A Decision problems of retirees and workers

We introduce some notation in order to make the derivations more readable. While we still solve the decision problems of individual retirees and workers, we drop the superscripts i and j. Furthermore, $V_2^r(a_t^r, b_{t+1}^r)$ denotes the derivative of the value function of a retiree in period $t+1$ with respect to per-period pension benefits b_{t+1}^r (i.e. the second state variable). We only show the derivations for the real accounting framework since those for the nominal accounting framework are analogous.

A.1 Retiree decision problem

A retiree maximises the following objective in period t:

$$V^r(a_{t-1}^r, b_t^r) = \max_{c_t^r, l_t^r, a_t^r, b_{t+1}^r} \left(\left((c_t^r)^v (1-l_t^r)^{1-v} \right)^\rho + \gamma\beta \left(V^r(a_t^r, b_{t+1}^r) \right)^\rho \right)^{\frac{1}{\rho}}$$

subject to:

$$a_t^r = \frac{1+r_t}{\gamma} a_{t-1}^r + (1-\tau_t)\xi w_t l_t^r + \mu_t b_t^r - c_t^r,$$
$$b_{t+1}^r = \mu_t b_t^r + \nu_t \xi w_t l_t^r.$$

Substituting the constraints:

$$V^r(a_{t-1}^r, b_t^r) =$$
$$\max_{c_t^r, l_t^r} \left(\left((c_t^r)^v (1-l_t^r)^{1-v} \right)^\rho + \gamma\beta \left(V^r \left(\frac{1+r_t}{\gamma} a_{t-1}^r + (1-\tau_t)\xi w_t l_t^r + \mu_t b_t^r - c_t^r, \mu_t b_t^r + \nu_t \xi w_t l_t^r \right) \right)^\rho \right)^{\frac{1}{\rho}}.$$

A.1.1 First-order conditions

The first-order condition with respect to c_t^r:

$$v \left(c_t^r\right)^{v\rho-1} \left(1-l_t^r\right)^{(1-v)\rho} = \beta\gamma \left(V^r \left(a_t^r, b_{t+1}^r\right)\right)^{\rho-1} V_1^r \left(a_t^r, b_{t+1}^r\right). \qquad \text{(A.1)}$$

Using the envelope theorem:

$$V_1^r\left(a_{t-1}^r, b_t^r\right) = \left(V^r\left(a_{t-1}^r, b_t^r\right)\right)^{1-\rho} v \frac{1+r_t}{\gamma}\left(c_t^r\right)^{v\rho-1}\left(1-l_t^r\right)^{(1-v)\rho}. \tag{A.2}$$

Shifting (A.2) one period forward and combining with (A.1) gives the Euler equation:

$$\frac{c_{t+1}^r}{c_t^r} = \beta(1+r_{t+1})\frac{((c_{t+1}^r)^v(1-l_{t+1}^r)^{1-v})^\rho}{((c_t^r)^v(1-l_t^r)^{1-v})^\rho}. \tag{A.3}$$

The first-order condition with respect to l_t^r:

$$(1-v)\left(c_t^r\right)^{v\rho}\left(1-l_t^r\right)^{(1-v)(\rho-1)} =$$

$$\beta\gamma\left(V^r\left(a_t^r, b_{t+1}^r\right)\right)^{\rho-1}\left(V_1^r\left(a_t^r, b_{t+1}^r\right)(1-\tau_t)\xi w_t + V_2^r\left(a_t^r, b_{t+1}^r\right)\mu_{t+1}\nu_t\xi w_t\right) \Leftrightarrow$$

$$(1-v)\left(c_t^r\right)^{v\rho}\left(1-l_t^r\right)^{(1-v)(\rho-1)} = \beta\gamma\left(V^r\left(a_t^r, b_{t+1}^r\right)\right)^{\rho-1} V_1^r\left(a_t^r, b_{t+1}^r\right)\left(1-\tau_t^r\right)\xi w_t, \tag{A.4}$$

