Online Appendix for Growth, Unemployment, and Fiscal Policy: A Political Economy Analysis

Tetsuo Ono*

Osaka University

November 15, 2017

*Graduate School of Economics, Osaka University, 1-7 Machikaneyama, Toyonaka, Osaka 560-0043, Japan. Tel.: +81-6-6850-6111; Fax: +81-6-6850-5256. E-mail: tono@econ.osaka-u.ac.jp

1 Supplementary Materials (Not for Publication)

1.1 Microfoundation of Political Objective Function

This appendix explains the microfoundation of the political objective function. The following presentation focuses on the unbalanced-budget case. The same argument also holds for the balanced-budget case.

Recall that the population of each generation has a unit measure, and that a young generation consists of two groups of voters—namely, employed persons (with a fraction of l) and unemployed persons (with a fraction of $1-l$). The generation comprising elderly persons also consists of two groups—namely, agents who were employed in youth and those who were unemployed in youth. However, these two types of elderly agents are included in a single group of voters, because they have the same policy preferences regarding public services.

The electoral competition takes place between two office-seeking candidates, L (left) and R (right). Each candidate announces a policy vector (τ, g, b) under the unbalanced-budget rule, subject to the government budget constraint. Elections are held in every period, and so today's candidates cannot make credible promises regarding future policies.

Let V^o, V^{ye}, and V^{yu} denote the indirect utility functions of elderly persons, young employed persons, and young unemployed persons, respectively. An older voter prefers candidate R over L; if given the inherited capital and debt, K and D, respectively, the following condition holds:

$$V^o\left(p^L; K, D\right) < V^o\left(p^R; K, D\right) + \sigma^{io} + \delta,$$

where p^L (p^R) is the policy vector proposed by candidate L (R). Likewise, given K and D and the equilibrium policy functions $\left\langle \tilde{T}, \tilde{G}, \tilde{B}, \tilde{D} \right\rangle$, an employed young voter prefers candidate R over L if

$$V^{ye}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) < V^{ye}\left(p^R, \tilde{G}\left(p^R\right); K, D\right) + \sigma^{ie} + \delta,$$

and an unemployed young voter prefers candidate R over L if

$$V^{yu}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) < V^{yu}\left(p^R, \tilde{G}\left(p^R\right); K, D\right) + \sigma^{iu} + \delta.$$

σ^{ij} (where $j \in \{o, e, u\}$) is an individual-specific parameter drawn from a symmetric group-specific distribution that is assumed to be uniform in support $[-1/2\phi^j, 1/2\phi^j]$. It measures voter i's individual ideological bias toward candidate R. Intuitively, a positive (negative) σ^{ij} implies that voter i has a bias in favor of party R (L). The parameter δ measures the relative popularity of candidate R in the population, which is assumed to be uniform in support $[-1/2\psi, 1/2\psi]$ with density ψ. Therefore, the sum of the terms σ^{ij} and δ captures the relative appeal of candidate R.

To calculate the vote share of each candidate, we identify the swing voter in group $j \in \{o, e, u\}$—that is, a voter whose ideological bias, given the candidate's platforms, makes him or her indifferent between the two parties:

$$\begin{aligned}
\sigma^o &= V^o\left(p^L; K, D\right) - V^o\left(p^R; K, D\right) - \delta, \\
\sigma^j &= V^{yj}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) - V^{yj}\left(p^R, \tilde{G}\left(p^R\right); K, D\right) - \delta, \quad j = e, u.
\end{aligned}$$

All voters i in group j with $\sigma^{ij} > \sigma^{j}$ prefer party R. Hence, given the distributional assumptions, candidate R's actual vote share, π^R, is

$$\pi^R = \frac{1}{2}\Bigg[\phi^o \cdot \left\{\frac{1}{2\phi^o} - \left(V^o\left(p^L; K, D\right) - V^o\left(p^R; K, D\right) - \delta\right)\right\}$$
$$+\, l\phi^e \cdot \left\{\frac{1}{2\phi^e} - \left(V^{ye}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) - V^{ye}\left(p^R, \tilde{G}\left(p^R\right); K, D\right) - \delta\right)\right\}$$
$$+ (1-l)\phi^u \cdot \left\{\frac{1}{2\phi^u} - \left(V^{yu}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) - V^{yu}\left(p^R, \tilde{G}\left(p^R\right); K, D\right) - \delta\right)\right\}\Bigg].$$

