $\widetilde{V}_3$ equipment capital and $\widetilde{V}_4$ is skilled labor. Under this interpretation the two first equations (factor share equations) of the three equation system estimated by Krusell et al. (2000) are direct transformations of the first-order conditions (B.3)-(B.4). Their third equation (the rate of return equality condition), in turn, may be linked to the conditions (B.1)-(B.2). However, as they do not show its explicit derivation the possible correspondence remains ambiguous. As regards the underlying production function (4′′) Krusell et al. (2000) left it outside their estimated 3-equation system.

D DECOMPOSITIONS: SKILL PREMIA AND OUTPUT

The individual growth contributions to output and the skill premium are derived as follows. We first have (using time subscripts for clarity),

$$\widehat{y}_t = ces\left(K_t^e, K_t^b, S_t, U_t\,;\, \widehat{\gamma}^{K^b}\tilde{t}, \widehat{\gamma}^{K^e}\tilde{t}, \widehat{\gamma}^{S}\tilde{t}, \widehat{\gamma}^{U}\tilde{t}\,;\, \widehat{\Sigma}\right) \tag{D.1}$$

$$\widehat{\omega}_t = f\left(K_t^e, K_t^b, S_t, U_t\,;\, \widehat{\gamma}^{K^b}\tilde{t}, \widehat{\gamma}^{K^e}\tilde{t}, \widehat{\gamma}^{S}\tilde{t}, \widehat{\gamma}^{U}\tilde{t}\,;\, \widehat{\Sigma}\right) \tag{D.2}$$

where $\widehat{y}$ and $\widehat{\omega}$ are, respectively, the estimated fits of the log of output and of the log skill premium conditional on the estimated technical change parameters, the substitution elasticity parameters, Σ, and on *ces* and f (the case-specific functional forms).

Then the growth contributions at time t, of, say, equipment capital to both output and the skill premium involve assuming that it remains at its previous level. Taking differences from estimated fits yield growth contributions of equipment capital to current period output and the skill premium from the previous period:

$$y_t^{K^e} = \widehat{y}_t - ces\left(K_{t-1}^e, K_t^b, S_t, U_t\,;\, \widehat{\gamma}^{K^b}\tilde{t}, \widehat{\gamma}^{K^e}\tilde{t}, \widehat{\gamma}^{S}\tilde{t}, \widehat{\gamma}^{U}\tilde{t}\,;\, \widehat{\Sigma}\right) \tag{D.3}$$

$$\omega_t^{K^e} = \widehat{\omega} - f\left(K_{t-1}^e, K_t^b, S_t, U_t\,;\, \widehat{\gamma}^{K^b}\tilde{t}, \widehat{\gamma}^{K^e}\tilde{t}, \widehat{\gamma}^{S}t, \widehat{\gamma}^{U}t\,;\, \widehat{\Sigma}\right) \tag{D.4}$$

Likewise, for the technical progress terms, the contribution of skill-augmenting technical change is found by assuming that there was no change between $t-1$ and t, i.e. replacing everywhere $\gamma^S \cdot \tilde{t}$ by

$\gamma^S \cdot (\tilde{t} - 1)$:

$$y_t^{TC^S} = \widehat{y}_t - ces\left(K_t^e, K_t^b, S_t, U_t\,;\, \widehat{\gamma}^{K^b}\tilde{t}, \widehat{\gamma}^{K^e}\tilde{t}, \widehat{\gamma}^S\left(\tilde{t}-1\right), \widehat{\gamma}^U\tilde{t}\,;\, \widehat{\Sigma}\right) \tag{D.5}$$

$$\omega_t^{TC^S} = \widehat{\omega}_t - f\left(K_t^e, K_t^b, S_t, U_t\,;\, \widehat{\gamma}^{K^b}\tilde{t}, \widehat{\gamma}^{K^e}\tilde{t}, \widehat{\gamma}^S\left(\tilde{t}-1\right), \widehat{\gamma}^U\tilde{t}\,;\, \widehat{\Sigma}\right) \tag{D.6}$$

