A EHL algebra

A.1 Calvo

The Lagrangian for the EHL Calvo setup is given by

$$L = \sum_{k=0}^{\infty} (\beta\theta_w)^k \, E_t \Bigg[U \left(C_{t+k|t} \; , \left(\frac{\Gamma^{ind}_{t,t+k} W^*_t}{W_{t+k}} \right)^{-\varepsilon_w} N^d_{t+k}, \cdot \right) - \lambda_{t+k|t} \left\{ (1+\tau^c_{t+k}) P_{t+k} C_{t+k|t} - (1-\tau^n_{t+k}) \Gamma^{ind}_{t,t+k} W^*_t \left(\frac{\Gamma^{ind}_{t,t+k} W^*_t}{W_{t+k}} \right)^{-\varepsilon_w} N^d_{t+k} - X_{t+k} \right\} \Bigg] , \tag{A.1}$$

where $\lambda_{t+k|t}$ is the Lagrange multiplier and the j index has been suppressed. The FOC for consumption is given by

$$(1+\tau^c_{t+k}) \lambda_{t+k|t} P_{t+k} = V_{C,t+k|t} \; . \tag{A.2}$$

The FOC for W^*_t is given by

$$0 = \sum_{k=0}^{\infty} (\beta\theta_w)^k \, E_t \Bigg[U_N \left(C_{t+k|t}, \left(\frac{\Gamma^{ind}_{t,t+k} W^*_t}{W_{t+k}} \right)^{-\varepsilon_w} N^d_{t+k}, \cdot \right) (-\varepsilon_w) \left(\frac{\Gamma^{ind}_{t,t+k} W^*_t}{W_{t+k}} \right)^{-\varepsilon_w} \frac{N^d_{t+k}}{W^*_t} + \lambda_{t+k|t} \left\{ (1-\varepsilon_w)(1-\tau^n_{t+k}) \Gamma^{ind}_{t,t+k} \left(\frac{\Gamma^{ind}_{t,t+k} W^*_t}{W_{t+k}} \right)^{-\varepsilon_w} N^d_{t+k} \right\} \Bigg] , \tag{A.3}$$

where U_N denotes the partial derivative of the felicity function with respect to N. Using

$$N^j_{t+k|t} = \left(\frac{W^j_{t+k|t}}{W_{t+k}} \right)^{-\varepsilon_w} N^d_{t+k} \; , \tag{2.2}$$

$$W^j_{t+k|t} = \Gamma^{ind}_{t,t+k} W^*_t \; , \tag{2.3}$$

and suppressing the arguments of the felicity function this can be rewritten as:

$$0 = \sum_{k=0}^{\infty} (\beta\theta_w)^k \, E_t \left[N_{t+k|t} \lambda_{t+k|t} \left(\frac{\varepsilon_w}{\varepsilon_w - 1} \frac{U_{N,t+k|t}}{\lambda_{t+k|t}} + (1-\tau^n_{t+k}) \Gamma^{ind}_{t,t+k} W^*_t \right) \right] \; . \tag{A.4}$$

Replacing $\lambda_{t+k|t}$ using (A.2) yields

$$0 = \sum_{k=0}^{\infty} (\beta\theta_w)^k \, E_t \left[N_{t+k|t} \frac{V_{C,t+k|t}(1-\tau^n_{t+k})}{(1+\tau^c_{t+k})} \left(\frac{\varepsilon_w}{\varepsilon_w - 1} \frac{U_{N,t+k|t}(1+\tau^c_{t+k})}{V_{C,t+k|t}(1-\tau^n_{t+k})} + \frac{\Gamma^{ind}_{t,t+k} W^*_t}{P_{t+k}} \right) \right] \; . \tag{A.5}$$

Making use of the definition of the after-tax marginal rate of substitution

$$MRS_{t+k|t} = - \frac{\left(1+\tau^c_{t+k}\right)}{\left(1-\tau^n_{t+k}\right)} \frac{U_{N,t+k|t}}{V_{C,t+k|t}} \tag{2.5}$$

this yields

$$0 = \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left[N_{t+k|t} \frac{V_{C,t+k|t}(1-\tau^n_{t+k})}{(1+\tau^c_{t+k})} \left(\frac{\varepsilon_w}{\varepsilon_w - 1} MRS_{t+k|t} - \frac{\Gamma^{ind}_{t,t+k} W^*_t}{P_{t+k}} \right) \right] . \quad (A.6)$$

Performing a log-linearization around the deterministic steady state yields[26]

$$0 = \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left[\frac{\varepsilon_w}{\varepsilon_w - 1} MRS \times \widehat{MRS}_{t+k|t} - \Gamma^{ind}_k \frac{W^*}{P} \left(\hat{W}^*_t - \hat{P}_{t+k} + \hat{\Gamma}^{ind}_{t,t+k} \right) \right] \quad (A.7)$$

or

$$\hat{W}^*_t = (1-\beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left[\widehat{MRS}_{t+k|t} + \hat{P}_{t+k} - \hat{\Gamma}^{ind}_{t,t+k} \right] . \quad (2.6)$$

Expand $MRS_{t+k|t}(C_{t+k|t}, N_{t+k|t})$ by the average MRS in the economy

$$MRS_{t+k|t} = \frac{MRS_{t+k|t}}{MRS_{t+k}} MRS_{t+k} \quad (A.8)$$

and log-linearize around the deterministic steady state:[27]

$$\widehat{MRS}_{t+k|t} = \varepsilon^{mrs}_c \left(\hat{C}_{t+k|t} - \hat{C}_{t+k} \right) + \varepsilon^{mrs}_n \left(\hat{N}_{t+k|t} - \hat{N}_{t+k} \right) + \widehat{MRS}_{t+k} , \quad (A.9)$$

where $\varepsilon^{mrs}_c \equiv (MRS_C \times C)/MRS$ and $\varepsilon^{mrs}_n \equiv (MRS_N \times N)/MRS$ denote the elasticities of the MRS with respect to C and N, respectively. Due to the required assumption of complete markets and equal initial wealth, marginal utilities are equal across households. Therefore

$$V_{C,t+k} = V_{C,t+k|t} \quad (A.10)$$

and log-linearized

$$V_{CC} C \hat{C}_{t+k} + V_{CN} N \hat{N}_{t+k} = V_{CC} C \hat{C}_{t+k|t} + V_{CN} N \hat{N}_{t+k|t} . \quad (A.11)$$

Rearranging

$$V_{CC} C \left(\hat{C}_{t+k|t} - \hat{C}_{t+k} \right) = -V_{CN} N \left(\hat{N}_{t+k|t} - \hat{N}_{t+k} \right) \quad (A.12)$$

[26] Depending on the exact conduct of monetary policy, e.g., in case of an interest rate rule, the steady state of nominal variables like P_t and W_t may not be well-defined (see e.g. Galí 2015). Linearization in this case can be interpreted as being done around the long-run trend of the nominal variables. Linearization around a proper steady state would involve rewriting the problem in terms of stationary variables like the real wage W_t/P_t and inflation rates, but would yield the same results as trend changes only appear as ratios and therefore cancel out.

[27] The computational steps here follow Sbordone (2006). If the MRS depends on additional variables like housing or durables, the same approach can be followed to replace the idiosyncratic MRS by the aggregate one.

and plugging into (A.9) yields

$$\widehat{MRS}_{t+k|t} = \widehat{MRS}_{t+k} + \underbrace{\left[-\frac{V_{CN}N}{V_{CC}C}\varepsilon_c^{mrs} + \varepsilon_n^{mrs}\right]}_{\equiv \varepsilon_{tot}^{mrs}} \left(\hat{N}_{t+k|t} - \hat{N}_{t+k}\right) . \tag{2.7}$$

This together with the linearized labor demand

$$\hat{N}_{t+k|t} = -\varepsilon_w \left(\hat{\Gamma}^{ind}_{t,t+k} + \hat{W}^*_t - \hat{W}_{t+k}\right) + \hat{N}^d_{t+k} \tag{A.13}$$

and the fact that up to first-order wage dispersion is zero and therefore $N^d_{t+k} = N_{t+k}$ can be used to express the idiosyncratic MRS as

$$\widehat{MRS}_{t+k|t} = \widehat{MRS}_{t+k} - \varepsilon_w \varepsilon_{tot}^{mrs} \left(\hat{\Gamma}^{ind}_{t,t+k} + \hat{W}^*_t - \hat{W}_{t+k}\right) . \tag{A.14}$$

Plug into (2.6) to get

$$\hat{W}^*_t = (1-\beta\theta_w)\left(\hat{W}_t + \frac{1}{1+\varepsilon_w\varepsilon_{tot}^{mrs}}\left(\widehat{MRS}_t - \left(\hat{W}_t - \hat{P}_t\right)\right)\right) + \beta\theta_w E_t\left(\hat{W}^*_{t+1} - \hat{\Gamma}^{ind}_{t,t+1}\right) . \tag{A.15}$$

where we have made use of $\hat{\Gamma}^{ind}_{t,t+k} = \hat{\Gamma}^{ind}_{t,t+1} + \hat{\Gamma}^{ind}_{t+1,t+k}$ and $\hat{\Gamma}^{ind}_{t,t} = 0$.

Next, plug in from the linearized LOM for wages in the economy

$$\hat{W}^*_t = \frac{1}{1-\theta_w}\hat{W}_t - \frac{\theta_w}{1-\theta_w}(\hat{\Gamma}^{ind}_{t-1,t} + \hat{W}_{t-1}) \tag{A.16}$$

to get

$$\begin{aligned} \frac{1}{1-\theta_w}\hat{W}_t - \frac{\theta_w}{1-\theta_w}\left(\hat{\Gamma}^{ind}_{t-1,t} + \hat{W}_{t-1}\right) \;&=\; (1-\beta\theta_w)\left(\hat{W}_t - \frac{1}{1+\varepsilon_w\varepsilon_{tot}^{mrs}}\hat{\mu}^w_t\right) \\ &+ \beta\theta_w E_t\left(-\hat{\Gamma}^{ind}_{t,t+1} + \frac{1}{1-\theta_w}\hat{W}_{t+1} - \frac{\theta_w}{1-\theta_w}\left(\hat{\Gamma}^{ind}_{t,t+1} + \hat{W}_t\right)\right) . \end{aligned} \tag{A.17}$$

Now add 0 to the left-hand side and expand the right-hand side:

$$\begin{aligned} &\frac{1}{1-\theta_w}\hat{W}_t - \frac{\theta_w}{1-\theta_w}\left(\hat{\Gamma}^{ind}_{t-1,t} + \hat{W}_{t-1}\right) + \left(\frac{1}{1-\theta_w}\hat{W}_{t-1} - \frac{1}{1-\theta_w}\hat{W}_{t-1}\right) \\ &= (1-\beta\theta_w)\hat{W}_t - \beta\theta_w\left(\frac{\theta_w}{1-\theta_w}\left(E_t\hat{\Gamma}^{ind}_{t,t+1} + \hat{W}_t\right) + \frac{1-\theta_w}{1-\theta_w}E_t\hat{\Gamma}^{ind}_{t,t+1}\right) \\ &+ \frac{\beta\theta_w}{1-\theta_w}E_t\left(\hat{W}_{t+1}\right) - \frac{(1-\beta\theta_w)}{1+\varepsilon_w\varepsilon_{tot}^{mrs}}\hat{\mu}^w_t . \end{aligned} \tag{A.18}$$

Factor the left-hand side and collect terms related to W_t on the right-hand side

$$\begin{aligned}
&\frac{1}{1-\theta_w}\left(\hat{W}_t - \hat{W}_{t-1}\right) + \hat{W}_{t-1} - \frac{\theta_w}{1-\theta_w}\hat{\Gamma}^{ind}_{t-1,t} \\
&= \underbrace{\left(\frac{1-\beta\theta_w - \theta_w\left(1-\beta\theta_w\right) - \beta\theta_w\theta_w}{1-\theta_w}\right)}_{1-\frac{\beta\theta_w}{1-\theta_w}}\hat{W}_t \\
&- \frac{\beta\theta_w}{1-\theta_w}E_t\hat{\Gamma}^{ind}_{t,t+1} + \frac{\beta\theta_w}{1-\theta_w}E_t\left(\hat{W}_{t+1}\right) - \frac{\left(1-\beta\theta_w\right)}{1+\varepsilon_w\varepsilon^{mrs}_{tot}}\hat{\mu}^w_t \ .
\end{aligned} \tag{A.19}$$

Subtract W_t from both sides

$$\begin{aligned}
&\frac{1}{1-\theta_w}\left(\hat{W}_t - \hat{W}_{t-1}\right) - \frac{\theta_w}{1-\theta_w}\hat{\Gamma}^{ind}_{t-1,t} - \left(\hat{W}_t - \hat{W}_{t-1}\right) \\
&= \frac{\beta\theta_w}{1-\theta_w}E_t\left(\hat{W}_{t+1} - \hat{W}_t\right) - \frac{\left(1-\beta\theta_w\right)}{1+\varepsilon_w\varepsilon^{mrs}_{tot}}\hat{\mu}^w_t - \frac{\beta\theta_w}{1-\theta_w}E_t\hat{\Gamma}^{ind}_{t,t+1} \ .
\end{aligned} \tag{A.20}$$

Collecting terms:

$$\begin{aligned}
&\frac{\theta_w}{1-\theta_w}\left(\hat{W}_t - \hat{W}_{t-1}\right) - \frac{\theta_w}{1-\theta_w}\hat{\Gamma}^{ind}_{t-1,t} \\
&= \frac{\beta\theta_w}{1-\theta_w}E_t\left(\hat{W}_{t+1} - \hat{W}_t\right) - \frac{\left(1-\beta\theta_w\right)}{1+\varepsilon_w\varepsilon^{mrs}_{tot}}\hat{\mu}^w_t - \frac{\beta\theta_w}{1-\theta_w}E_t\hat{\Gamma}^{ind}_{t,t+1} \ .
\end{aligned} \tag{A.21}$$

Solve for wage inflation:

$$\hat{\Pi}^w_t = \beta E_t\hat{\Pi}_{w,t+1} - \frac{\left(1-\theta_w\right)\left(1-\beta\theta_w\right)}{\theta_w\left(1+\varepsilon_w\varepsilon^{mrs}_{tot}\right)}\hat{\mu}^w_t - \frac{\beta\theta_w}{1-\theta_w}E_t\hat{\Gamma}^{ind}_{t,t+1} + \frac{\theta_w}{1-\theta_w}\hat{\Gamma}^{ind}_{t-1,t} \ . \tag{2.8}$$

A.2 Rotemberg

The Lagrangian for the EHL Rotemberg setup is given by

$$L = \sum_{k=0}^{\infty}\beta^k E_t\left[\begin{array}{l} U\left(C_{t+k}, \left(\dfrac{W^j_{t+k}}{W_{t+k}}\right)^{-\varepsilon_w} N^d_{t+k}\right) \\ -\lambda_{t+k}\left\{\begin{array}{l} \left(1+\tau^c_{t+k}\right)P_{t+k}C^j_{t+k} - \left(1-\tau^n_{t+k}\right)W^j_{t+k}\left(\dfrac{W^j_{t+k}}{W_{t+k}}\right)^{-\varepsilon_w} N^d_{t+k} \\ +\dfrac{\phi_w}{2}\left(\dfrac{1}{\Gamma^{ind}_{t+k-1,t+k}}\dfrac{W^j_{t+k}}{W^j_{t+k-1}} - 1\right)^2 \Xi_{t+k} - X_{t+k} \end{array}\right\} \end{array}\right] . \tag{A.22}$$