Candidate R's probability of winning the election is

$$\text{Prob}\left[\pi^R \geq \frac{1}{2}\right]$$
$$= \text{Prob}\Bigg[\delta \geq \hat{\delta} \equiv \frac{1}{\phi^o + l\phi^e + (1-l)\phi^u} \cdot \left\{\phi^o \cdot \left(V^o\left(p^L; K, D\right) - V^o\left(p^R; K, D\right)\right)\right.$$
$$+\, l\phi^e \cdot \left(V^{ye}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) - V^{ye}\left(p^R, \tilde{G}\left(p^R\right); K, D\right)\right)$$
$$\left. + (1-l)\phi^u \cdot \left(V^{yu}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) - V^{yu}\left(p^R, \tilde{G}\left(p^R\right); K, D\right)\right)\right]$$
$$= \frac{1}{2} - \frac{\psi}{\phi^o + l\phi^e + (1-l)\phi^u} \cdot \left[\phi^o \cdot \left(V^o\left(p^L; K, D\right) - V^o\left(p^R; K, D\right)\right)\right.$$
$$+\, l\phi^e \cdot \left(V^{ye}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) - V^{ye}\left(p^R, \tilde{G}\left(p^R\right); K, D\right)\right)$$
$$\left. + (1-l)\phi^u \cdot \left(V^{yu}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) - V^{yu}\left(p^R, \tilde{G}\left(p^R\right); K, D\right)\right)\right]$$
$$= \frac{1}{2} - \omega \cdot \left(V^o\left(p^L; K, D\right) - V^o\left(p^R; K, D\right)\right)$$
$$- (1-\omega) \cdot \left[l \cdot \left(\left(V^{ye}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) - V^{ye}\left(p^R, \tilde{G}\left(p^R\right); K, D\right)\right)\right)\right.$$
$$\left. - (1-l) \cdot \left(V^{yu}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) - V^{yu}\left(p^R, \tilde{G}\left(p^R\right); K, D\right)\right)\right],$$

where $\phi^e = \phi^u$ is assumed, and ω is defined as

$$\omega \equiv \frac{\psi\phi^o}{\phi^o + l\phi^e + (1-l)\phi^u}.$$

Because both candidates seek to maximize their probability of winning the election, the Nash equilibrium is characterized by the following equation:

$$\left(p^{L*}\right) = \max_{p^L}\left\{\begin{array}{c} \omega \cdot \left(V^o\left(p^L; K, D\right) - V^o\left(p^R; K, D\right)\right) \\ + (1-\omega) \cdot \left[l \cdot \left(\left(V^{ye}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) - V^{ye}\left(p^R, \tilde{G}\left(p^R\right); K, D\right)\right)\right)\right. \\ \left. + (1-l) \cdot \left(V^{yu}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) - V^{yu}\left(p^R, \tilde{G}\left(p^R\right); K, D\right)\right)\right] \end{array}\right\},$$

$$\left(p^{R*}\right) = \max_{p^R}\left\{\begin{array}{c} -\omega \cdot \left(V^o\left(p^L; K, D\right) - V^o\left(p^R; K, D\right)\right) \\ - (1-\omega) \cdot \left[l \cdot \left(\left(V^{ye}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) - V^{ye}\left(p^R, \tilde{G}\left(p^R\right); K, D\right)\right)\right)\right. \\ \left. - (1-l) \cdot \left(V^{yu}\left(p^L, \tilde{G}\left(p^L\right); K, D\right) - V^{yu}\left(p^R, \tilde{G}\left(p^R\right); K, D\right)\right)\right] \end{array}\right\}.$$

Therefore, the two candidates' platforms converge in equilibrium to the same fiscal policy that maximizes the weighted-average utility of elderly persons, young employed persons, and young unemployed persons, $\omega V^o + (1-\omega)\cdot(lV^{ye} + (1-l)V^{yu})$, subject to the government budget constraint. This is the political objective function given in the main body of the paper.

■

1.2 Supplement to Appendix A.7

The objective here is to show the following proposition.

Proposition B.1. *Consider a debt constraint $D' \leq \Psi(K,D)$. There is a Markov-perfect political equilibrium with the debt constraint if $\Psi(K,D)$ is specified by*

$$\Psi(K,D) \equiv \mu \cdot (K+D) + l\phi AD,$$

where $\mu \in \Re$ is constant.