where we use the linearity of the consumption function in total lifetime wealth to determine that $V_2^r\left(a_t^r, b_{t+1}^r\right) = R_{t+1}^r \frac{\gamma}{1+r_{t+1}} V_1^r\left(a_t^r, b_{t+1}^r\right)$ and define $\tau_t^r = \tau_t - (R_t^r - 1)\nu_t$. Working one extra unit of time in period t gives $\mu_{t+1}\nu_t\xi w_t$ additional per-period pension benefits from period $t+1$ onwards. $V_2^r\left(a_t^r, b_{t+1}^r\right)$ denotes the proper valuation of one additional accrued unit of per-period pension benefits. Recall that the annuity factor $R_{t+1}^r = 1 + \mu_{t+2}\frac{\gamma}{1+r_{t+1}}R_{t+2}^r$ represents the present discounted value to a retiree in period $t+1$ of receiving one consumption good each period from period $t+1$ until death (corrected for future revaluation). One additional accrued unit of per-period pension benefits from period $t+1$ onwards is therefore equally valuable to a retiree as having $R_{t+1}^r\frac{\gamma}{1+r_{t+1}}$ additional units of a_t^r. Combining (A.4) with (A.1):

$$1 - l_t^r = \frac{1-v}{v}\frac{c_t^r}{(1-\tau_t^r)\xi w_t}. \tag{A.5}$$

A.1.2 Writing the Euler equation solely in terms of consumption

Substituting (A.5) into (A.3):

$$\frac{c_{t+1}^r}{c_t^r} = \left(\beta(1+r_{t+1})\left(\frac{(1-\tau_t^r)w_t}{(1-\tau_{t+1}^r)w_{t+1}}\right)^{(1-v)\rho}\right)^{\sigma}, \tag{A.6}$$

where we have used that $\sigma = \frac{1}{1-\rho}$. We define retiree full consumption as $x_t^r \equiv c_t^r + (1-l_t^r)(1-\tau_t^r)\xi w_t = \frac{c_t^r}{v}$, which follows the same Euler equation as c_t^r:

$$x_\tau^r = x_t^r \prod_{s=t}^{\tau-1}\left(\beta(1+r_{s+1})\left(\frac{(1-\tau_s^r)w_s}{(1-\tau_{s+1}^r)w_{s+1}}\right)^{(1-v)\rho}\right)^{\sigma}, \forall\ \tau = t, t+1, \ldots$$

A.1.3 Deriving the full consumption function and indirect value function

Let retiree full income d_t^r and retiree human wealth h_t^r be defined as:

$$\begin{aligned} d_t^r &= (1-\tau_t^r)\xi w_t, \\ h_t^r &= d_t^r + \frac{\gamma}{1+r_{t+1}} h_{t+1}^r. \end{aligned}$$

Iterating the budget constraint forwards and imposing a transversality condition gives the lifetime budget constraint and full consumption function:

$$\begin{aligned} \sum_{\tau=t}^{\infty}\left(\prod_{s=t}^{\tau-1}\frac{\gamma}{1+r_{s+1}}\right)x_\tau^r &= \frac{1+r_t}{\gamma}a_{t-1}^r + h_t^r + \mu_t b_t^r R_t^r \Leftrightarrow \\ x_t^r &= \frac{1}{\Delta_t^r}\left(\frac{1+r_t}{\gamma}a_{t-1}^r + h_t^r + \mu_t b_t^r R_t^r\right), \end{aligned}$$

with Δ_t^r the inverse MPCW of retirees (using that $\sigma = \frac{1}{1-\rho}$ and $\sigma\rho = \sigma - 1$):

$$\Delta_t^r = 1 + \gamma\beta^\sigma \Delta_{t+1}^r \left((1+r_{t+1})\left(\frac{(1-\tau_t^r)w_t}{(1-\tau_{t+1}^r)w_{t+1}}\right)^{1-v}\right)^{\sigma-1}.$$