The FOC for consumption is given by

$$(1+\tau^c_{t+k})\lambda^j_{t+k}P_{t+k} = V_{C,t+k} \ . \tag{A.23}$$

The corresponding FOC for the optimal wage is given by

$$
\begin{aligned}
0 \;=\; & U_N\left(C_t^j,\left(\frac{W_t^j}{W_t}\right)^{-\varepsilon_w} N_t^d,\cdot\right)(-\varepsilon_w)\left(\frac{W_t^j}{W_t}\right)^{-\varepsilon_w}\frac{N_t^d}{W_t^j} \\
& + \lambda_t^j\left\{(1-\varepsilon_w)\left(1-\tau_t^n\right)\left(\frac{W_t^j}{W_t}\right)^{-\varepsilon_w} N_t^d - \phi_w\left(\frac{1}{\Gamma_{t-1,t}^{ind}}\frac{W_t^j}{W_{t-1}^j}-1\right)\frac{\Xi_t}{\Gamma_{t-1,t}^{ind}W_{t-1}^j}\right\} \\
& - E_t\lambda_{t+1}^j\left\{\phi_w\left(\frac{1}{\Gamma_{t,t+1}^{ind}}\frac{W_{t+1}^j}{W_t^j}-1\right)(-1)\frac{W_{t+1}^j}{\left(W_t^j\right)^2}\frac{1}{\Gamma_{t,t+1}^{ind}}\Xi_{t+1}\right\}\;.
\end{aligned} \tag{A.24}
$$

As there is no wage dispersion in the Rotemberg case, imposing symmetry means that $N_t^j = N_t^d = N_t$. Additionally substituting for λ_t from (A.23) and dividing by $V_{C,t}/(1+\tau_t^c)$, the above equation can be written as

$$
\begin{aligned}
0 \;=\; & \frac{U_{N,t}}{V_{C,t}}(1+\tau_t^c)(-\varepsilon_w)\frac{N_t}{W_t}+\frac{1}{P_t}\left\{(1-\varepsilon_w)(1-\tau_t^n)N_t-\phi_w\left(\frac{1}{\Gamma_{t-1,t}^{ind}}\frac{W_t}{W_{t-1}}-1\right)\frac{\Xi_t}{\Gamma_{t-1,t}^{ind}W_{t-1}}\right\} \\
& + E_t\beta\frac{V_{C,t+1}}{V_{C,t}}\frac{(1+\tau_t^c)}{(1+\tau_{t+1}^c)}\frac{1}{W_t}\left\{\phi_w\left(\frac{1}{\Gamma_{t,t+1}^{ind}}\frac{W_{t+1}}{W_t}-1\right)\frac{W_{t+1}}{W_t}\frac{1}{\Gamma_{t,t+1}^{ind}}\frac{\Xi_{t+1}}{P_{t+1}}\right\}\;,
\end{aligned} \tag{A.25}
$$

or, dividing by N_t, multiplying by P_t, and making use of the definition of the after-tax MRS (2.5), as

$$
\begin{aligned}
0 \;=\; & \varepsilon_w\frac{MRS_t}{\frac{W_t}{P_t}}(1-\tau_t^n)+\left\{(1-\varepsilon_w)(1-\tau_t^n)-\phi_w\left(\frac{\Pi_{w,t}}{\Gamma_{t-1,t}^{ind}}-1\right)\Pi_t\frac{1}{N_t}\frac{1}{\Gamma_{t-1,t}^{ind}}\frac{\frac{\Xi_t}{P_t}}{\frac{W_{t-1}}{P_{t-1}}}\right\} \\
& + E_t\beta\frac{V_{C,t+1}}{V_{C,t}}\frac{(1+\tau_t^c)}{(1+\tau_{t+1}^c)}\frac{1}{N_t}\frac{1}{\frac{W_t}{P_t}}\left\{\phi_w\left(\frac{\Pi_{w,t+1}}{\Gamma_{t,t+1}^{ind}}-1\right)\frac{\Pi_{w,t+1}}{\Gamma_{t,t+1}^{ind}}\frac{\Xi_{t+1}}{P_{t+1}}\right\}\;.
\end{aligned} \tag{2.13}
$$

Linearizing (2.13) around the steady state and making use of

$$\hat{\mu}_t^w \equiv \left(\hat{W}_t - \hat{P}_t\right) - \widehat{MRS}_t \tag{2.9}$$

and $\Gamma_1^{ind} = \Pi$ yields

$$
\begin{aligned}
0 \;=\;& \underbrace{\varepsilon_w \frac{MRS}{\frac{W}{P}}}_{\frac{\varepsilon_w - 1}{\varepsilon_w}}(1-\tau_t^n)(-1)\hat{\mu}_t^w \\
&+ \underbrace{\left[\varepsilon_w \frac{MRS}{\frac{W}{P}}(-\tau^n) + (1-\varepsilon_w)(-\tau^n)\right]}_{0} \hat{\tau}_t^n \\
&- \phi_w \underbrace{\left(\Pi^{-1}\Pi_w - 1\right)}_{0} \Pi \frac{1}{N} \Pi^{-1} \frac{\Xi^{real}}{W^{real}} \left(\hat{\Pi}_t - \hat{N}_t - \hat{\Gamma}_{t-1,t}^{ind} + \hat{\Xi}_t^{real} - \hat{W}_{t-1}^{real}\right) \\
&- \phi_w \Pi \frac{1}{N} \Pi^{-1} \frac{\Xi^{real}}{W^{real}} \Pi^{-1}\Pi_w \hat{\Pi}_{w,t} \\
&+ E_t \beta \frac{1}{N} \frac{1}{W^{real}} \phi_w \underbrace{\left(\Pi^{-1}\Pi_w - 1\right)}_{0} \Pi^{-1}\Pi_w \Xi^{real} \left(\hat{V}_{C,t+1} - \hat{V}_{C,t} + \hat{\tau}_t^c - \hat{\tau}_{t+1}^c - \hat{N}_t - \hat{W}_t^{real} - \hat{\Xi}_{t+1}^{real}\right) \\
&+ E_t \beta \frac{1}{N} \frac{1}{W^{real}} \phi_w \Pi^{-1} \Xi^{real} \left(2\Pi^{-1}\Pi_w^2 - \Pi_w\right) \hat{\Pi}_{w,t+1} \\
&+ E_t \beta \frac{1}{N} \frac{1}{W^{real}} \phi_w \Pi_w \Xi^{real} \left(-2\Pi^{-2}\Pi_w + \Pi^{-1}\right) \hat{\Gamma}_{t,t+1}^{ind} \; .
\end{aligned}
\tag{A.26}
$$

Simplifying and using the steady state relation $\Pi = \Pi_w$ yields

$$
0 = (-1)\varepsilon_w \frac{\varepsilon_w - 1}{\varepsilon_w}(1-\tau^n)\hat{\mu}_t^w - \phi_w \underbrace{\frac{\Xi^{real}}{NW^{real}}}_{\frac{1}{\aleph}} \hat{\Pi}_{w,t} + E_t \beta \frac{\Xi^{real}}{NW^{real}} \phi_w \left(\hat{\Pi}_{w,t+1} - \hat{\Gamma}_{t,t+1}^{ind}\right) \tag{A.27}
$$

and thus

$$
\hat{\Pi}_{w,t} = \beta E_t \left(\hat{\Pi}_{w,t+1} - \hat{\Gamma}_{t,t+1}^{ind}\right) - \frac{(\varepsilon_w - 1)(1-\tau^n)\aleph}{\phi_w} \hat{\mu}_t^w \; . \tag{2.14}
$$

B SGU algebra

B.1 Calvo

The associated Lagrangian is given by

$$
\begin{aligned}
L = \sum_{k=0}^{\infty} \beta^k E_t \Bigg[& U\left(C_{t+k}\,, N_{t+k}, \cdot\right) \\
- \lambda_{t+k} \Big\{ & (1+\tau_{t+k}^c) P_{t+k} C_{t+k} - \left(1-\tau_{t+k}^n\right) W_{t+k}^{\varepsilon_w} N_{t+k}^d \theta_w^k \left(\Gamma_{t,t+k}^{ind} W_t^*\right)^{1-\varepsilon_w} - X_{t+k} \Big\} \Bigg] \, ,
\end{aligned}
\tag{B.1}
$$

where in the budget constraint we have made use of

$$\int_0^1 W_{t+k}^j N_{t+k}^j dj = \int_0^1 W_{t+k}^j \left(\frac{W_{t+k}^j}{W_{t+k}}\right)^{-\varepsilon_w} N_{t+k}^d dj$$
$$= W_{t+k}^{\varepsilon_w} N_{t+k}^d \int_0^1 \left(W_{t+k}^j\right)^{1-\varepsilon_w} dj = W_{t+k}^{\varepsilon_w} N_{t+k}^d \left(\theta_w^k \left(\Gamma_{t,t+k}^{ind} W_t^*\right)^{1-\varepsilon_w} + \left(1-\theta_w^k\right) X_{1,t+k}\right) . \tag{B.2}$$

The last term, $X_{1,t+k}$, captures the wage level in the other labor markets where price resetting has taken place. Hence, it is independent of W_t^* and can be omitted as it drops out when taking the derivative.

The FOC for consumption is given by

$$(1+\tau_{t+k}^c)\lambda_{t+k} P_{t+k} = V_{C,t+k} \ , \tag{B.3}$$

while the FOC for W_t^* is given by

$$0 = \sum_{k=0}^{\infty} \beta^k E_t \Bigg[U_{N,t+k} \frac{\partial N_{t+k}}{\partial W_t^*}$$
$$+ \lambda_{t+k} \left\{ (1-\varepsilon_w)(1-\tau_{t+k}^n) W_{t+k}^{\varepsilon_w} N_{t+k}^d \theta_w^k \left(\Gamma_{t,t+k}^{ind}\right)^{1-\varepsilon_w} (W_t^*)^{-\varepsilon_w} \right\} \Bigg] . \tag{B.4}$$

Making use of

$$N_{t+k} \equiv \int_0^1 N_{t+k}^j dj = \int_0^1 \left(\frac{W_{t+k}^j}{W_{t+k}}\right)^{-\varepsilon_w} N_{t+k}^d \ dj = W_{t+k}^{\varepsilon_w} N_{t+k}^d \int_0^1 \left(W_{t+k}^j\right)^{-\varepsilon_w} dj$$
$$= W_{t+k}^{\varepsilon_w} N_{t+k}^d \left(\theta_w^k \left(\Gamma_{t,t+k}^{ind} W_t^*\right)^{-\varepsilon_w} + \left(1-\theta_w^k\right) X_{2,t+k}\right) \ , \tag{B.5}$$

we can evaluate the inner derivative in the first line of (B.4) to get

$$0 = \sum_{k=0}^{\infty} \beta^k E_t \Bigg[U_{N,t+k} \left(-\varepsilon^w\right) \frac{N_{t+k}^d}{W_{t+k}^{-\varepsilon^w}} \theta_w^k \left(\Gamma_{t,t+k}^{ind}\right)^{-\varepsilon_w} (W_t^*)^{-\varepsilon_w - 1}$$
$$+ \lambda_{t+k} \left\{ (1-\varepsilon_w)(1-\tau_{t+k}^n) W_{t+k}^{\varepsilon_w} N_{t+k}^d \theta_w^k \left(\Gamma_{t,t+k}^{ind}\right)^{1-\varepsilon_w} (W_t^*)^{-\varepsilon_w} \right\} \Bigg] . \tag{B.6}$$

Factoring out, and multiplying by $(W_t^*)^{-\varepsilon_w - 1}$ yields

$$0 = \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \lambda_{t+k} N_{t+k}^d W_{t+k}^{\varepsilon_w} \left(\Gamma_{t,t+k}^{ind}\right)^{-\varepsilon_w}$$
$$\times \left[\frac{U_{N,t+k}}{\lambda_{t+k}} (-\varepsilon_w) + \left(1-\tau_{t+k}^n\right)(1-\varepsilon_w) \Gamma_{t,t+k}^{ind} W_t^* \right] \tag{B.7}$$

or

$$0 = \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \lambda_{t+k} N_{t+k}^d W_{t+k}^{\varepsilon_w} \left(1-\tau_{t+k}^n\right) \left(\Gamma_{t,t+k}^{ind}\right)^{-\varepsilon_w}$$
$$\times \left[\frac{U_{N,t+k}\left(1+\tau_{t+k}^c\right) P_{t+k}}{V_{C,t+k}\left(1-\tau_{t+k}^n\right)} (-\varepsilon_w) + (1-\varepsilon_w)\,\Gamma_{t,t+k}^{ind} W_t^*\right] . \tag{B.8}$$

Using the after-tax MRS definition, this is equal to

$$0 = E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k V_{C,t+k} N_{t+k}^d W_{t+k}^{\varepsilon_w} \frac{1-\tau_{t+k}^n}{1+\tau_{t+k}^c} \left(\Gamma_{t,t+k}^{ind}\right)^{-\varepsilon_w} \left[MRS_{t+k}\frac{\varepsilon_w}{\varepsilon_w-1} - \Gamma_{t,t+k}^{ind}\frac{W_t^*}{P_{t+k}}\right] . \tag{B.9}$$

Performing a log-linearization around the deterministic steady state yields

$$0 = \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left[\frac{\varepsilon_w}{\varepsilon_w-1} \times MRS\widehat{MRS}_{t+k} - \Gamma_k^{ind}\frac{W^*}{P}\left(\hat{W}_t^* - \hat{P}_{t+k} + \hat{\Gamma}_{t,t+k}^{ind}\right)\right] \tag{B.10}$$

or

$$\hat{W}_t^* = (1-\beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left[\widehat{MRS}_{t+k} + \hat{P}_{t+k} - \hat{\Gamma}_{t,t+k}^{ind}\right] . \tag{B.11}$$

Note that compared to the EHL case, it is the economy-wide MRS that shows up here, not the individual one. Subtracting $\hat{W}_t$ from both sides and using (2.9), we can write this recursively as

$$\hat{W}_t^* - \hat{W}_t = -\beta\theta_w \hat{W}_t + (1-\beta\theta_w)\,\hat{\mu}_t^w(-1) + \beta\theta_w E_t\left(\hat{W}_{t+1}^* - \hat{\Gamma}_{t,t+1}^{ind}\right) . \tag{B.12}$$

Using (A.16) we obtain

$$\begin{aligned}\left(\frac{1}{1-\theta_w}\hat{W}_t - \frac{\theta_w}{1-\theta_w}\left(\hat{\Gamma}_{t-1,t}^{ind} + \hat{W}_{t-1}\right)\right) - \hat{W}_t \;&=\; -\beta\theta_w \hat{W}_t + (1-\beta\theta_w)\,\hat{\mu}_t^w(-1)\\ &\quad + \beta\theta_w E_t\left(\frac{1}{1-\theta_w}\hat{W}_{t+1} - \frac{\theta_w}{1-\theta_w}\left(\hat{\Gamma}_{t,t+1}^{ind} + \hat{W}_t\right) - \hat{\Gamma}_{t,t+1}^{ind}\right)\end{aligned} \tag{B.13}$$

from which the New Keynesian Wage Phillips Curve follows as

$$\hat{\Pi}_t^w = \beta E_t \hat{\Pi}_{t+1}^w - \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w}\hat{\mu}_t^w - \frac{\beta\theta_w}{1-\theta_w}E_t\hat{\Gamma}_{t,t+1}^{ind} + \frac{\theta_w}{1-\theta_w}\hat{\Gamma}_{t-1,t}^{ind} . \tag{3.4}$$

B.2 Rotemberg

The Lagrangian is

$$L=\sum_{k=0}^{\infty}\beta^{k}E_{t}\left[\begin{array}{l}U\left(C_{t+k},N_{t+k},\cdot\right)\\ -\lambda_{t+k}\left\{\begin{array}{l}\left(1+\tau_{t+k}^{c}\right)P_{t+k}C_{t+k}-\left(1-\tau_{t+k}^{n}\right)N_{t+k}^{d}\int_{0}^{1}W_{t+k}^{j}\left(\frac{W_{t+k}^{j}}{W_{t+k}}\right)^{-\varepsilon_{w}}dj\\ +\frac{\phi_{w}}{2}\int_{0}^{1}\left(\frac{1}{\Gamma_{t,t+k}^{ind}}\frac{W_{t+k}^{j}}{W_{t+k-1}^{j}}-1\right)^{2}dj\Xi_{t+k}-X_{t+k}\end{array}\right\}\end{array}\right]. \tag{B.14}$$

The corresponding first order condition for the optimal wage is given by

$$\begin{aligned}0 \;=\; & U_{N,t}\left(-\varepsilon_{w}\right)\int_{0}^{1}\left(\frac{W_{t}^{j}}{W_{t}}\right)^{-\varepsilon_{w}}\frac{N_{t}^{d}}{W_{t}^{j}}\,dj\\ & +\lambda_{t}\left\{\left(1-\varepsilon_{w}\right)\left(1-\tau_{t}^{n}\right)N_{t}^{d}\int_{0}^{1}\left(\frac{W_{t}^{j}}{W_{t}}\right)^{-\varepsilon_{w}}dj-\phi_{w}\int_{0}^{1}\left(\frac{1}{\Gamma_{t-1,t}^{ind}}\frac{W_{t}^{j}}{W_{t-1}^{j}}-1\right)\frac{1}{\Gamma_{t-1,t}^{ind}W_{t-1}^{j}}\,dj\;\Xi_{t}\right\}\\ & -E_{t}\lambda_{t+1}\left\{\phi_{w}\int_{0}^{1}\left(\frac{1}{\Gamma_{t,t+1}^{ind}}\frac{W_{t+1}^{j}}{W_{t}^{j}}-1\right)\left(-1\right)\frac{W_{t+1}^{j}}{\Gamma_{t,t+1}^{ind}\left(W_{t}^{j}\right)^{2}}\,dj\;\Xi_{t+1}\right\}.\end{aligned} \tag{B.15}$$

Imposing symmetry

$$\begin{aligned}0=U_{N}\left(C_{t},N_{t},\cdot\right)\left(-\varepsilon_{w}\right)\frac{N_{t}^{d}}{W_{t}}+\lambda_{t}\left\{\left(1-\varepsilon_{w}\right)\left(1-\tau_{t}^{n}\right)N_{t}^{d}-\phi_{w}\left(\frac{1}{\Gamma_{t-1,t}^{ind}}\frac{W_{t}}{W_{t-1}}-1\right)\frac{\Xi_{t}}{\Gamma_{t-1,t}^{ind}W_{t-1}}\right\}\\ -E_{t}\lambda_{t+1}\left\{\phi_{w}\left(\frac{1}{\Gamma_{t,t+1}^{ind}}\frac{W_{t+1}}{W_{t}}-1\right)\left(-1\right)\frac{W_{t+1}}{\Gamma_{t,t+1}^{ind}\left(W_{t}\right)^{2}}\Xi_{t+1}\right\},\end{aligned} \tag{B.16}$$

which is identical to equation (A.25).