Growth contributions of aggregated labor, capital, total factor productivity and total contribution of these factors are respectively obtained as (dropping time subscripts for convenience),

$$y^N = y^S + y^U \tag{D.7}$$

$$y^K = y^{K^e} + y^{K^b} \tag{D.8}$$

$$y^{TFP} = y^{TC^S} + y^{TC^U} + y^{TC^{K^e}} + y^{TC^{K^b}} \tag{D.9}$$

$$\widehat{g^Y} = y^N + y^K + y^{TFP} \tag{D.10}$$

Similarly the contributions of relative labor supply, capital skill complementarity, total technical change as well as their total contribution to the skill premium is obtained as,

$$\omega^{RSS} = \omega^S + \omega^U \tag{D.11}$$

$$\omega^{KSC} = \omega^{K^e} + \omega^{K^b} \tag{D.12}$$

$$\omega^{TC} = \omega^{TC^S} + \omega^{TC^U} + \omega^{TC^{K^e}} + \omega^{TC^{K^b}} \tag{D.13}$$

$$\widehat{g^{\omega}} = \omega^{RSS} + \omega^{KSC} + \omega^{TC} \tag{D.14}$$

In the context of three-level-CES production function the skill premium is independent from the development of structures capital stock and, therefore, $\omega_t^{K^b} = 0$

In the case of Four-Factor-Nested CD-CES production function total technical change (or TFP) contributions to output growth and to the growth of the skill premium are expressed in terms of common Hicks-neutral technical change component and the growth contributions of the deviations of unskilled labor and equipment capital augmenting technical changes from that of skilled labor augmenting technical change as follows,

$$y^{TFP} = y^{TC^H} + y^{TC^{(U-S)}} + y^{TC^{(K^e-S)}} \tag{D.15}$$

$$\omega^{TC} = \omega^{TC^H} + \omega^{TC^{(U-S)}} + \omega^{TC^{(K^e-S)}} \tag{D.16}$$

Tables 6 and 9 present average growth contributions of inputs and respective augmenting technical change components. The most striking is how similar growth contribution estimates each specification alternative gives. In all cases labor, capital and TFP have corresponded around 47%, 21-22% and 31-32%, respectively, of the growth in the sample period. Around 89% of labor contribution and around 82% of capital contribution except specification 1 of 2-level case where it is 92% – is related to the growth of skilled labor and the growth of equipment capital, respectively. Only the allocation TFP contributions varies across cases except the contribution of skilled labor augmenting technical change that uniformly exceeds somewhat that of the TFP implying somewhat negative net contributions from other augmenting technical components.

E Disaggregated Fit Measures of Estimations

Table E.1: Residual Standard Errors of The estimated Production Systems

	Table 2				
	(a)	(a')	(a'')	(b)	(c)
σ_{K_b}	0.140	0.136	0.083	0.094	0.100
σ_{K_e}	0.122	0.181	0.087	0.159	0.089
σ_S	0.043	0.041	0.027	0.136	0.022
σ_U	0.038	0.036	0.025	0.115	0.022
σ_Y	0.042	0.044	0.027	0.041	0.026
	Table 3				
	(a)			(b)	(c)
σ_{K_b}	0.097			0.097	0.100
σ_{K_e}	0.099			0.127	0.097
σ_S	0.036			0.038	0.034
σ_U	0.030			0.046	0.029
σ_Y	0.027			0.035	0.027
	Table 4				
	(a)			(b)	(c)
σ_{K_b}	0.081			0.094	0.100
σ_{K_e}	0.057			0.069	0.067
σ_S	0.035			0.058	0.022
σ_U	0.027			0.084	0.021
σ_Y	0.028			0.038	0.028
	Table 7				
	(a)			(b)	(c)
σ_{K_b}	0.128			0.131	0.091
σ_{K_e}	0.091			0.095	0.060
σ_S	0.029			0.028	0.048
σ_U	0.025			0.027	0.031
σ_Y	0.027			0.027	0.029

F HOW DOES THE SKILL PREMIUM WORK?

It is further interesting to note what our various systems work *out of sample*. To examine that, we extrapolate the exogenous variables in our five-equation system. We consider the following five scenarios:

1. $g^{\Lambda}_{T_+} = g^{\Lambda}$, "hist"

2. 1. except S : $g^{S}_{T_+} = g^{U}$

3. 1. except U : $g^{U}_{T_+} = g^{S}$

4. 1. except K^b : $g^{K^b}_{T_+} = g^{K^e}$

5. 1. except K^e : $g^{K^e}_{T_+} = g^{K^b}$

where $\Lambda = S, U, K^e, K^b$ and $T_+ \in [2009,\ 2050]$.