Consider the following debt constraint:

$$D' \leq \mu_K K + \mu_D D, \tag{1}$$

where $\mu_K, \mu_D \in \Re$. Assume that this constraint is binding, and guess the following linear policy function of g':

$$g' = G_K K' + G_D D',$$

where $G_K, G_D \in \Re$. Given this guess, we solve the maximization problem of the political objective and obtain the policy function for g. We then calculate μ_K and μ_D that verify the initial guess.

Suppose that the constraint is binding: $D' = \mu_K K + \mu_D D$. The government budget constraint, $D' = \tau l\phi(1-\alpha)AK = g + (1-l)b + RD$, is reformulated as

$$1-\tau = \frac{l\phi(1-\alpha)AK - RD - g - (1-l)b + \mu_K K + \mu_D D}{l\phi(1-\alpha)AK}. \tag{2}$$

We substitute (2) into the capital-market-clearing condition and obtain

$$K' = \frac{\beta}{1+\beta}\cdot[l\phi(1-\alpha)AK - RD - g + \mu_K K + \mu_D D] - D'. \tag{3}$$

With (1) and (3), the guess of g', $g' = G_K K' + G_D D'$, is reformulated as follows:

$$\begin{aligned} g' &= G_K\frac{\beta}{1+\beta}\cdot[l\phi(1-\alpha)AK - RD - g + \mu_K K + \mu_D D] - G_K D' + G_D D' \\ &= G_K\frac{\beta}{1+\beta}\cdot[l\phi(1-\alpha)AK - RD - g + \mu_K K + \mu_D D] \\ &+ (G_D - G_K)\cdot(\mu_K K + \mu_D D)\,. \end{aligned} \tag{4}$$

Thus, the problem is to maximize

$$P = \omega\eta \ln g + (1-\omega)(1+\beta)\, l \ln(1-\tau)\,\phi(1-\alpha)AK + (1-\omega)(1+\beta)(1-l)\ln b + (1-\omega)\,\beta\eta \ln g',$$

subject to (2) and (4).

The first-order conditions with respect to g and b are

$$g : \frac{\omega\eta}{g} = \frac{(1-\omega)(1+\beta)\, l}{l\phi(1-\alpha)AK - RD - g - (1-l)b + \mu_K K + \mu_D D} + \frac{(1-\omega)\,\beta\eta G_K \frac{\beta}{1+\beta}}{G_K \frac{\beta}{1+\beta}\left\{l\phi(1-\alpha)AK - RD - g + \mu_K K + \mu_D D\right\} + (G_D - G_K)\cdot(\mu_K K + \mu_D D)},$$

$$b : \frac{l}{l\phi(1-\alpha)AK - RD - (1-l)b + \mu_K K + \mu_D D} = \frac{1}{b}.$$

The first-order condition with respect to b leads to

$$b = l\phi(1-\alpha)AK - RD + \mu_K K + \mu_D D.$$

Plugging this into the first-order condition with respect to g, we obtain

$$\frac{\omega\eta}{g} = \frac{(1-\omega)(1+\beta)}{l\phi(1-\alpha)AK - RD + \mu_K K + \mu_D D} + \frac{(1-\omega)\,\beta\eta G_K \frac{\beta}{1+\beta}}{G_K \frac{\beta}{1+\beta}\left\{l\phi(1-\alpha)AK - RD - g + \mu_K K + \mu_D D\right\} + (G_D - G_K)\cdot(\mu_K K + \mu_D D)}.$$

If $G_D = G_K$, this condition leads to the following linear policy function of g:

$$g = \frac{\omega\eta}{(1-\omega)(1+\beta(1+\eta)) + \omega\eta} \cdot \left[l\phi(1-\alpha)AK - RD + \mu_K K + \mu_D D\right]. \tag{5}$$

Eq. (5) suggests that the initial guess, $g' = G_K K' + G_D D' = G_K \cdot (K' + D')$, is correct if

$$l\phi A(1-\alpha) + \mu_K = -R + \mu_D$$

—that is, if

$$\mu_D = l\phi A + \mu_K, \tag{6}$$
$$G_D = G_K = \frac{\omega\eta}{(1-\omega)(1+\beta(1+\eta)) + \omega\eta} \cdot \left(l\phi A(1-\alpha) + \mu_K\right).$$

Condition (6) implies that we obtain a Markov-perfect political equilibrium if the constraint is given by

$$D' \leq \mu_K K + (l\phi A + \mu_K)\, D,$$

or

$$D' \leq \mu \cdot (K + D) + l\phi A D,$$

where $\mu \in \Re$.