C Elasticities of the after-tax MRS

C.1 Habits

C.1.1 Additively separable

First consider additively separable preferences with habits of the form

$$\frac{\left(C_t - \phi_c C_{t-1}\right)^{1-\sigma} - 1}{1-\sigma} - \psi \frac{N^{1+\varphi}}{1+\varphi} \ , \tag{C.1}$$

where $0 \leq \phi_c \leq 1$ measures the degree of habits, $\varphi \geq 0$ is the inverse of the Frisch elasticity, $\sigma \geq 0$ determines the intertemporal elasticity of substitution, and $\psi > 0$ determines the weight of the disutility of labor.

If habits are internal, we get

$$V_{C_t} = \left(C_t - \phi_c C_{t-1}\right)^{-\sigma} - \beta \phi_c \left(C_{t+1} - \phi_c C_t\right)^{-\sigma}$$

and in steady state

$$V_C = \left(1 - \beta \phi_c\right) \left(\left(1 - \phi_c\right) C\right)^{-\sigma} \ .$$

Similarly, the other partial derivatives are given by

$$\begin{aligned} U_{N_t} &= -\psi N_t^{\varphi} \\ U_N &= -\psi N^{\varphi} \\ V_{C_t C_t} &= -\sigma \left(C_t - \phi_c C_{t-1}\right)^{-\sigma-1} + \beta \phi_c^2 \left(-\sigma\right) \left(C_{t+1} - \phi_c C_t\right)^{-\sigma-1} \\ V_{CC} &= \left(1 + \beta \phi_c^2\right) \left(-\sigma\right) \left(\left(1 - \phi_c\right) C\right)^{-\sigma-1} \\ V_{CN} &= 0 \end{aligned}$$

The marginal rate of substitution and its derivatives follow as

$$\begin{aligned} MRS &= \frac{1+\tau^c}{1-\tau^n} \frac{\psi N^{\varphi}}{\left(1 - \beta \phi_c\right) \left(\left(1 - \phi_c\right) C\right)^{-\sigma}} \\ MRS_N &= \varphi \frac{1+\tau^c}{1-\tau^n} \frac{\psi N^{\varphi-1}}{\left(1 - \beta \phi_c\right) \left(\left(1 - \phi_c\right) C\right)^{-\sigma}} \\ MRS_C &= \frac{1+\tau^c}{1-\tau^n} \psi N^{\varphi} \left(-1\right) \frac{1}{\left(V_C\right)^2} V_{CC} \end{aligned}$$

Therefore,

$$\varepsilon_n^{mrs} = \varphi \tag{C.2}$$

Table 6: Elasticities ε_n^{mrs},ε_c^{mrs}, and ε_{tot}^{mrs} for different felicity functions

	Add. separable	Add. sep. log leisure	Mult. separable	GHH
U	$\frac{C^{1-\sigma}-1}{1-\sigma} - \psi\frac{N^{1+\varphi}}{1+\varphi}$	$\frac{C^{1-\sigma}-1}{1-\sigma} + \psi\log(1-N_t)$	$\frac{\left(C^{\eta}(1-N)^{1-\eta}\right)^{1-\sigma}}{1-\sigma}$	$\frac{\left(C-\psi N^{1+\varphi}\right)^{1-\sigma}-1}{1-\sigma}$
V_N	$-\psi N^{\varphi}$	$-\psi\frac{1}{1-N}$	$-(1-\eta)(1-\sigma)\frac{U}{1-N}$	$\left(C-\psi N^{1+\varphi}\right)^{-\sigma}$ $\times(-\psi)(1+\varphi)N^{\varphi}$
V_C	$C^{-\sigma}$	$C^{-\sigma}$	$\eta(1-\sigma)\frac{U}{C}$	$\left(C-\psi N^{1+\varphi}\right)^{-\sigma}$
V_{CC}	$-\sigma C^{-\sigma-1}$	$-\sigma C^{-\sigma-1}$	$\eta(1-\sigma)(\eta(1-\sigma)-1)\frac{U}{C^2}$	$-\sigma\left(C-\psi N^{1+\varphi}\right)^{-\sigma-1}$
V_{CN}	0	0	$\eta(1-\sigma)(1-\eta)(\sigma-1)$ $\times\frac{U}{C(1-N)}$	$\sigma\left(C-\psi N^{1+\varphi}\right)^{-\sigma-1}$ $\times\psi(1+\varphi)N^{\varphi}$
MRS	$\frac{1+\tau^c}{1-\tau^n}\frac{\psi N^{\varphi}}{C^{-\sigma}}$	$\frac{1+\tau^c}{1-\tau^n}\frac{\psi(1-N_t)^{-1}}{C^{-\sigma}}$	$\frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{C}{1-N}$	$\frac{1+\tau^c}{1-\tau^n}\psi(1+\varphi)N^{\varphi}$
MRS_N	$\varphi\frac{1+\tau^c}{1-\tau^n}\frac{\psi N^{\varphi-1}}{C^{-\sigma}}$	$\frac{1+\tau^c}{1-\tau^n}\frac{\psi(1-N_t)^{-2}}{C^{-\sigma}}$	$\frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{C}{(1-N)^2}$	$\frac{1+\tau^c}{1-\tau^n}\psi(1+\varphi)\varphi N^{\varphi-1}$
MRS_C	$\frac{1+\tau^c}{1-\tau^n}\sigma\frac{\psi N^{\varphi}}{C^{1-\sigma}}$	$\frac{1+\tau^c}{1-\tau^n}\sigma\frac{\psi(1-N)^{-1}}{C^{1-\sigma}}$	$\frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{(1-\phi_c)}{(1-N)}$	0
ε_n^{mrs}	φ	$\frac{N}{1-N}$	$\frac{N}{1-N}$	φ
ε_c^{mrs}	σ	σ	1	0
ε_{tot}^{mrs}	φ	$\frac{N}{1-N}$	$\left[1-\frac{(1-\eta)(\sigma-1)}{\eta(1-\sigma)-1}\right]\frac{N}{1-N}$	φ
ε_{tot}^{mrs} (int. habits)	φ	$\frac{N}{1-N}$	$\left[1-\frac{(1-\eta)(\sigma-1)}{\eta(1-\sigma)-1}\frac{(1-\phi_c)}{(1+\phi_c^2\beta)}\right]\frac{N}{1-N}$	φ
ε_{tot}^{mrs} (ext. habits)	φ	$\frac{N}{1-N}$	$\left[1-\frac{(1-\eta)(\sigma-1)}{\eta(1-\sigma)-1}(1-\phi_c)\right]\frac{N}{1-N}$	φ

Table 6: *Notes*: Elasticities of the after-tax marginal rate of substitution with respect to hours worked, ε_n^{mrs}, with respect to consumption, ε_c^{mrs}, and the total elasticity, ε_{tot}^{mrs}, for additively separable preferences in consumption and hours worked (first column), additively separable preferences in consumption and log leisure (second column), for multiplicative preferences (third column), and Greenwood, Hercowitz, and Huffman (1988)-type preferences (fourth column). The last two rows display the total elasticity when internal or external habits in consumption of the form $C_t - \phi_c C_{t-1}$ are assumed.

and

$$\varepsilon_c^{mrs} = \frac{\frac{1+\tau^c}{1-\tau^n}(-U_N)\frac{-1}{(V_C)^2}V_{CC}}{\frac{1+\tau^c}{1-\tau^n}\frac{U_N}{V_C}}C = (-1)\frac{V_{CC}C}{V_C} = (-1)\frac{(1+\beta\phi_c^2)(-\sigma)((1-\phi_c)C)^{-\sigma-1}C}{(1-\beta\phi_c)((1-\phi_c)C)^{-\sigma}}$$
$$= \frac{1+\beta\phi_c^2}{1-\beta\phi_c}\frac{\sigma}{(1-\phi_c)}\,, \tag{C.3}$$

Because of $V_{CN} = 0$, we also have[28]

$$\varepsilon_{tot}^{mrs} = \varepsilon_n^{mrs}\,, \tag{C.6}$$

If habits are external, we get the partial derivatives

$$V_{N_t} = -\psi N_t^{\varphi}$$
$$V_N = -\psi N^{\varphi}$$
$$V_{C_t} = (C_t - \phi_c C_{t-1})^{-\sigma}$$
$$V_C = ((1-\phi_c)C)^{-\sigma}$$
$$V_{C_tC_t} = (-\sigma)(C_t - \phi_c C_{t-1})^{-\sigma-1}$$
$$V_{CC} = (-\sigma)((1-\phi_c)C)^{-\sigma-1}$$
$$V_{CN} = 0$$

and the marginal rate of substitution

$$MRS = \frac{1+\tau^c}{1-\tau^n}\frac{\psi N^{\varphi}}{((1-\phi_c)C)^{-\sigma}}$$
$$MRS_N = \varphi\frac{1+\tau^c}{1-\tau^n}\frac{\psi N^{\phi-1}}{((1-\phi_c)C)^{-\sigma}}$$
$$MRS_C = \frac{1+\tau^c}{1-\tau^n}\frac{\psi N^{\varphi}}{((1-\phi_c)C)^{-\sigma+1}}$$

and therefore

$$\varepsilon_c^{mrs} = (-1)\frac{V_{CC}C}{V_C} = (-1)\frac{(-\sigma)((1-\phi_c)C)^{-\sigma-1}C}{((1-\phi_c)C)^{-\sigma}} = \frac{\sigma}{(1-\phi_c)} \tag{C.7}$$

[28] A related functional form with unitary Frisch elasticity considers log utility in leisure:

$$\frac{(C_t - \phi_c C_{t-1})^{1-\sigma} - 1}{1-\sigma} - \psi\log(1-N)\,. \tag{C.4}$$

and yields

$$\varepsilon_{tot}^{mrs} = \varepsilon_n^{mrs} = N/(1-N) \tag{C.5}$$

.

with similar expressions for log leisure. As a consequence, ε_n^{mrs} is the same as in the case of internal habits and, because of $V_{CN} = 0$, we also have

$$\varepsilon_{tot}^{mrs} = \varepsilon_n^{mrs} \,. \tag{C.8}$$

C.1.2 Multiplicatively separable

Consider a multiplicative felicity function[29] with habits

$$U_t = \frac{\left((C_t - \phi_c C_{t-1})^{\eta}\,(1-N)^{1-\eta}\right)^{1-\sigma}}{1-\sigma} = \frac{(C_t - \phi_c C_{t-1})^{\eta(1-\sigma)}(1-N)^{(1-\eta)(1-\sigma)}}{1-\sigma}\,, \tag{C.9}$$

where $0 \le \phi_c \le 1$ measures the degree of habits, $0 \le \eta \le 1$ determines the weight of leisure, and $\sigma \ge 0$ determines the intertemporal elasticity of substitution.

If habits are internal, we have

$$\begin{aligned}
V_{N_t} &= (1-\eta)\,(C_t - \phi_c C_{t-1})^{\eta(1-\sigma)}\,(-1)\,(1-N_t)^{(1-\eta)(1-\sigma)-1} \\
&= -\,(1-\eta)\,(1-\sigma)\,\frac{U_t}{(1-N_t)} \\
V_N &= -\,(1-\eta)\,((1-\phi_c)\,C)^{\eta(1-\sigma)}(1-N)^{(1-\eta)(1-\sigma)-1} \\
&= -\,(1-\eta)\,(1-\sigma)\,\frac{U}{(1-N)} \\
V_{C_t} &= \eta\,(C_t - \phi_c C_{t-1})^{\eta(1-\sigma)-1}\,(1-N_t)^{(1-\eta)(1-\sigma)} \\
&\quad -\phi_c\beta\eta\,(C_{t+1} - \phi_c C_t)^{\eta(1-\sigma)-1}\,(1-N_{t+1})^{(1-\eta)(1-\sigma)} \\
&= \eta\,(1-\sigma)\left(\frac{U_t}{C_t - \phi_c C_{t-1}} - \beta\phi_c\frac{U_{t+1}}{C_{t+1} - \phi_c C_t}\right) \\
V_C &= \eta\,(1-\phi_c\beta)\,((1-\phi_c)\,C)^{\eta(1-\sigma)-1}\,(1-N_t)^{(1-\eta)(1-\sigma)} \\
&= \eta\,(1-\sigma)\,(1-\phi_c\beta)\,\frac{U}{(1-\phi_c)C} \\
V_{C_tC_t} &= \eta\,(\eta\,(1-\sigma)-1)\,(C_t - \phi_c C_{t-1})^{\eta(1-\sigma)-2}(1-N_t)^{(1-\eta)(1-\sigma)} \\
&\quad -\phi_c\beta\eta\,(\eta\,(1-\sigma)-1)\,(-\phi_c)\,(C_{t+1} - \phi_c C_t)^{\eta(1-\sigma)-2}(1-N_{t+1})^{(1-\eta)(1-\sigma)} \\
V_{CC} &= \eta\,(\eta\,(1-\sigma)-1)\left(1+\phi_c^2\beta\right)((1-\phi_c)\,C)^{\eta(1-\sigma)-2}(1-N_t)^{(1-\eta)(1-\sigma)} \\
&= \frac{(\eta\,(1-\sigma))\,(\eta\,(1-\sigma)-1)\,(1+\phi_c^2\beta)\,U}{((1-\phi_c)C)^2} \\
V_{C_tN_t} &= \eta\,(C_t - \phi_c C_{t-1})^{\eta(1-\sigma)-1}\,(1-\eta)\,(1-\sigma)\,(-1)\,(1-N_t)^{(1-\eta)(1-\sigma)-1} \\
V_{CN} &= \eta\,((1-\phi_c)\,C)^{\eta(1-\sigma)-1}\,(1-\eta)\,(\sigma-1)\,(1-N)^{(1-\eta)(1-\sigma)-1} \\
&= (\eta\,(1-\sigma))\,(1-\eta)\,(\sigma-1)\,\frac{U}{(1-\phi_c)C\,(1-N)}
\end{aligned}$$

[29] It has e.g. been used by Backus, Kehoe, and Kydland (1992).