In scenario 1., we extrapolate all exogenous variables by their in-sample historical mean growth rates, g^{Λ}, forward to a symmetric future date, $T_+ = 2050$. The other scenarios do likewise but single out one particular growth pattern for special interest.[19]

Scenarios 2. and 3. eliminate the RSS effect, albeit in different ways. In 2. we set the out-of-sample growth rate of skilled labor, $g^{S}_{T_+}$, to the historical growth of unskilled labor, g^{U}. This implies a substantial reduction in the growth of skilled labor (recall Table 1). Scenario 3 reverses this, so that growth in unskilled labor becomes as high as skilled labor, which in turn implies a substantial increase in the growth of unskilled labor.

Scenario 4. raises the growth in structures capital to that of the rate of equipment capital. By contrast, in scenario 5. the growth in equipment capital falls to that of structures. Notice though that under projection scenarios 4. and 5., that the *ratio* of equipment to structures capital is the same in both cases (equal to its 2008 level) but that the individual levels in each scenario will differ:

$$\frac{g^{K^e}_{T_+} = g^{K^e}}{g^{K^b}_{T_+} = g^{Ke}} \equiv \frac{g^{K^e}_{T_+} = g^{K^b}}{g^{K^b}_{T_+} = g^{K^b}} : \begin{cases} 4. \ \left(g^{K^b}_{T_+} = g^{K^e}\right) > \left(g^{K^b}_{T_+} = g^{K^b}\right) \\ \\ 5. \ \left(g^{K^e}_{T_+} = g^{K^b}\right) < \left(g^{K^e}_{T_+} = g^{K^e}\right) \end{cases}$$

Figures F.1 and F.2 take the models in tables 3 and 4 as laboratories to examine extrapolated paths. These are the best performing models (the latter being the best). But also their choice is instructive since it illustrates systems with and without KSC, respectively. The figures plot, as before, the different projections for the log premium, ω, the relative log user cost, r, and the real GDP growth rate.