■

1.3 Proof of Proposition A.1

The problem the government faces in the presence of the debt constraint is

$$\begin{aligned}
\max P = {} & \omega\eta \ln g + (1-\omega)(1+\beta) l \ln (1-\tau)\,\phi(1-\alpha)AK \\
& + (1-\omega)(1+\beta)(1-l)\ln b + (1-\omega)\beta\eta \ln g' \\
& \text{s.t. } K' + D' = \frac{\beta}{1+\beta} \cdot \left[l\,(1-\tau)\,\phi(1-\alpha)AK + (1-l)b \right], \\
& g + (1-l)b + RD = \tau l \phi(1-\alpha)AK + D', \\
& D' \leq Al\phi D, \\
& \text{given } K \text{ and } D.
\end{aligned}$$

Suppose that the debt constraint is binding: $D' = Al\phi D$. Then, the government budget constraint is $g + (1-l)b + RD = \tau l \phi(1-\alpha)AK + Al\phi D$, or

$$1 - \tau = \frac{l\phi(1-\alpha)A\,(K+D) - g - (1-l)b}{l\phi(1-\alpha)AK}. \tag{7}$$

The substitution of (7) into the capital-market-clearing condition leads to

$$K' = \frac{\beta}{1+\beta} \cdot \left[l\phi(1-\alpha)A\,(K+D) - g \right] - D'. \tag{8}$$

Here, we conjecture that $g' = G \cdot (K' + D')$, where $G(>0)$ is constant. With (8), we can reformulate this conjecture as

$$g' = G \cdot \frac{\beta}{1+\beta} \cdot \left[l\phi(1-\alpha)A\,(K+D) - g \right].$$

Plugging this into P leads to

$$\begin{aligned}
P = {} & \omega\eta \ln g + (1-\omega)(1+\beta) l \ln (1-\tau)\,\phi(1-\alpha)AK \\
& + (1-\omega)(1+\beta)(1-l)\ln b + (1-\omega)\beta\eta \ln \left[l\phi(1-\alpha)A\,(K+D) - g \right],
\end{aligned}$$

where unrelated terms are omitted from the expression.

The first-order conditions with respect to g and b are

$$g : \frac{\omega\eta}{g} = \frac{(1-\omega)\,(1+\beta)\,l}{l\phi(1-\alpha)A\,(K+D) - g - (1-l)b} + \frac{(1-\omega)\,\beta\eta}{l\phi(1-\alpha)A\,(K+D) - g}, \tag{9}$$

$$b : \frac{(1-\omega)\,(1+\beta)\,l(1-l)}{l\phi(1-\alpha)A\,(K+D) - g - (1-l)b} = \frac{(1-\omega)\,(1+\beta)\,(1-l)}{b}. \tag{10}$$

Condition (10) is reformulated as

$$b = l\phi(1-\alpha)A\,(K+D) - g. \tag{11}$$

Using (9) and (11), we can obtain the policy functions of g and b as demonstrated in Proposition A.1.

We substitute the policy functions of g and b into the government budget constraint in (7) and obtain the tax rate as in Proposition A.1. We also substitute them into (8) and obtain the capital-market-clearing condition as in Proposition A.1. By using $D' = Al\phi D$ and the capital-market-clearing condition, we can calculate $D'K'$ as a function of D/K.

Taking into account the nonbinding case, we can write the policy function of D' as

$$D' = \min\left\{ Al\phi D, \frac{1-\omega}{(1-\omega)(1+\beta(1+\eta))+\omega\eta} \cdot \beta \cdot (\tilde{\eta}-\eta) \cdot (l\phi(1-\alpha)AK - RD) \right\}.$$

Thus, the debt constraint is binding if

$$Al\phi D \leq \frac{1-\omega}{(1-\omega)(1+\beta(1+\eta))+\omega\eta} \cdot \beta \cdot (\tilde{\eta}-\eta) \cdot (l\phi(1-\alpha)AK - RD)$$

—that is, if

$$\frac{D}{K} \leq d \equiv \frac{(1-\omega)(1-\alpha)\beta(\tilde{\eta}-\eta)}{\{(1-\omega)(1+\beta(1+\eta))+\omega\eta\} + (1-\omega)\beta\alpha(\tilde{\eta}-\eta)}.$$

■