Therefore,

$$
\begin{aligned}
MRS &= \frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{(1-\phi_c)C}{(1-\phi_c\beta)(1-N)} \\
MRS_N &= \frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{(1-\phi_c)C}{(1-\phi_c\beta)(1-N)^2} \\
MRS_C &= \frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{1-\phi_c}{(1-\phi_c\beta)(1-N)}
\end{aligned}
$$

and

$$
\varepsilon_n^{mrs} = \frac{\frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{(1-\phi_c)C}{(1-\phi_c\beta)(1-N)^2}N}{\frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{(1-\phi_c)C}{(1-\phi_c\beta)(1-N)}} = \frac{N}{1-N} \tag{C.10}
$$

$$
\varepsilon_c^{mrs} = \frac{\frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{1-\phi_c}{1-\phi_c\beta}\frac{1}{1-N}}{\frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{1-\phi_c}{1-\phi_c\beta}\frac{C}{1-N}}C = 1 \tag{C.11}
$$

Finally

$$
\begin{aligned}
\frac{V_{CN}}{V_{CC}} &= \frac{(\eta(1-\sigma))(1-\eta)(\sigma-1)\frac{U}{(1-\phi_c)C(1-N)}}{\frac{(\eta(1-\sigma))(\eta(1-\sigma)-1)(1+\phi_c^2\beta)U}{((1-\phi_c)C)^2}} \\
&= \frac{(1-\eta)(\sigma-1)}{\eta(1-\sigma)-1}\frac{(1-\phi_c)}{(1+\phi_c^2\beta)}\frac{C}{1-N}
\end{aligned}
$$

and

$$
\begin{aligned}
\varepsilon_{tot}^{mrs} &= -\frac{V_{CN}N}{V_{CC}C}\varepsilon_c^{mrs} + \varepsilon_n^{mrs} \\
&= -\frac{(1-\eta)(\sigma-1)}{\eta(1-\sigma)-1}\frac{(1-\phi_c)}{(1+\phi_c^2\beta)}\frac{CN}{(1-N)C}\times 1 + \frac{N}{1-N} \\
&= \left[1-\frac{(1-\eta)(\sigma-1)}{\eta(1-\sigma)-1}\frac{(1-\phi_c)}{(1+\phi_c^2\beta)}\right]\frac{N}{1-N}
\end{aligned} \tag{C.12}
$$

In case of $\sigma = 1$, i.e. log utility, utility becomes separable again and (C.12) reduces to (C.5).

With external habits,

$$
\begin{aligned}
V_{C_t} &= \eta\left(C_t - \phi_c C_{t-1}\right)^{\eta(1-\sigma)-1}\left(1-N_t\right)^{(1-\eta)(1-\sigma)} \\
&= \eta\left(1-\sigma\right)\frac{U_t}{C_t - \phi_c C_{t-1}} \\
V_C &= \eta\left(\left(1-\phi_c\right)C\right)^{\eta(1-\sigma)-1}\left(1-N_t\right)^{(1-\eta)(1-\sigma)} \\
&= \eta\left(1-\sigma\right)\frac{U}{(1-\phi_c)C} \\
V_{C_tC_t} &= \eta\left(\eta\left(1-\sigma\right)-1\right)\left(C_t - \phi_c C_{t-1}\right)^{\eta(1-\sigma)-2}\left(1-N_t\right)^{(1-\eta)(1-\sigma)} \\
V_{CC} &= \eta\left(\eta\left(1-\sigma\right)-1\right)\left(\left(1-\phi_c\right)C\right)^{\eta(1-\sigma)-2}(1-N)^{(1-\eta)(1-\sigma)} \\
&= \frac{\left(\eta\left(1-\sigma\right)\right)\left(\eta\left(1-\sigma\right)-1\right)U}{((1-\phi_c)C)^2} \\
V_{C_tN_t} &= \eta\left(C_t - \phi_c C_{t-1}\right)^{\eta(1-\sigma)-1}\left(1-\eta\right)\left(1-\sigma\right)\left(-1\right)\left(1-N_t\right)^{(1-\eta)(1-\sigma)-1} \\
V_{CN} &= \eta\left(\left(1-\phi_c\right)C\right)^{\eta(1-\sigma)-1}\left(1-\eta\right)\left(\sigma-1\right)\left(1-N\right)^{(1-\eta)(1-\sigma)-1} \\
&= \left(\eta\left(1-\sigma\right)\right)\left(1-\eta\right)\left(\sigma-1\right)\frac{U}{(1-\phi_c)C\left(1-N\right)}\ .
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
MRS &= \frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{(1-\phi_c)C}{1-N} \\
MRS_N &= \frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{(1-\phi_c)C}{1-N} \\
MRS_C &= \frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{1-\phi_c}{(1-N)}
\end{aligned}
$$

and

$$
\varepsilon_n^{mrs} = \frac{\frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{(1-\phi_c)C}{(1-N)^2}N}{\frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{(1-\phi_c)C}{(1-N)}} = \frac{N}{1-N} \tag{C.13}
$$

$$
\varepsilon_c^{mrs} = \frac{\frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{1-\phi_c}{1-N}}{\frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{(1-\phi_c)C}{1-N}}C = 1 \tag{C.14}
$$

Finally

$$
\begin{aligned}
\frac{V_{CN}}{V_{CC}} &= \frac{\left(\eta\left(1-\sigma\right)\right)\left(1-\eta\right)\left(\sigma-1\right)\frac{U}{(1-\phi_c)C(1-N)}}{\frac{(\eta(1-\sigma))(\eta(1-\sigma)-1)U}{((1-\phi_c)C)^2}} \\
&= \frac{\left(1-\eta\right)\left(\sigma-1\right)}{\eta\left(1-\sigma\right)-1}\frac{(1-\phi_c)C}{1-N}
\end{aligned}
$$

$$
\begin{aligned}
\varepsilon^{mrs}_{tot} &= -\frac{V_{CN}N}{V_{CC}C}\varepsilon^{mrs}_{c} + \varepsilon^{mrs}_{n} \\
&= -\frac{(1-\eta)\,(\sigma-1)}{\eta(1-\sigma)-1}(1-\phi_c)\frac{CN}{(1-N)C}\times 1 + \frac{N}{1-N} \\
&= \left[1 - \frac{(1-\eta)\,(\sigma-1)}{\eta(1-\sigma)-1}(1-\phi_c)\right]\frac{N}{1-N} \qquad \text{(C.15)}
\end{aligned}
$$

C.1.3 GHH

Consider GHH preferences with habits of the form

$$
U = \frac{\left(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\right)^{1-\sigma} - 1}{1-\sigma}\ , \qquad \text{(C.16)}
$$

where $0 \leq \phi_c \leq 1$ measures the degree of habits, $\varphi \geq 0$ is related to the Frisch elasticity, $\sigma \geq 0$ determines the intertemporal elasticity of substitution ($\sigma = 1$ corresponds to log utility), and $\psi > 0$ determines weight of the disutility of labor. In case of internal habits we get

$$
\begin{aligned}
V_{N_t} &= \left(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\right)^{-\sigma}(-\psi)\,(1+\varphi)N_t^{\varphi} \\
V_N &= \left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma}(-\psi)\,(1+\varphi)N^{\varphi} \\
V_{C_t} &= \left(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\right)^{-\sigma} - \beta\phi_c\left(C_{t+1} - \phi_c C_t - \psi N_{t+1}^{1+\varphi}\right)^{-\sigma} \\
V_C &= (1-\beta\phi_c)\left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma} \\
V_{C_tC_t} &= -\sigma\left(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\right)^{-\sigma-1} - \beta\phi_c\,(-\sigma)\,(-\phi_c)\left(C_{t+1} - \phi_c C_t - \psi N_{t+1}^{1+\varphi}\right)^{-\sigma-1} \\
V_{CC} &= -\sigma\left(1+\beta\phi_c^2\right)\left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma-1} \\
V_{C_tN_t} &= -\sigma\left(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\right)^{-\sigma-1}(-\psi)\,(1+\varphi)\,N_t^{\varphi} \\
V_{CN} &= \sigma\left((1-\phi_c)\,C - \psi N^{1+\varphi}\right)^{-\sigma-1}\psi\,(1+\varphi)\,N^{\varphi}
\end{aligned}
$$

and therefore

$$
\begin{aligned}
MRS &= \frac{1+\tau^c}{1-\tau^n}\frac{\left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma}\psi(1+\varphi)N^{\varphi}}{(1-\beta\phi_c)\left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma}} = \frac{1+\tau^c}{1-\tau^n}\frac{\psi(1+\varphi)N^{\varphi}}{(1-\beta\phi_c)} \\
MRS_N &= \frac{1+\tau^c}{1-\tau^n}\psi(1+\varphi)\varphi\frac{N^{\varphi-1}}{(1-\beta\phi_c)} \\
MRS_C &= 0
\end{aligned}
$$

and

$$\varepsilon_n^{mrs} = \frac{\frac{1+\tau^c}{1-\tau^n}\frac{\psi(1+\varphi)\varphi}{(1-\beta\phi_c)}N^{\varphi-1}N}{\frac{1+\tau^c}{1-\tau^n}\frac{\psi(1+\varphi)}{(1-\beta\phi_c)}N^{\varphi}} = \varphi \tag{C.17}$$

$$\varepsilon_c^{mrs} = 0 \tag{C.18}$$

and therefore

$$\varepsilon_{tot}^{mrs} = \varepsilon_n^{mrs}\,. \tag{C.19}$$

For external habits

$$\begin{aligned}
V_{N_t} &= \left(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\right)^{-\sigma}(-\psi)\,(1+\varphi)N_t^{\varphi} \\
V_N &= \left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma}(-\psi)\,(1+\varphi)N^{\varphi} \\
V_{C_t} &= \left(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\right)^{-\sigma} \\
V_C &= \left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma} \\
V_{C_tC_t} &= -\sigma\left(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\right)^{-\sigma-1} \\
V_{CC} &= -\sigma\left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma-1} \\
V_{C_tN_t} &= -\sigma\left(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\right)^{-\sigma-1}(-\psi)\,(1+\varphi)\,N_t^{\varphi} \\
V_{CN} &= \sigma\left((1-\phi_c)\,C - \psi N^{1+\varphi}\right)^{-\sigma-1}\psi\,(1+\varphi)\,N^{\varphi}
\end{aligned}$$

and therefore

$$\begin{aligned}
MRS &= \frac{1+\tau^c}{1-\tau^n}\frac{\left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma}\psi(1+\varphi)N^{\varphi}}{\left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma}} = \frac{1+\tau^c}{1-\tau^n}\psi(1+\varphi)N^{\varphi} \\
MRS_N &= \frac{1+\tau^c}{1-\tau^n}\psi(1+\varphi)\varphi N^{\varphi-1} \\
MRS_C &= 0
\end{aligned}$$

and

$$\varepsilon_n^{mrs} = \frac{\frac{1+\tau^c}{1-\tau^n}\psi(1+\varphi)\varphi N^{\varphi-1}N}{\frac{1+\tau^c}{1-\tau^n}\psi(1+\varphi)N^{\varphi}} = \varphi \tag{C.20}$$

$$\varepsilon_c^{mrs} = 0 \tag{C.21}$$

and therefore

$$\varepsilon_{tot}^{mrs} = \varepsilon_n^{mrs}\,. \tag{C.22}$$

D Welfare

To keep the exposition simple, we in the following abstract from sticky prices. As long as i) price dispersion/price adjustment costs are 0 in steady state and ii) we consider an efficient steady state, they could easily be added without affecting the conclusions derived.[30] Without loss of generality, we also omit the preference shocks for notational brevity.

D.1 SGU framework

In the SGU framework, household members supply the same homogenous labor good to unions so that the aggregate utility function is given by

$$U^{SGU} = \int_0^1 U_t(j)\, dj = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \tag{D.1}$$

A second-order Taylor approximation around the deterministic steady state yields

$$\hat{U}^{SGU} = C^{-\sigma}\hat{C}_t - \frac{1}{2}C^{-\sigma-1}\sigma\hat{C}_t^2 - N^{\varphi}\hat{N}_t - \frac{1}{2}N^{\varphi-1}\varphi\hat{N}_t^2 \,. \tag{D.2}$$

D.1.1 Rotemberg

In a symmetric Rotemberg equilibrium, we can drop all indices j. The aggregate production function and the resource constraint are

$$Y_t = A_t N_t^{1-\alpha} \tag{D.3}$$

$$Y_t = C_t + \frac{\phi_w^{SGU}}{2}(\Pi_t^w - 1)^2 Y_t \,. \tag{D.4}$$

We can use them to express consumption as a function of production and wage adjustment costs:

$$C_t = A_t N_t^{1-\alpha}\left(1 - \frac{\phi_w^{SGU}}{2}(\Pi_t^w - 1)^2\right) \tag{D.5}$$

[30] In case nominal rigidities affect the deterministic steady state or a second order approximation to the model dynamics is required, there would be interaction effects between price and wage rigidities.

A second-order Taylor expansion yields

$$
\begin{aligned}
C_t - C =& N^{1-\alpha}\left(1-\frac{\phi_w^{SGU}}{2}(\Pi^w-1)^2\right)(A_t-A)+(1-\alpha)\,AN^{-\alpha}\left(1-\frac{\phi_w^{SGU}}{2}(\Pi^w-1)^2\right)(N_t-N)\\
&- AN^{1-\alpha}\phi_w^{SGU}\left(\Pi^w-1\right)\left(\Pi_t^w-\Pi^w\right)\\
&+\frac{1}{2}2\left(1-\alpha\right)N^{-\alpha}\left(1-\frac{\phi_w^{SGU}}{2}(\Pi^w-1)^2\right)(N_t-N)\left(A_t-A\right)-\frac{1}{2}AN^{1-\alpha}\phi_w^{SGU}\left(\Pi_t^w-\Pi^w\right)^2\\
&+\frac{1}{2}\left(1-\alpha\right)\left(-\alpha\right)AN^{-\alpha-1}\left(1-\frac{\phi_w^{SGU}}{2}(\Pi^w-1)^2\right)(N_t-N)^2 .
\end{aligned}
\tag{D.6}
$$

Letting hats denote deviations from steady state, $\hat{X}_t = X_t - X$, and imposing a zero inflation steady state, we can write hours worked as

$$
\begin{aligned}
\hat{N}_t =& \frac{1}{(1-\alpha)}N^{-\alpha}\Bigg[\hat{C}_t - N^{1-\alpha}\hat{A}_t-(1-\alpha)\,N^{-\alpha}\hat{N}_t-(1-\alpha)\,N^{-\alpha}\hat{N}_t\hat{A}_t\\
&+\frac{1}{2}\alpha\left(1-\alpha\right)N^{-\alpha-1}\hat{N}_t^2+\frac{\phi_w^{SGU}}{2}C\left(\hat{\Pi}_t^w\right)^2\Bigg] .
\end{aligned}
\tag{D.7}
$$

Plugging into (D.2) yields

$$
\begin{aligned}
\hat{U}^{SGU}_{Rotemberg} =& C^{-\sigma}\hat{C}_t-\frac{1}{2}C^{-\sigma-1}\sigma\hat{C}_t^2-N^{\varphi}\hat{N}_t-\frac{1}{2}N^{\varphi-1}\varphi\hat{N}_t^2\\
=& C^{-\sigma}\hat{C}_t-\frac{1}{2}C^{-\sigma-1}\sigma\hat{C}_t^2\\
&-N^{\varphi}\left(\frac{1}{(1-\alpha)\,N^{-\alpha}}\begin{bmatrix}\hat{C}_t-N^{1-\alpha}\hat{A}_t-(1-\alpha)\,N^{-\alpha}\hat{N}_t-(1-\alpha)\,N^{-\alpha}\hat{N}_t\hat{A}_t\\+\frac{1}{2}\alpha\left(1-\alpha\right)N^{-\alpha-1}\hat{N}_t^2+\frac{\phi_w^{SGU}}{2}C\left(\hat{\Pi}_t^w\right)^2\end{bmatrix}\right)\\
&-\frac{1}{2}N^{\varphi-1}\varphi\hat{N}_t^2\\
=& \left(C^{-\sigma}-\frac{N^{\varphi}}{(1-\alpha)\,N^{-\alpha}}\right)\hat{C}_t-\frac{1}{2}C^{-\sigma-1}\sigma\hat{C}_t^2\\
&-\left(\frac{1}{2}N^{\varphi-1}\varphi-\frac{N^{\varphi}}{(1-\alpha)\,N^{-\alpha}}\frac{1}{2}\alpha\left(1-\alpha\right)N^{-\alpha-1}\right)\hat{N}_t^2\\
&+\frac{N^{\varphi}N^{\alpha}}{(1-\alpha)\,N^{-\alpha}}\hat{A}_t+N^{\varphi}\hat{N}_t\hat{A}_t-\frac{N^{\varphi}}{(1-\alpha)\,N^{-\alpha}}C\frac{\phi_w^{SGU}}{2}\left(\hat{\Pi}_t^w\right)^2 .
\end{aligned}
\tag{D.8}
$$

Note that in an efficient steady state, the linear term in consumption will drop out as

$$
C^{-\sigma}-\frac{N^{\varphi}}{(1-\alpha)\,N^{-\alpha}}=0\,. \tag{D.9}
$$

This is nothing else than the condition that the marginal rate of substitution is equal to the marginal product of labor. This is the well-known result that with an efficient steady state, a linear approximation to the policy functions is sufficient to get a second order accurate welfare measure. Second order terms from the policy functions plugged into the second order approximated utility function would result in terms of order higher than two.