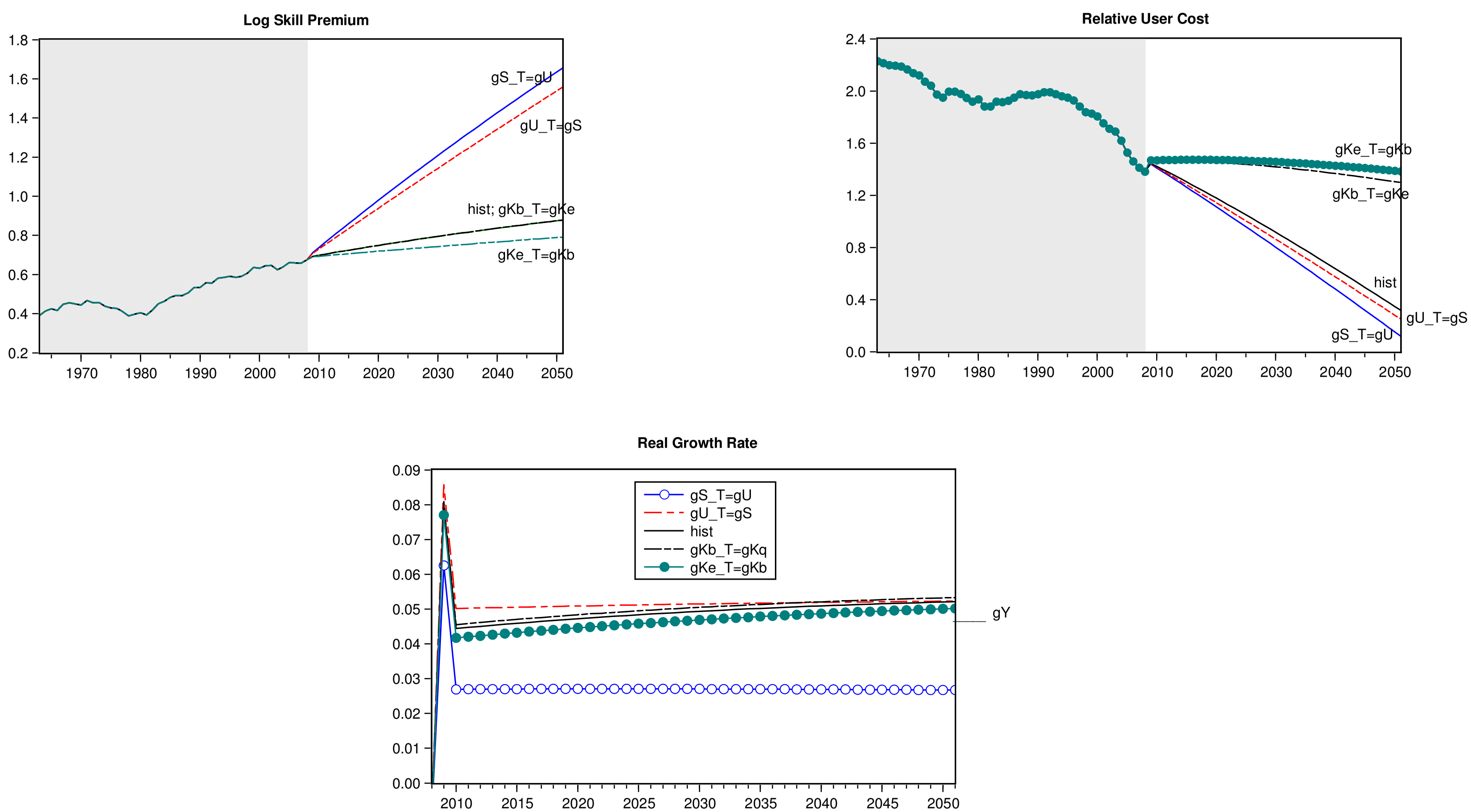
– Figures F.1 and F.2 here –

Projections based on *historical* trends are qualitatively similar across the two models. The log skill premium continues its upward trend (from around 0.68 in 2008 to 0.98 (model 4a), 0.88 (model 3a)). The relative user cost continues to decline on historical projections although with large differences across models (from around 1.4 to 0.8 (model 4), to 0.3 (model 3). The same is true for growth rates. Expanding factors will pull up growth since growth for factors is increasing, the differences in the growth trajectory reflect then the relative growth rates. Note since our framework is essentially static with constant positive factor growth rates, growth is thus acyclical and mechanically above its historical median.

Constellation 3 is the easiest to analyze since neither g^{K^e} nor g^{K^b} affect the premium. Moreover, the premium is always higher when $g^S = g^U$ or $g^U = g^S$ since the negative RSS component is removed entirely. Regarding user costs, in scenarios 4. and 5. the upward trend in K^e/K^b becomes fixed at its 2008 level. With capital more scarce relative to the other growing factors in the economy, user costs rise. In the case where the level of structures capital rises well above its historical growth rate (scenario 4.), r^{K^b} falls and widens the real relative user cost, $r^{K^e} - r^{K^b}$. The two labor scenarios lead to user cost trajectories similar to globally historical projections. On growth, where skilled labor grows only at the rate of unskilled labor, growth falls below that even of history. Likewise growth suffers when equipment capital is constrained to a lower growth rate. Otherwise, there are relatively small differences.

Let us now move to constellation 2. Notwithstanding qualitatively similar features, there are some large size differences with respect to model 3. Here, recalling equation (18), unless $g^S = g^U = 0$, the RSS will operate on the skill premium. Given this, we see high projections for the premium to be 0.1-0.2 higher in the labor series projections relative to the previous case. The path for relative user costs is, by comparison, much weaker. For the two capital growth expansions, the user cost term becomes essentially flat. Whilst for the historical and labor projections, the reduction in the user cost becomes more pronounced. In terms of growth, the outcomes are quite similar to before, except that now with KSC present, the $g^{K^e} = g^{K^b}$ has a less negative impact on growth relative to the historical trends case.

Figure F.1: Model Projections: Skill Premium, Relative Users Costs, Growth, Constellation 2: Model $K^b, \psi, [U, \sigma, (K^e, \eta, S)]$



Notes: The grey area denotes history. For real growth graphs, to maximize legibility, we started from the out-of-sample period; the g^Y line in the lowest panel is the median real growth rate, 0.039. Both this figure and figure F.2 are drawn on pairwise common axes for comparability.

Figure F.2: Model Projections: Skill Premium, Relative Users Costs, Growth, constellation 3: Model $K^b, \psi, [K^e, \sigma, (U, \eta, S)]$