D.1.2 Calvo

In the symmetric Calvo equilibrium, aggregate output and the resource constraint are given by

$$C_t = Y_t = A_t \left(\frac{N_t}{S_t^w} \right)^{1-\alpha} , \tag{D.10}$$

where we aggregated over labor services:

$$N_t \equiv \int_0^1 N_t^j dj = \int_0^1 \left(\frac{W_t^j}{W_t} \right)^{-\varepsilon_w} N_t^d dj = N_t^d \int_0^1 \left(\frac{W_t^j}{W_t} \right)^{-\varepsilon_w} dj \, . \tag{D.11}$$

Defining the auxiliary variable $S_t^W \equiv \int_0^1 \left(\frac{W_t^j}{W_t} \right)^{-\varepsilon_w} dj$, which captures wage dispersion, implies

$$N_t^d = \frac{N_t}{S_t^W} \, . \tag{D.12}$$

A second order approximation to (D.10) yields

$$\begin{aligned}
\hat{C}_t =& \left(\frac{N}{S^w} \right)^{1-\alpha} (A_t - A) + \frac{(1-\alpha) A N^{-\alpha}}{(S^w)^{1-\alpha}} (N_t - N) - (1-\alpha) A N^{1-\alpha} (S^w)^{-(1-\alpha)-1} (S_t^w - S^w) \\
&+ \frac{1}{2} 2 \begin{bmatrix} \frac{(1-\alpha) N^{-\alpha}}{(S^w)^{1-\alpha}} (N_t - N)(A_t - A) - N^{1-\alpha} (S^w)^{-(1-\alpha)-1} (S_t^w - S^w)(A_t - A) \\ - (1-\alpha)(-(1-\alpha)) A N^{-\alpha} (S^w)^{-(1-\alpha)-1} (S_t^w - S^w)(N_t - N) \end{bmatrix} \\
&+ \frac{1}{2} \begin{pmatrix} A N^{1-\alpha} (-(1-\alpha)(-(1-\alpha)-1)) (S^w)^{-(1-\alpha)-2} (S_t^w - S^w)^2 \\ + (1-\alpha)(-\alpha) A \frac{N^{-\alpha-1}}{(S^w)^{1-\alpha}} (N_t - N)^2 \end{pmatrix} \\
&+ \left[(1-\alpha) N^{-\alpha} \hat{N}_t \hat{A}_t - N^{1-\alpha} \hat{S}_t^w \hat{A}_t - (1-\alpha)(-(1-\alpha)) N^{-\alpha} \hat{S}_t^w \hat{N}_t \right] \\
&- (-(1-\alpha)(-(1-\alpha)-1)) N^{1-\alpha} \left(\hat{S}_t^w \right)^2 + \frac{1}{2} \alpha (\alpha - 1) N^{\alpha-2} \hat{N}_t^2 \, ,
\end{aligned} \tag{D.13}$$

where the second equality uses that in steady state $A = 1$ and $S_t^w = 1$. We will show below that $\hat{S}_t^w = 0$ up to first order, so that

$$\hat{N}_t = \frac{1}{(1-\alpha) N^{-\alpha}} \begin{bmatrix} \hat{C}_t + (1-\alpha) C \hat{S}_t^w - N^{1-\alpha} \hat{A}_t - (1-\alpha) N^{-\alpha} \hat{N}_t \\ - (1-\alpha) N^{-\alpha} \hat{N}_t \hat{A}_t + \frac{1}{2} \alpha (1-\alpha) N^{-\alpha-1} \hat{N}_t^2 \end{bmatrix} . \tag{D.14}$$

Plugging this into the approximated felicity function (D.2) yields

$$
\begin{aligned}
\hat{U}_{Calvo}^{SGU} =& C^{-\sigma}\hat{C}_t - \frac{1}{2}C^{-\sigma-1}\sigma\hat{C}_t^2 - N^{\varphi}\left(\frac{1}{(1-\alpha)\,N^{-\alpha}}\begin{bmatrix}\hat{C}_t + (1-\alpha)C\hat{S}_t^w - N^{1-\alpha}\hat{A}_t - (1-\alpha)\,N^{-\alpha}\hat{N}_t \\ -(1-\alpha)\,N^{-\alpha}\hat{N}_t\hat{A}_t + \frac{1}{2}\alpha\,(1-\alpha)\,N^{-\alpha-1}\hat{N}_t^2\end{bmatrix}\right) \\
& - \frac{1}{2}N^{\varphi-1}\varphi\hat{N}_t^2 \\
=& \left(C^{-\sigma} - \frac{N^{\varphi}}{(1-\alpha)\,N^{-\alpha}}\right)\hat{C}_t - \frac{1}{2}C^{-\sigma-1}\sigma\hat{C}_t^2 \\
& - \left(\frac{1}{2}N^{\varphi-1}\varphi - \frac{N^{\varphi}}{(1-\alpha)\,N^{-\alpha}}\frac{1}{2}\alpha\,(1-\alpha)\,N^{-\alpha-1}\right)\hat{N}_t^2 \\
& + \frac{N^{\varphi}N^{\alpha}}{(1-\alpha)\,N^{-\alpha}}\hat{A}_t + N^{\varphi}\hat{N}_t\hat{A}_t - \frac{N^{\varphi}}{(1-\alpha)\,N^{-\alpha}}(1-\alpha)C\hat{S}_t^w \,.
\end{aligned}
\tag{D.15}
$$

Next, consider the law of motion for the wage dispersion term S_t^w:

$$
\begin{aligned}
S_t^W &= \int_0^1 \left(\frac{W_t^j}{W_t}\right)^{-\varepsilon_w} dj = (1-\theta)\left(\frac{W_t^*}{W_t}\right)^{-\varepsilon_w} + \theta\int_0^1\left(\frac{W_{t-1}^j}{W_t}\right)^{-\varepsilon_w} dj \\
&= (1-\theta)\left(\frac{W_t^*}{W_t}\right)^{-\varepsilon_w} + \theta\left(\frac{W_{t-1}}{W_t}\right)^{-\varepsilon_w}\int_0^1\left(\frac{W_{t-1}^j}{W_{t-1}}\right)^{-\varepsilon_w} dj \\
&= (1-\theta)\,(\Pi_t^{w*})^{-\varepsilon_w} + \theta\left(\frac{W_{t-1}}{W_t}\right)^{-\varepsilon_w} S_{t-1}^W \\
&= (1-\theta)\,(\Pi_t^{w*})^{-\varepsilon_w} + \theta\,(\Pi_t^w)^{\varepsilon_w}\, S_{t-1}^W \,.
\end{aligned}
\tag{D.16}
$$

A second order Taylor approximation yields

$$
\begin{aligned}
\hat{S}_t^W =& (1-\theta_w)\,(-\varepsilon_w)\,(\Pi_t^{w*})^{-\varepsilon_w-1}\,(\Pi_t^{w*} - \Pi^{w*}) + \varepsilon_w\theta_w(\Pi^w)^{\varepsilon_w-1}S^W\,(\Pi_t^w - \Pi^w) + \theta_w(\Pi_t^w)^{\varepsilon_w}\left(S_t^W - S^w\right) \\
& + \frac{1}{2}\begin{bmatrix}(1-\theta_w)\,(-\varepsilon_w)\,(-\varepsilon_w-1)\,(\Pi_t^{w*})^{-\varepsilon_w-2}(\Pi_t^{w*} - \Pi^{w*})^2 \\ +\varepsilon_w\,(\varepsilon_w-1)\,\theta_w(\Pi_t^w)^{\varepsilon_w-2}S^W(\Pi_t^w - \Pi^w)^2\end{bmatrix} \\
& + \varepsilon_w\theta_w(\Pi^w)^{\varepsilon_w-1}\,(\Pi_t^w - \Pi^w)\left(S_{t-1}^W - S^w\right) \\
=& (1-\theta_w)\,(-\varepsilon_w)\,(\Pi_t^{w*} - \Pi^{w*}) + \varepsilon_w\theta_w\,(\Pi_t^w - \Pi^w) + \theta_w\left(S_{t-1}^W - S^w\right) \\
& + \frac{1}{2}\left[(1-\theta_w)\,\varepsilon_w\,(\varepsilon_w+1)\,(\Pi_t^{w*} - \Pi^{w*})^2 + \varepsilon_w\,(\varepsilon_w-1)\,\theta_w(\Pi_t^w - \Pi^w)^2\right] \\
& + \varepsilon_w\theta_w\,(\Pi_t^w - \Pi^w)\left(S_{t-1}^W - S^w\right) \\
=& \varepsilon_w\left(\theta_w\,(\Pi_t^w - \Pi^w) - (1-\theta_w)\,(\Pi_t^{w*} - \Pi^{w*})\right) + \theta_w\left(S_{t-1}^W - S^w\right) \\
& + \frac{1}{2}\left[(1-\theta_w)\,\varepsilon_w\,(\varepsilon_w+1)\,(\Pi_t^{w*} - \Pi^{w*})^2 + \varepsilon_w\,(\varepsilon_w-1)\,\theta_w(\Pi_t^w - \Pi^w)^2\right] \\
& + \varepsilon_w\theta_w\,(\Pi_t^w - \Pi^w)\left(S_{t-1}^W - S^w\right)\,,
\end{aligned}
\tag{D.17}
$$

where we again imposed a zero inflation steady state. The evolution of wages is given by

$$W_t = \left[(1-\theta_w)\left(W_t^*\right)^{1-\varepsilon_w} + \theta_w W_{t-1}^{1-\varepsilon_w}\right]^{\frac{1}{1-\varepsilon_w}} \tag{D.18}$$

so that

$$1 = (1-\theta_w)\left(\Pi_t^{w*}\right)^{1-\varepsilon_w} + \theta_w (\Pi_t^w)^{\varepsilon_w - 1}\,. \tag{D.19}$$

A first order approximation yields

$$0 = (1-\theta_w)(1-\varepsilon_w)\left(\Pi_t^{w*}\right)^{-\varepsilon_w}\left(\Pi_t^{w*} - \Pi^{w*}\right) + \theta_w(\varepsilon_w - 1)\left(\Pi^w\right)^{\varepsilon_w - 2}\left(\Pi_t^w - \Pi^w\right) \tag{D.20}$$

so that

$$\theta_w\left(\Pi_t^w - \Pi^w\right) = (1-\theta_w)\left(\Pi_t^{w*} - \Pi^{w*}\right)\,, \tag{D.21}$$

which implies

$$\left(\frac{\theta_w}{(1-\theta_w)}\right)^2 \left(\Pi_t^w - \Pi^w\right)^2 = \left(\Pi_t^{w*} - \Pi^{w*}\right)^2 \tag{D.22}$$

A second-order approximation of (D.19) yields

$$\begin{aligned}
0 =& (1-\theta_w)(1-\varepsilon_w)\left(\Pi_t^{w*}\right)^{-\varepsilon_w}\left(\Pi_t^{w*} - \Pi^{w*}\right) + \theta_w(\varepsilon_w - 1)\left(\Pi^w\right)^{\varepsilon_w - 2}\left(\Pi_t^w - \Pi^w\right) \\
&+ \frac{1}{2}\begin{bmatrix} (1-\theta_w)(1-\varepsilon_w)(-\varepsilon_w)\left(\Pi_t^{w*}\right)^{-\varepsilon_w - 1}\left(\Pi_t^{w*} - \Pi^{w*}\right)^2 \\ +\theta_w(\varepsilon_w - 1)(\varepsilon_w - 2)\left(\Pi^w\right)^{\varepsilon_w - 3}\left(\Pi_t^w - \Pi^w\right)^2 \end{bmatrix} \\
=& (1-\theta_w)\left(\Pi_t^{w*} - \Pi^{w*}\right) - \theta_w\left(\Pi_t^w - \Pi^w\right) \\
&+ \frac{1}{2}\left[(1-\theta_w)(-\varepsilon_w)\left(\Pi_t^{w*} - \Pi^{w*}\right)^2 - \theta_w(\varepsilon_w - 2)\left(\Pi_t^w - \Pi^w\right)^2\right]
\end{aligned} \tag{D.23}$$

so that

$$\begin{aligned}
&\theta_w\left(\Pi_t^w - \Pi^w\right) - (1-\theta_w)\left(\Pi_t^{w*} - \Pi^{w*}\right) \\
&= \frac{1}{2}\left[(1-\theta_w)(-\varepsilon_w)\left(\Pi_t^{w*} - \Pi^{w*}\right)^2 - \theta_w(\varepsilon_w - 2)\left(\Pi_t^w - \Pi^w\right)^2\right]\,.
\end{aligned} \tag{D.24}$$

Inserting into (D.17), we get:

$$\begin{aligned}
\hat{S}_t^W =& \frac{1}{2}\varepsilon_w\left[(1-\theta_w)(-\varepsilon_w)\left(\Pi_t^{w*} - \Pi^{w*}\right)^2 - \theta_w(\varepsilon_w - 2)\left(\Pi_t^w - \Pi^w\right)^2\right] + \theta_w\left(S_{t-1}^W - S^w\right) \\
&+ \frac{1}{2}\left[(1-\theta_w)\varepsilon_w(\varepsilon_w + 1)\left(\Pi_t^{w*} - \Pi^{w*}\right)^2 + \varepsilon_w(\varepsilon_w - 1)\theta_w\left(\Pi_t^w - \Pi^w\right)^2\right] \\
&+ \varepsilon_w\theta_w\left(\Pi_t^w - \Pi^w\right)\left(S_{t-1}^W - S^w\right)\,.
\end{aligned} \tag{D.25}$$

This shows the well-known result that $\hat{S}_t^W$ is 0 up to first order in a zero inflation steady state. Thus, up to second order, we can drop the last term as it is of third order. Now we can use (D.22) to get

$$
\begin{aligned}
\hat{S}_t^W =& \theta_w \left(S_{t-1}^W - S^w\right) + \frac{1}{2}\varepsilon_w \left[(1-\theta_w)(-\varepsilon_w) \left(\frac{\theta_w}{(1-\theta_w)}\right)^2 (\Pi_t^w - \Pi^w)^2 - \theta_w(\varepsilon_w - 2)(\Pi_t^w - \Pi^w)^2 \right] \\
&+ \frac{1}{2}\left[(1-\theta_w)\varepsilon_w(\varepsilon_w+1)\left(\frac{\theta_w}{(1-\theta_w)}\right)^2 (\Pi_t^w - \Pi^w)^2 + \varepsilon_w(\varepsilon_w - 1)\theta_w(\Pi_t^w - \Pi^w)^2 \right] \\
=& \theta_w \left(S_{t-1}^W - S^w\right) + \frac{1}{2}\left[\begin{array}{l} (1-\theta_w)\left(\frac{\theta_w}{(1-\theta_w)}\right)^2 (\Pi_t^w - \Pi^w)^2 \left(-\varepsilon_w^2 + \varepsilon_w^2 + \varepsilon_w\right) \\ -\theta_w(\Pi_t^w - \Pi^w)^2 \left(-\varepsilon_w^2 - 2\varepsilon_w + \varepsilon_w^2 - \varepsilon_w\right) \end{array} \right] \\
=& \theta_w \left(S_{t-1}^W - S^w\right) + \frac{1}{2}\left[\frac{\theta_w^2}{(1-\theta_w)}(\Pi_t^w - \Pi^w)^2 \varepsilon_w + \theta_w(\Pi_t^w - \Pi^w)^2 \varepsilon_w \right] \\
=& \theta_w \left(S_{t-1}^W - S^w\right) + \frac{1}{2}\varepsilon_w\theta_w \left[\frac{\theta_w}{(1-\theta_w)} + 1 \right] (\Pi_t^w - \Pi^w)^2 \\
=& \theta_w \left(S_{t-1}^W - S^w\right) + \frac{1}{2}\varepsilon_w\theta_w \left[\frac{\theta_w}{(1-\theta_w)} + \frac{(1-\theta_w)}{(1-\theta_w)} \right] (\Pi_t^w - \Pi^w)^2 \\
=& \theta_w \left(S_{t-1}^W - S^w\right) + \frac{1}{2}\frac{\varepsilon_w\theta_w}{1-\theta_w}(\Pi_t^w - \Pi^w)^2 \\
=& \theta_w \hat{S}_{t-1}^W + \frac{1}{2}\frac{\varepsilon_w\theta_w}{1-\theta_w}\left(\hat{\Pi}_t^w\right)^2 .
\end{aligned}
\tag{D.26}
$$

Iterating this equation forward from time t_0 onwards yields

$$
\begin{aligned}
\hat{S}_{t_0}^W =& \theta_w \hat{S}_{t_0-1}^W + \frac{1}{2}\frac{\varepsilon_w\theta_w}{1-\theta_w}\left(\hat{\Pi}_{t_0}^w\right)^2 \\
\hat{S}_{t_0+1}^W =& \theta_w \left(\theta_w \hat{S}_{t_0-1}^W + \frac{1}{2}\frac{\varepsilon_w\theta_w}{1-\theta_w}\left(\hat{\Pi}_{t_0}^w\right)^2\right) + \frac{1}{2}\frac{\varepsilon_w\theta_w}{1-\theta_w}\left(\hat{\Pi}_{t_0+1}^w\right)^2 \\
\hat{S}_{t_0+2}^W =& \left(\theta_w \left(\theta_w \hat{S}_{t_0-1}^W + \frac{1}{2}\frac{\varepsilon_w\theta_w}{1-\theta_w}\left(\hat{\Pi}_{t_0}^w\right)^2\right) + \frac{1}{2}\frac{\varepsilon_w\theta_w}{1-\theta_w}\left(\hat{\Pi}_{t_0+1}^w\right)^2\right) + \frac{1}{2}\frac{\varepsilon_w\theta_w}{1-\theta_w}\left(\hat{\Pi}_{t_0+2}^w\right)^2
\end{aligned}
\tag{D.27}
$$

so that the discounted sum is given by

$$\begin{aligned}
&\sum_{t=t_0}^{\infty}\beta^{t-t_0}\hat{S}_t^W \\
&=\sum_{t=t_0}^{\infty}(\beta\theta_w)^{t-t_0}\left(\frac{1}{2}\frac{\varepsilon_w\theta_w}{1-\theta_w}\left(\hat{\Pi}_{t_0}^w\right)^2+\theta_w\hat{S}_{t_0-1}^W\right)+\sum_{t=t_0+1}^{\infty}\beta^{t-t_0}\theta_w^{t-t_0+1}\left(\frac{1}{2}\frac{\varepsilon_w\theta_w}{1-\theta_w}\left(\hat{\Pi}_{t_0+1}^w\right)^2\right)+\ldots \\
&=\sum_{t=t_0}^{\infty}(\beta\theta_w)^{t-t_0}\theta_w\hat{S}_{t_0-1}^W+\left(\frac{1}{2}\frac{\varepsilon_w\theta_w}{1-\theta_w}\left(\hat{\Pi}_{t_0}^w\right)^2\right)\sum_{t=t_0}^{\infty}(\beta\theta_w)^{t-t_0} \\
&\quad+\left(\frac{1}{2}\frac{\varepsilon_w\theta_w}{1-\theta_w}\left(\hat{\Pi}_{t_0+1}^w\right)^2\right)\sum_{t=t_0+1}^{\infty}(\beta\theta_w)^{t-t_0}\theta_w+\ldots \\
&=\frac{\theta_w}{1-\beta\theta_w}\hat{S}_{t_0-1}^W+\left(\frac{1}{2}\frac{\varepsilon_w\theta_w}{1-\theta_w}\left(\hat{\Pi}_{t_0}^w\right)^2\right)\frac{1}{1-\beta\theta_w}+\left(\frac{1}{2}\frac{\varepsilon_w\theta_w}{1-\theta_w}\left(\hat{\Pi}_{t_0+1}^w\right)^2\right)\frac{\beta}{1-\beta\theta_w}+\ldots \\
&=\frac{\theta_w}{1-\beta\theta_w}\hat{S}_{t_0-1}^W+\frac{1}{1-\beta\theta_w}\left[\left(\frac{1}{2}\frac{\varepsilon_w\theta_w}{1-\theta_w}\left(\hat{\Pi}_{t_0}^w\right)^2\right)+\left(\frac{1}{2}\frac{\varepsilon_w\theta_w}{1-\theta_w}\left(\hat{\Pi}_{t_0+1}^w\right)^2\right)\beta+\ldots\right] \\
&=\frac{\theta_w}{1-\beta\theta_w}\hat{S}_{t_0-1}^W+\frac{1}{1-\beta\theta_w}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(\frac{1}{2}\frac{\varepsilon_w\theta_w}{1-\theta_w}\left(\hat{\Pi}_t^w\right)^2\right) \\
&=\frac{\theta_w}{1-\beta\theta_w}\hat{S}_{t_0-1}^W+\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{1}{2}\left(\frac{\varepsilon_w\theta_w}{(1-\beta\theta_w)(1-\theta_w)}\left(\hat{\Pi}_t^w\right)^2\right).
\end{aligned}\tag{D.28}$$

D.2 Comparison

Comparing welfare under Calvo (D.15) and Rotemberg (D.8) shows the difference is given by

$$\begin{aligned}
\Delta U^{SGU} &=\hat{U}_{Calvo}^{SGU}-\hat{U}_{Rotemberg}^{SGU} \\
&=\frac{N^{\varphi}}{(1-\alpha)N^{-\alpha}}C\frac{\phi_w^{SGU}}{2}\left(\hat{\Pi}_t^w\right)^2-\frac{N^{\varphi}}{(1-\alpha)N^{-\alpha}}(1-\alpha)C\hat{S}_t^w \\
&=\frac{N^{\varphi}}{(1-\alpha)N^{-\alpha}}C\left(\frac{\phi_w^{SGU}}{2}\left(\hat{\Pi}_t^w\right)^2-(1-\alpha)\hat{S}_t^w\right).
\end{aligned}\tag{D.29}$$

Using (D.28) and

$$\frac{N^{\varphi}C}{(1-\alpha)N^{-\alpha}}=\frac{N^{\varphi}N^{1-\alpha}}{(1-\alpha)N^{-\alpha}}=\frac{N^{1+\varphi}}{(1-\alpha)},\tag{D.30}$$

we can write the welfare difference as

$$
\begin{aligned}
&\sum_{t=t_0}^{\infty} \beta^{t-t_0} \Delta U^{SGU} \\
&= \frac{N^{1+\varphi}}{(1-\alpha)} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left(\frac{\phi_w^{SGU}}{2} \left(\hat{\Pi}_t^w \right)^2 - (1-\alpha)\, \hat{S}_t^w \right) \\
&= \frac{N^{1+\varphi}}{(1-\alpha)} \left[\begin{array}{l} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{\phi_w^{SGU}}{2} \left(\hat{\Pi}_t^w \right)^2 - (1-\alpha)\, \frac{\theta_w}{1-\beta\theta_w} \hat{S}_{t_0-1}^W \\ -(1-\alpha) \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\frac{\varepsilon_w \theta_w}{(1-\beta\theta_w)(1-\theta_w)} \left(\hat{\Pi}_t^w \right)^2 \right) \end{array} \right] \\
&= \frac{N^{1+\varphi}}{(1-\alpha)} \left[-(1-\alpha)\, \frac{\theta_w}{1-\beta\theta_w} \hat{S}_{t_0-1}^W + \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\phi_w^{SGU} - (1-\alpha)\, \frac{\varepsilon_w \theta_w}{(1-\beta\theta_w)(1-\theta_w)} \right) \left(\hat{\Pi}_t^w \right)^2 \right].
\end{aligned}
\tag{D.31}
$$

Assuming that initial price dispersion is 0, i.e. $\hat{S}_{t_0-1}^W = 0$, this simplifies to

$$
\begin{aligned}
\sum_{t=t_0}^{\infty} \beta^{t-t_0} \Delta U^{SGU} =& \frac{N^{1+\varphi}}{(1-\alpha)} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\phi_w^{SGU} - (1-\alpha)\, \frac{\varepsilon_w \theta_w}{(1-\beta\theta_w)(1-\theta_w)} \right) \left(\hat{\Pi}_t^w \right)^2 \\
=& \frac{N^{1+\varphi}}{(1-\alpha)} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\frac{(\varepsilon_w - 1)(1-\tau^n)\aleph}{(1-\theta_w)(1-\beta\theta_w)} \theta_w - (1-\alpha)\, \frac{\varepsilon_w \theta_w}{(1-\beta\theta_w)(1-\theta_w)} \right) \left(\hat{\Pi}_t^w \right)^2 \\
=& N^{1+\varphi} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\frac{(\varepsilon_w - 1)(1-\tau^n)\aleph}{(1-\alpha)(1-\theta_w)(1-\beta\theta_w)} \theta_w - \frac{\varepsilon_w \theta_w}{(1-\beta\theta_w)(1-\theta_w)} \right) \left(\hat{\Pi}_t^w \right)^2,
\end{aligned}
\tag{D.32}
$$

where the second line uses that the slope of the wage Phillips Curve is identical with

$$
\phi_w^{SGU} = \frac{(\varepsilon_w - 1)(1-\tau^n)\aleph}{(1-\theta_w)(1-\beta\theta_w)} \theta_w \tag{D.33}
$$

With $\aleph = (1-\alpha)$ and $(1-\tau^n) = \frac{\varepsilon_w}{(\varepsilon_w - 1)}$, i.e. if the monopolistic distortion is counteracted by appropriate subsidies:

$$
\sum_{t=t_0}^{\infty} \beta^{t-t_0} \Delta U^{SGU} = N^{1+\varphi} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\frac{\varepsilon_w \theta_w}{(1-\theta_w)(1-\beta\theta_w)} - \frac{\varepsilon_w \theta_w}{(1-\beta\theta_w)(1-\theta_w)} \right) \left(\hat{\Pi}_t^w \right)^2 = 0. \tag{D.34}
$$

Thus, in the case of an undistorted steady state, i.e. when the labor tax undoes the effect of monopolistic competition, the welfare losses conditional on initial wage dispersion being 0 are identical between the Rotemberg and Calvo frameworks. This completes the proof of Proposition 2 for the SGU framework.

Finally, consider unconditional welfare. Taking the unconditional expectations of equation (D.26) we get

$$E\hat{S}_t^W = E\theta_w \hat{S}_{t-1}^W + \frac{1}{2}\frac{\varepsilon_w \theta_w}{1-\theta_w} E\left(\hat{\Pi}_t^w\right)^2 \tag{D.35}$$

so that the unconditional mean of the price dispersion term is given by:

$$E\hat{S}_t^W = \frac{1}{2}\frac{\varepsilon_w \theta_w}{\left(1-\theta_w\right)^2} E\left(\hat{\Pi}_t^w\right)^2 . \tag{D.36}$$

Hence, the average wage dispersion in the stochastic model is not 0. Taking the unconditional expectations in (D.31) and using the previous result, we obtain:

$$\begin{aligned}
E\sum_{t=t_0}^{\infty}\beta^{t-t_0}\Delta U^{SGU} =& \frac{N^{1+\varphi}}{(1-\alpha)}\left[\begin{array}{l} -\left(1-\alpha\right)\dfrac{\theta_w}{1-\beta\theta_w}E\hat{S}_{t_0-1}^W \\ +\displaystyle\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{1}{2}\left(\phi_w^{SGU}-\left(1-\alpha\right)\frac{\varepsilon_w\theta_w}{\left(1-\beta\theta_w\right)\left(1-\theta_w\right)}\right)E\left(\hat{\Pi}_t^w\right)^2 \end{array}\right] \\
=& \frac{N^{1+\varphi}}{(1-\alpha)}\left[\begin{array}{l} -\left(1-\alpha\right)\dfrac{\theta_w}{1-\beta\theta_w}\dfrac{1}{2}\dfrac{\varepsilon_w\theta_w}{\left(1-\theta_w\right)^2}E\left(\hat{\Pi}_t^w\right)^2 \\ +\dfrac{1}{2}\left(-\left(1-\alpha\right)\dfrac{\varepsilon_w\theta_w}{\left(1-\beta\theta_w\right)\left(1-\theta_w\right)}\right)\dfrac{1}{1-\beta}E\left(\hat{\Pi}_t^w\right)^2 \end{array}\right] \\
=& \frac{1}{2}\frac{N^{1+\varphi}}{(1-\alpha)}\left(\begin{array}{l} \phi_w^{SGU}-\left(1-\alpha\right)\dfrac{\varepsilon_w\theta_w}{\left(1-\beta\theta_w\right)\left(1-\theta_w\right)} \\ -\left(1-\beta\right)\left(1-\alpha\right)\dfrac{\theta_w}{1-\beta\theta_w}\dfrac{\varepsilon_w\theta_w}{\left(1-\theta_w\right)^2} \end{array}\right)\frac{1}{1-\beta}E\left(\hat{\Pi}_t^w\right)^2 .
\end{aligned} \tag{D.37}$$

Again imposing identical wage PC slopes via (D.33), we get

$$E\sum_{t=t_0}^{\infty}\beta^{t-t_0}\Delta U^{SGU} = \frac{1}{2}\frac{N^{1+\varphi}}{(1-\alpha)}\phi_w^{SGU}\left(\begin{array}{l} 1-\dfrac{\left(1-\alpha\right)\varepsilon_w}{\left(\varepsilon_w-1\right)\aleph\left(1-\tau^n\right)} \\ -\dfrac{\left(1-\beta\right)\left(1-\alpha\right)\theta_w}{\left(1-\theta_w\right)}\dfrac{\left(1-\alpha\right)\varepsilon_w}{\left(\varepsilon_w-1\right)\aleph\left(1-\tau^n\right)} \end{array}\right)\frac{1}{1-\beta}E\left(\hat{\Pi}_t^w\right)^2. \tag{D.38}$$

With an undistorted steady state characterized by $\aleph = (1-\alpha)$ and $(1-\tau^n) = \frac{\varepsilon_w}{(\varepsilon_w-1)}$:

$$
\begin{aligned}
&E\sum_{t=t_0}^{\infty}\beta^{t-t_0}\Delta U^{SGU} \\
&=\frac{1}{2}\frac{N^{1+\varphi}}{(1-\alpha)}\phi_w^{SGU}\left(1-\frac{(1-\alpha)\,\varepsilon_w}{(\varepsilon_w-1)\,\aleph\,(1-\tau^n)}-\frac{(1-\beta)\,(1-\alpha)\,\theta_w}{(1-\theta_w)}\frac{(1-\alpha)\,\varepsilon_w}{(\varepsilon_w-1)\,\aleph\,(1-\tau^n)}\right)\frac{1}{1-\beta}E\left(\hat{\Pi}_t^w\right)^2 \\
&=\frac{1}{2}\frac{N^{1+\varphi}}{(1-\alpha)}\phi_w^{SGU}\left(-\frac{(1-\beta)\,(1-\alpha)\,\theta_w}{(1-\theta_w)}\right)\frac{1}{1-\beta}E\left(\hat{\Pi}_t^w\right)^2 \\
&=\frac{1}{2}N^{1+\varphi}\phi_w^{SGU}\left(-\frac{(1-\beta)\,\theta_w}{(1-\theta_w)}\right)\frac{1}{1-\beta}E\left(\hat{\Pi}_t^w\right)^2 .
\end{aligned}
\tag{D.39}
$$

Noting that

$$
1-\frac{1-\beta\theta_w}{1-\theta_w}=\frac{1-\theta_w}{1-\theta_w}-\frac{1-\beta\theta_w}{1-\theta_w}=\frac{-\theta_w+\beta\theta_w}{1-\theta_w}=\frac{-\theta_w\,(1-\beta)}{1-\theta_w}\,, \tag{D.40}
$$

we get that

$$
E\sum_{t=t_0}^{\infty}\beta^{t-t_0}\Delta U^{SGU}=\frac{1}{2}N^{1+\varphi}\phi_w^{SGU}\left(1-\frac{1-\beta\theta_w}{1-\theta_w}\right)\frac{1}{1-\beta}E\left(\hat{\Pi}_t^w\right)^2<0\,. \tag{D.41}
$$

Hence, Calvo wage setting is associated with higher unconditional welfare losses. This completes the proof of Proposition 3for the SGU case.

D.3 EHL

In the EHL case, workers supply differentiated goods, which complicates aggregation in the Calvo case. In the symmetric Rotemberg equilibrium, the aggregate felicity function is still

$$
U_{Rotemberg}^{EHL}=\int_0^1 U_t\,(j)\,dj=\frac{C_t^{1-\sigma}-1}{1-\sigma}-\frac{N_t^{1+\varphi}}{1+\varphi}=U_{Rotemberg}^{SGU} \tag{D.42}
$$

Aggregation in the Calvo case is more involved. We obtain

$$
\begin{aligned}
U_{Calvo}^{EHL} &= \int_0^1 \frac{\left(C_t^j\right)^{1-\sigma}-1}{1-\sigma} - \frac{\left(N_t^j\right)^{1+\varphi}}{1+\varphi} dj \\
&= \frac{C_t^{1-\sigma}-1}{1-\sigma} - \int_0^1 \left(\frac{\left(N_t^j\right)^{1+\varphi}}{1+\varphi}\right) dj \\
&= \frac{C_t^{1-\sigma}-1}{1-\sigma} - \int_0^1 \left(\frac{\left(\left(\frac{W_t^j}{W_t}\right)^{-\varepsilon} N_t^d\right)^{1+\varphi}}{1+\varphi}\right) dj \\
&= \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{\left(N_t^d\right)^{1+\varphi}}{1+\varphi} \int_0^1 \left(\left(\frac{W_t^j}{W_t}\right)^{-\varepsilon}\right)^{1+\varphi} dj \\
&= \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{\left(N_t^d\right)^{1+\varphi}}{1+\varphi} \int_0^1 \left(\frac{W_t^j}{W_t}\right)^{-\varepsilon(1+\varphi)} dj \\
&= \left(\frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{\left(N_t^d\right)^{1+\varphi}}{1+\varphi} X_t^W\right) \\
&= \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{\left(\frac{N_t}{S_t^W}\right)^{1+\varphi}}{1+\varphi} X_t^W , \qquad \text{(D.43)}
\end{aligned}
$$

where the second equality uses the complete markets assumption and where the auxiliary variable X_t^W has the recursive representation

$$
\begin{aligned}
X_t^W &\equiv \int_0^1 \left(\frac{W_t^j}{W_t}\right)^{-\varepsilon_w(1+\varphi)} dj = (1-\theta)\left(\frac{W_t^*}{W_t}\right)^{-\varepsilon_w(1+\varphi)} + \theta \int_0^1 \left(\frac{W_{t-1}^j}{W_t}\right)^{-\varepsilon_w(1+\varphi)} dj \\
&= (1-\theta)\left(\frac{W_t^*}{W_t}\right)^{-\varepsilon_w(1+\varphi)} + \theta\left(\frac{W_{t-1}}{W_t}\right)^{-\varepsilon_w} \int_0^1 \left(\frac{W_{t-1}^j}{W_{t-1}}\right)^{-\varepsilon_w(1+\varphi)} dj \\
&= (1-\theta)\left(\Pi_t^{w*}\right)^{-\varepsilon_w(1+\varphi)} + \theta\left(\frac{W_{t-1}}{W_t}\right)^{-\varepsilon_w(1+\varphi)} X_{t-1}^W \\
&= (1-\theta)\left(\Pi_t^{w*}\right)^{-\varepsilon_w(1+\varphi)} + \theta(\Pi_t^w)^{\varepsilon_w(1+\varphi)} X_{t-1}^W , \qquad \text{(D.44)}
\end{aligned}
$$

Thus, compared to the Rotemberg case in (D.42), the disutility of labor term is different. A second-order approximation to the disutility of labor term yields

$$\begin{aligned}\frac{\left(\frac{N_t}{S_t^W}\right)^{1+\varphi}}{1+\varphi}X_t^W \approx&\frac{N^{1+\varphi}}{1+\varphi}+\frac{N^{\varphi}}{\left(S^W\right)^{1+\varphi}}X^W\left(N_t-N\right)-N^{1+\varphi}\left(S^W\right)^{-(1+\varphi)-1}X^W\left(S_t^W-S^W\right)\\&+\frac{1}{2}\frac{X^W}{\left(S^W\right)^{1+\varphi}}N^{\varphi-1}\varphi(N_t-N)^2+\frac{\left(\frac{N}{S^W}\right)^{1+\varphi}}{1+\varphi}\left(X_t^W-X^W\right)\\=&\frac{N^{1+\varphi}}{1+\varphi}+N^{\varphi}\hat{N}_t-N^{1+\varphi}\hat{S}_t^W+\frac{1}{2}N^{\varphi-1}\varphi\hat{N}_t^2+\frac{N^{1+\varphi}}{1+\varphi}\hat{X}_t^W\ ,\end{aligned} \tag{D.45}$$

where we used that the dispersion terms $\hat{S}_t^w$ and $\hat{X}_t^w$ are 0 up to first order, as we will show below. As the resource constraint and the production function are the same as in the SGU case, we get

$$\begin{aligned}\hat{U}_{Calvo}^{EHL}=&C^{-\sigma}\hat{C}_t-\frac{1}{2}C^{-\sigma-1}\sigma\hat{C}_t^2-N^{\varphi}\hat{N}_t+N^{1+\varphi}\left(S_t^W-S^W\right)-\frac{1}{2}N^{\varphi-1}\varphi\hat{N}_t^2-\frac{N^{1+\varphi}}{1+\varphi}\left(X_t^W-X^W\right)\\=&C^{-\sigma}\hat{C}_t-\frac{1}{2}C^{-\sigma-1}\sigma\hat{C}_t^2-N^{\varphi}\left(\frac{1}{(1-\alpha)N^{-\alpha}}\left[\begin{array}{l}\hat{C}_t+(1-\alpha)C\hat{S}_t^w-N^{1-\alpha}\hat{A}_t-(1-\alpha)N^{-\alpha}\hat{N}_t\\-(1-\alpha)N^{-\alpha}\hat{N}_t\hat{A}_t+\frac{1}{2}\alpha(1-\alpha)N^{-\alpha-1}\hat{N}_t^2\end{array}\right]\right)\\&+N^{1+\varphi}\left(S_t^W-S^W\right)-\frac{1}{2}N^{\varphi-1}\varphi\hat{N}_t^2-\frac{N^{1+\varphi}}{1+\varphi}\left(X_t^W-X^W\right)\\=&\left(C^{-\sigma}-\frac{N^{\varphi}}{(1-\alpha)N^{-\alpha}}\right)\hat{C}_t-\frac{1}{2}C^{-\sigma-1}\sigma\hat{C}_t^2-\left(\frac{1}{2}N^{\varphi-1}\varphi-\frac{N^{\varphi}}{(1-\alpha)N^{-\alpha}}\frac{1}{2}\alpha(1-\alpha)N^{-\alpha-1}\right)\hat{N}_t^2\\&+\frac{N^{\varphi}N^{\alpha}}{(1-\alpha)N^{-\alpha}}\hat{A}_t+N^{\varphi}\hat{N}_t\hat{A}_t-\left(\frac{N^{\varphi}}{N^{-\alpha}}C-N^{1+\varphi}\right)\hat{S}_t^w-\frac{N^{1+\varphi}}{1+\varphi}\hat{X}_t^W\ .\end{aligned} \tag{D.46}$$

Thus, the period utility difference between Calvo and Rotemberg is given by

$$\begin{aligned}\hat{U}_{Calvo}^{EHL}-\hat{U}_{Rotemberg}^{EHL}=&\frac{N^{\varphi}}{(1-\alpha)N^{-\alpha}}C\frac{\phi_w^{EHL}}{2}\left(\hat{\Pi}_t^w\right)^2+\left(\frac{N^{\varphi}}{N^{-\alpha}}C-N^{1+\varphi}\right)\hat{S}_t^w-\frac{N^{1+\varphi}}{1+\varphi}\hat{X}_t^W\\=&\frac{N^{\varphi}}{(1-\alpha)N^{-\alpha}}C\left(\frac{\phi_w^{EHL}}{2}\left(\hat{\Pi}_t^w\right)^2-(1-\alpha)\hat{S}_t^w\right)+N^{1+\varphi}\left(\hat{S}_t^w-\frac{1}{1+\varphi}\hat{X}_t^W\right)\ .\end{aligned} \tag{D.47}$$

As in steady state

$$\frac{N^{\varphi}C}{(1-\alpha)N^{-\alpha}}=\frac{N^{\varphi}N^{1-\alpha}}{(1-\alpha)N^{-\alpha}}=\frac{N^{1+\varphi}}{(1-\alpha)} \tag{D.48}$$

we can simplify the welfare loss to

$$\Delta U^{EHL}=\hat{U}_{Calvo}^{EHL}-\hat{U}_{Rotemberg}^{EHL}=\frac{N^{1+\varphi}}{(1-\alpha)}\left(\frac{\phi_w^{EHL}}{2}\left(\hat{\Pi}_t^w\right)^2-\frac{(1-\alpha)}{1+\varphi}\hat{X}_t^W\right)\ . \tag{D.49}$$

The derivation of the second-order approximation to the auxiliary variable

$$X_t^W = (1-\theta_W)\left(\Pi_t^{w*}\right)^{-\varepsilon_w(1+\varphi)} + \theta_W\left(\Pi_t^w\right)^{\varepsilon_w(1+\varphi)} X_{t-1}^W \tag{D.50}$$

follows the lines of the one for S_t^w. A second-order approximation yields:

$$\begin{aligned}
\hat{X}_t^W =& (1-\theta_w)(-\varepsilon_w(1+\varphi))(\Pi_t^{w*})^{-\varepsilon_w(1+\varphi)-1}(\Pi_t^{w*}-\Pi^{w*}) + \varepsilon_w(1+\varphi)\,\theta_w(\Pi^w)^{\varepsilon_w(1+\varphi)-1}X^W(\Pi_t^w-\Pi^w)\\
&+\theta_w(\Pi_t^w)^{\varepsilon_w(1+\varphi)}\left(X_{t-1}^W - X^w\right)\\
&+\frac{1}{2}\begin{bmatrix}(1-\theta_w)(-\varepsilon_w(1+\varphi))(-\varepsilon_w(1+\varphi)-1)(\Pi_t^{w*})^{-\varepsilon_w(1+\varphi)-2}(\Pi_t^{w*}-\Pi^{w*})^2\\ +\varepsilon_w(1+\varphi)(\varepsilon_w(1+\varphi)-1)\,\theta_w(\Pi_t^w)^{\varepsilon_w(1+\varphi)-2}X^W(\Pi_t^w-\Pi^w)^2\end{bmatrix}\\
&+\varepsilon_w(1+\varphi)\,\theta_w(\Pi^w)^{\varepsilon_w(1+\varphi)-1}(\Pi_t^w-\Pi^w)\left(X_{t-1}^W-X^w\right)\\
=& (1-\theta_w)(-\varepsilon_w(1+\varphi))(\Pi_t^{w*}-\Pi^{w*}) + \varepsilon_w(1+\varphi)\,\theta_w(\Pi_t^w-\Pi^w) + \theta_w\left(X_{t-1}^W-X^w\right)\\
&+\frac{1}{2}\begin{bmatrix}(1-\theta_w)\,\varepsilon_w(1+\varphi)(\varepsilon_w(1+\varphi)+1)(\Pi_t^{w*}-\Pi^{w*})^2\\ +\varepsilon_w(1+\varphi)(\varepsilon_w(1+\varphi)-1)\,\theta_w(\Pi_t^w-\Pi^w)^2\end{bmatrix}\\
&+\varepsilon_w(1+\varphi)\,\theta_w(\Pi_t^w-\Pi^w)\left(X_{t-1}^W-X^w\right)\\
=&\varepsilon_w(1+\varphi)\left(\theta_w(\Pi_t^w-\Pi^w)-(1-\theta_w)(\Pi_t^{w*}-\Pi^{w*})\right)+\theta_w\left(X_{t-1}^W-X^w\right)\\
&+\frac{1}{2}\begin{bmatrix}(1-\theta_w)\,\varepsilon_w(1+\varphi)(\varepsilon_w(1+\varphi)+1)(\Pi_t^{w*}-\Pi^{w*})^2\\ +\varepsilon_w(1+\varphi)(\varepsilon_w(1+\varphi)-1)\,\theta_w(\Pi_t^w-\Pi^w)^2\end{bmatrix}\\
&+\varepsilon_w(1+\varphi)\,\theta_w(\Pi_t^w-\Pi^w)\left(X_{t-1}^W-X^w\right)\ .
\end{aligned} \tag{D.51}$$

We already know that

$$\theta_w(\Pi_t^w-\Pi^w)-(1-\theta_w)(\Pi_t^{w*}-\Pi^{w*}) = \frac{1}{2}\left[(1-\theta_w)(-\varepsilon_w)(\Pi_t^{w*}-\Pi^{w*})^2-\theta_w(\varepsilon_w-2)(\Pi_t^w-\Pi^w)^2\right], \tag{D.24}$$

so that

$$\begin{aligned}
\hat{X}_t^W =& \frac{1}{2}\varepsilon_w(1+\varphi)\left[(1-\theta_w)(-\varepsilon_w)(\Pi_t^{w*}-\Pi^{w*})^2-\theta_w(\varepsilon_w-2)(\Pi_t^w-\Pi^w)^2\right]\\
&+\theta_w\left(X_{t-1}^W-X^w\right)\\
&+\frac{1}{2}\begin{bmatrix}(1-\theta_w)\,\varepsilon_w(1+\varphi)(\varepsilon_w(1+\varphi)+1)(\Pi_t^{w*}-\Pi^{w*})^2\\ +\varepsilon_w(1+\varphi)(\varepsilon_w(1+\varphi)-1)\,\theta_w(\Pi_t^w-\Pi^w)^2\end{bmatrix}\\
&+\varepsilon_w(1+\varphi)\,\theta_w(\Pi_t^w-\Pi^w)\left(X_{t-1}^W-X^w\right)\ .
\end{aligned} \tag{D.52}$$

Like S_t^W, the dispersion related term $\hat{X}_t^W$ is zero up to first order. For that reason, we can immediately drop the last term in the previous equation as being third order. Using

$$\left(\frac{\theta_w}{(1-\theta_w)}\right)^2 (\Pi_t^w - \Pi^w)^2 = (\Pi_t^{w*} - \Pi^{w*})^2 \tag{D.22}$$

we obtain:

$$\begin{aligned}
\hat{X}_t^W =& \frac{1}{2}\varepsilon_w (1+\varphi) \begin{bmatrix} (1-\theta_w)(-\varepsilon_w)\left(\frac{\theta_w}{(1-\theta_w)}\right)^2 \left(\hat{\Pi}_t^w\right)^2 \\ -\theta_w(\varepsilon_w - 2)\left(\hat{\Pi}_t^w\right)^2 \end{bmatrix} + \theta_w\left(X_{t-1}^W - X^w\right) \\
&+ \frac{1}{2}\begin{bmatrix} (1-\theta_w)\varepsilon_w(1+\varphi)(\varepsilon_w(1+\varphi)+1)\left(\frac{\theta_w}{(1-\theta_w)}\right)^2\left(\hat{\Pi}_t^w\right)^2 \\ +\varepsilon_w(1+\varphi)(\varepsilon_w(1+\varphi)-1)\theta_w\left(\hat{\Pi}_t^w\right)^2 \end{bmatrix} \\
=&\theta_w\left(X_{t-1}^W - X^w\right) + \frac{1}{2}(1+\varphi)\begin{bmatrix} (1-\theta_w)\left(\frac{\theta_w}{(1-\theta_w)}\right)^2\left(\hat{\Pi}_t^w\right)^2\left(-\varepsilon_w^2 + \varepsilon_w^2(1+\varphi)+\varepsilon_w\right) \\ -\theta_w\left(\hat{\Pi}_t^w\right)^2\left(-\varepsilon_w^2 - 2\varepsilon_w + \varepsilon_w^2(1+\varphi) - \varepsilon_w\right) \end{bmatrix} \\
=&\theta_w\left(X_{t-1}^W - X^w\right) + \frac{1}{2}(1+\varphi)\begin{bmatrix} \frac{\theta_w^2}{(1-\theta_w)}\left(\hat{\Pi}_t^w\right)^2(\varepsilon_w(1+\varphi\varepsilon_w)) \\ +\theta_w\left(\hat{\Pi}_t^w\right)^2(\varepsilon_w(1+\varphi\varepsilon_w)) \end{bmatrix} \\
=&\theta_w\left(X_{t-1}^W - X^w\right) + \frac{1}{2}(\varepsilon_w(1+\varphi\varepsilon_w))\theta_w(1+\varphi)\left[\frac{\theta_w}{(1-\theta_w)}+1\right]\left(\hat{\Pi}_t^w\right)^2 \\
=&\theta_w\left(X_{t-1}^W - X^w\right) + \frac{1}{2}(\varepsilon_w(1+\varphi\varepsilon_w))\theta_w(1+\varphi)\left[\frac{\theta_w}{(1-\theta_w)}+\frac{(1-\theta_w)}{(1-\theta_w)}\right]\left(\hat{\Pi}_t^w\right)^2 \\
=&\theta_w\left(X_{t-1}^W - X^w\right) + \frac{1}{2}(1+\varphi)\frac{(\varepsilon_w(1+\varphi\varepsilon_w))\theta_w}{1-\theta_w}\left(\hat{\Pi}_t^w\right)^2
\end{aligned} \tag{D.53}$$

Analogous to (D.28), the discounted sum of the auxiliary price dispersion term is given by

$$\sum_{t=t_0}^{\infty}\beta^{t-t_0}\hat{X}_t^W = \frac{\theta_w}{1-\beta\theta_w}\hat{X}_{t_0-1}^W + \sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{1}{2}\left(\frac{(1+\varphi)(\varepsilon_w(1+\varphi\varepsilon_w))\theta_w}{(1-\beta\theta_w)(1-\theta_w)}\left(\hat{\Pi}_t^w\right)^2\right). \tag{D.54}$$

The present discounted welfare loss then follows as:

$$\begin{aligned}
\sum_{t=t_0}^{\infty}\beta^{t-t_0}\Delta U^{EHL} &= \frac{N^{1+\varphi}}{(1-\alpha)}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(\frac{\phi_w^{EHL}}{2}\left(\hat{\Pi}_t^w\right)^2 - \frac{(1-\alpha)}{1+\varphi}\hat{X}_t^W\right) \\
= \frac{N^{1+\varphi}}{(1-\alpha)}&\left[-\frac{\theta_w}{1-\beta\theta_w}\hat{X}_{t_0-1}^W + \sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{1}{2}\left(\phi_w^{EHL} - \frac{(1-\alpha)}{1+\varphi}\frac{(1+\varphi)(\varepsilon_w(1+\varphi\varepsilon_w))\theta_w}{(1-\beta\theta_w)(1-\theta_w)}\right)\left(\hat{\Pi}_t^w\right)^2\right].
\end{aligned} \tag{D.55}$$

Conditional on initial wage dispersion being 0 so that $\hat{X}^W_{t_0-1} = 0$, we get the conditional welfare difference as:

$$
\begin{aligned}
& E_t \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Delta U^{EHL} \\
&= \frac{N^{1+\varphi}}{(1-\alpha)} E_t \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left(\frac{\phi_w^{EHL}}{2} \left(\hat{\Pi}_t^w\right)^2 - \frac{(1-\alpha)}{1+\varphi} \hat{X}_t^W \right) \\
&= \frac{N^{1+\varphi}}{(1-\alpha)} \left[\sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\phi_w^{EHL} - \frac{(1-\alpha)}{1+\varphi} \frac{(1+\varphi)\left(\varepsilon_w \left(1+\varphi\varepsilon_w\right)\right)\theta_w}{\left(1-\beta\theta_w\right)\left(1-\theta_w\right)} \right) E_t \left(\hat{\Pi}_t^w\right)^2 \right] \\
&= \frac{N^{1+\varphi}}{(1-\alpha)} \left[\sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\phi_w^{EHL} - \frac{(1-\alpha)\left(\varepsilon_w \left(1+\varphi\varepsilon_w\right)\right)\theta_w}{\left(1-\beta\theta_w\right)\left(1-\theta_w\right)} \right) E_t \left(\hat{\Pi}_t^w\right)^2 \right] \\
&= N^{1+\varphi} \left[\sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\frac{\left(\varepsilon_w - 1\right)\theta_w \left(1-\tau^n\right) \aleph \left(1+\varphi\varepsilon_w\right)}{(1-\alpha)\left(1-\theta_w\right)\left(1-\beta\theta_w\right)} - \frac{\varepsilon_w \left(1+\varphi\varepsilon_w\right)\theta_w}{\left(1-\beta\theta_w\right)\left(1-\theta_w\right)} \right) E_t \left(\hat{\Pi}_t^w\right)^2 \right],
\end{aligned}
\tag{D.56}
$$

where the last line imposes identical slopes of the wage PC via:

$$
\phi_w^{EHL} = \frac{\left(\varepsilon_w - 1\right)\theta_w \left(1-\tau^n\right) \aleph \left(1+\varphi\varepsilon_w\right)}{\left(1-\theta_w\right)\left(1-\beta\theta_w\right)} . \tag{D.57}
$$

In an undistorted steady state with $\aleph = (1-\alpha)$ and $(1-\tau^n) = \frac{\varepsilon_w}{(\varepsilon_w - 1)}$:

$$
E_t \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Delta U^{EHL} = N^{1+\varphi} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\frac{\varepsilon_w \theta_w \left(1+\varphi\varepsilon_w\right)}{\left(1-\theta_w\right)\left(1-\beta\theta_w\right)} - \frac{\varepsilon_w \theta_w \left(1+\varphi\varepsilon_w\right)}{\left(1-\beta\theta_w\right)\left(1-\theta_w\right)} \right) E_t \left(\hat{\Pi}_t^w\right)^2 \right\} = 0. \tag{D.58}
$$

This completes the proof of Proposition 2 for the EHL framework.

Finally, consider unconditional welfare. Taking the unconditional expectations of equation (D.53) we get

$$
E\hat{X}_t^W = \theta_w \hat{X}_{t-1}^W + \frac{1}{2}(1+\varphi) \frac{\left(\varepsilon_w \left(1+\varphi\varepsilon_w\right)\right)\theta_w}{1-\theta_w} E\left(\hat{\Pi}_t^w\right)^2 \tag{D.59}
$$

so that the unconditional mean of the auxiliary price dispersion term is given by:

$$
E\hat{X}_t^W = \frac{1}{2}(1+\varphi) \frac{\left(\varepsilon_w \left(1+\varphi\varepsilon_w\right)\right)\theta_w}{\left(1-\theta_w\right)^2} E\left(\hat{\Pi}_t^w\right)^2 . \tag{D.60}
$$

Taking the unconditional expectations in (D.55) and using the previous result, we obtain:

$$
\begin{aligned}
&E\sum_{t=t_0}^{\infty}\beta^{t-t_0}\Delta U^{EHL}\\
&=\frac{N^{1+\varphi}}{(1-\alpha)}\left[\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{1}{2}\left(\phi_w^{EHL}-(1-\alpha)\frac{(\varepsilon_w(1+\varphi\varepsilon_w))\theta_w}{(1-\beta\theta_w)(1-\theta_w)}\right)E\left(\hat{\Pi}_t^w\right)^2-\frac{(1-\alpha)}{1+\varphi}\frac{\theta_w}{1-\beta\theta_w}E\hat{X}_{t_0-1}^W\right]\\
&=\frac{N^{1+\varphi}}{(1-\alpha)}\left[\begin{array}{l}\frac{1}{2}\left(\phi_w^{EHL}-(1-\alpha)\frac{(\varepsilon_w(1+\varphi\varepsilon_w))\theta_w}{(1-\beta\theta_w)(1-\theta_w)}\right)E\left(\hat{\Pi}_t^w\right)^2\frac{1}{1-\beta}\\ -\frac{(1-\alpha)}{1+\varphi}\frac{\theta_w}{1-\beta\theta_w}\frac{1}{2}(1+\varphi)\frac{(\varepsilon_w(1+\varphi\varepsilon_w))\theta_w}{(1-\theta_w)^2}E\left(\hat{\Pi}_t^w\right)^2\end{array}\right]\\
&=\frac{1}{2}\frac{N^{1+\varphi}}{(1-\alpha)}\left[\phi_w^{EHL}-(1-\alpha)\frac{(\varepsilon_w(1+\varphi\varepsilon_w))\theta_w}{(1-\beta\theta_w)(1-\theta_w)}-(1-\beta)(1-\alpha)\frac{\theta_w}{1-\beta\theta_w}\frac{(\varepsilon_w(1+\varphi\varepsilon_w))\theta_w}{(1-\theta_w)^2}\right].
\end{aligned}
\tag{D.61}
$$

Again imposing identical slopes of the wage Phillips curves, we get:

$$
E\sum_{t=t_0}^{\infty}\beta^{t-t_0}\Delta W^{EHL}=\frac{1}{2}\frac{N^{1+\varphi}}{(1-\alpha)}\phi_w^{EHL}\left(\begin{array}{l}1-\frac{(1-\alpha)\varepsilon_w}{(\varepsilon_w-1)\aleph(1-\tau^n)}\\ -\frac{(1-\beta)(1-\alpha)\theta_w}{(1-\theta_w)}\frac{(1-\alpha)\varepsilon_w}{(\varepsilon_w-1)\aleph(1-\tau^n)}\end{array}\right)\frac{1}{1-\beta}E\left(\hat{\Pi}_t^w\right)^2
\tag{D.62}
$$

In an undistorted steady state with $\aleph=(1-\alpha)$ and $(1-\tau^n)=\frac{\varepsilon_w}{(\varepsilon_w-1)}$:

$$
\begin{aligned}
E\sum_{t=t_0}^{\infty}\beta^{t-t_0}\Delta U^{EHL}&=\frac{1}{2}\frac{N^{1+\varphi}}{(1-\alpha)}\phi_w^{EHL}\left(\begin{array}{l}1-\frac{(1-\alpha)\varepsilon_w}{(\varepsilon_w-1)\aleph(1-\tau^n)}\\ -\frac{(1-\beta)(1-\alpha)\theta_w}{(1-\theta_w)}\frac{(1-\alpha)\varepsilon_w}{(\varepsilon_w-1)\aleph(1-\tau^n)}\end{array}\right)\frac{1}{1-\beta}E\left(\hat{\Pi}_t^w\right)^2\\
&=\frac{1}{2}\frac{N^{1+\varphi}}{(1-\alpha)}\phi_w^{EHL}\left(-\frac{(1-\beta)(1-\alpha)\theta_w}{(1-\theta_w)}\right)\frac{1}{1-\beta}E\left(\hat{\Pi}_t^w\right)^2\\
&=\frac{1}{2}N^{1+\varphi}\phi_w^{EHL}\left(-\frac{(1-\beta)\theta_w}{(1-\theta_w)}\right)\frac{1}{1-\beta}E\left(\hat{\Pi}_t^w\right)^2.
\end{aligned}
\tag{D.63}
$$

Again using (D.40) it follows that:

$$
E\sum_{t=t_0}^{\infty}\beta^{t-t_0}\Delta U^{EHL}=\frac{1}{2}N^{1+\varphi}\phi_w^{EHL}\left(1-\frac{1-\beta\theta_w}{1-\theta_w}\right)\frac{1}{1-\beta}E\left(\hat{\Pi}_t^w\right)^2<0
\tag{D.64}
$$

Thus, Calvo wage setting is associated with higher unconditional welfare losses. This completes the proof of Proposition 3 for the SGU case.

D.4 Comparison SGU/EHL with efficient steady state

The previous sections have compared Calvo and Rotemberg wage setting in the SGU and EHL setup, respectively, for a given amount for wage stickiness as measured by the Calvo wage setting parameter. But for a given Calvo wage setting parameter, the EHL and SGU setups produce very different slopes of the wage Phillips Curve, implying that amount of wage inflation variability will differ across the two insurance scheme. Thus, to make the EHL and SGU frameworks comparable, we need to fix the slope of the Wage Phillips Curve at the same level by setting the respective Calvo parameters to satisfy

$$\phi_w = \phi_w^{EHL} = \frac{(\varepsilon_w - 1)\,\theta_w^{EHL}\,(1-\tau^n)\,\aleph\,(1+\varphi\varepsilon_w)}{(1-\theta_w^{EHL})\,(1-\beta\theta_w^{EHL})} = \frac{(\varepsilon_w - 1)\,\theta_w^{SGU}\,(1-\tau^n)\,\aleph}{(1-\theta_w^{SGU})\,(1-\beta\theta_w^{SGU})} = \phi_w^{SGU}\,. \tag{D.65}$$

This in turn implies that $\theta_w^{EHL} < \theta_w^{SGU}$. Put differently, the slope of the wage Phillips Curve under Calvo wage setting is steeper in the EHL than in the SGU framework for a given amount of Calvo wage stickiness. In order to keep the slope of the Wage Phillips Curve the same, the EHL setup requires a lower degree of Calvo wage stickiness. The difference in unconditional welfare between the SGU and the EHL case under identical slopes of the Wage Phillips Curve then follows from (D.41) and (D.64) as

$$\begin{aligned} & E\sum_{t=t_0}^{\infty}\beta^{t-t_0}\Delta U^{SGU} - E\sum_{t=t_0}^{\infty}\beta^{t-t_0}\Delta U^{EHL} \\ &= \frac{1}{2}N^{1+\varphi}\frac{\phi_w}{1-\beta}\left(1-\frac{1-\beta\theta_w^{SGU}}{1-\theta_w^{SGU}} - \left(1-\frac{1-\beta\theta_w^{EHL}}{1-\theta_w^{EHL}}\right)\right)E\Big(\hat{\Pi}_t^w\Big)^2 \\ &= \frac{1}{2}N^{1+\varphi}\frac{\phi_w}{1-\beta}\left(\frac{1-\beta\theta_w^{EHL}}{1-\theta_w^{EHL}} - \frac{1-\beta\theta_w^{SGU}}{1-\theta_w^{SGU}}\right)E\Big(\hat{\Pi}_t^w\Big)^2 < 0\,, \end{aligned} \tag{D.66}$$

where we used that inflation variability is the same in both setups with $\phi_w = \phi_w^{EHL} = \phi_w^{SGU}$ and that $\theta_w^{EHL} < \theta_w^{SGU}$. This proves Proposition 4

Table 7: Model moments from the Galí (2015), Chapter 6 model

	Strict Targeting			Flexible Targeting		
	Price	Wage	Comp.	Price	Wage	Comp.
	Technology Shock					
$\sigma(\pi_p)$	0.000	0.135	0.123	0.298	0.243	0.246
$\sigma(\pi_w)$	0.266	0.000	0.021	0.238	0.165	0.169
$\sigma(\tilde{y})$	3.417	0.204	0.000	0.848	1.183	1.113
	Demand Shock					
$\sigma(\pi_p)$	0.000	0.000	0.000	0.026	0.041	0.038
$\sigma(\pi_w)$	0.000	0.000	0.000	0.054	0.066	0.064
$\sigma(\tilde{y})$	0.000	0.000	0.000	1.082	1.054	1.061

Table 7: *Notes*: Displayed are the variance of log price inflation π_p, log wage inflation π_w, and of the log output gap $\tilde{y}$. Numbers have been multiplied by 100.