Writing out the indirect retiree value function:

$$
\begin{aligned}
(V_t^r)^\rho &= \sum_{s=t}^{\infty}\left((\beta\gamma)^{s-t}\, c_s^r \left(\frac{1-v}{v}\frac{1}{(1-\tau_s^r)\xi w_s}\right)^{1-v}\right)^\rho \Leftrightarrow \\
V_t^r &= (\Delta_t^r)^{\frac{1}{\rho}}\, v x_t^r \left(\frac{1-v}{v}\frac{1}{(1-\tau_t^r)\xi w_t}\right)^{1-v}.
\end{aligned}
$$

A.2 Worker decision problem

A worker maximises the following objective in period t:

$$
V^w(a_{t-1}^w, b_t^w) = \max_{c_t^w, l_t^w, a_t^w, b_{t+1}^w} \left(\left((c_t^w)^v(1-l_t^w)^{1-v}\right)^\rho + \beta\left(\omega V^w(a_t^w, b_{t+1}^w) + (1-\omega)V^r(a_t^w, b_{t+1}^w)\right)^\rho\right)^{\frac{1}{\rho}},
$$

subject to the constraints that become operative once he retires and subject to:

$$
\begin{aligned}
a_t^w &= (1+r_t)\, a_{t-1}^w + (1-\tau_t) w_t l_t^w + f_t^w - c_t^w, \\
b_{t+1}^w &= \mu_t b_t^w + \nu_t w_t l_t^w.
\end{aligned}
$$

Substituting the constraints:

$$
\begin{aligned}
&V^w(a_{t-1}^w, b_t^w) = \max_{c_t^w, l_t^w} \\
&\Big(\left((c_t^w)^v(1-l_t^w)^{1-v}\right)^\rho + \beta\omega\Big(V^w\left((1+r_t)\, a_{t-1}^w + (1-\tau_t)w_t l_t^w + f_t^w - c_t^w, \mu_t b_t^w + \nu_t w_t l_t^w\right) + \\
&(1-\omega)V^r\left((1+r_t)\, a_{t-1}^w + (1-\tau_t)w_t l_t^w + f_t^w - c_t^w, \mu_t b_t^w + \nu_t w_t l_t^w\right)\Big)^\rho\Big)^{\frac{1}{\rho}}.
\end{aligned}
$$

A.2.1 First-order conditions

<u>The first-order condition with respect to c_t^r:</u>

$$
\begin{aligned}
&v\,(c_t^w)^{v\rho-1}\,(1-l_t^w)^{(1-v)\rho} = \\
&\beta\left(\omega V^w(a_t^w, b_{t+1}^w) + (1-\omega)V^r(a_t^w, b_{t+1}^w)\right)^{\rho-1}\left(\omega V_1^w(a_t^w, b_{t+1}^w) + (1-\omega)V_1^r\left(a_t^w, b_{t+1}^w\right)\right),
\end{aligned}
\tag{A.7}
$$

where we can find $V_1^w\left(a_t^w, b_{t+1}^w\right)$ and $V_1^r\left(a_t^w, b_{t+1}^w\right)$ using the envelope theorem and shifting the conditions one period forward:

$$V_1^w\left(a_t^w, b_{t+1}^w\right) = \left(V^w\left(a_t^w, b_{t+1}^w\right)\right)^{1-\rho} v\left(1+r_{t+1}\right)\left(c_{t+1}^w\right)^{v\rho-1}\left(1-l_{t+1}^w\right)^{(1-v)\rho}, \tag{A.8}$$

$$V_1^r\left(a_t^w, b_{t+1}^w\right) = \left(V^r\left(a_t^w, b_{t+1}^w\right)\right)^{1-\rho} v\left(1+r_{t+1}\right)\left(c_{t+1}^r\right)^{v\rho-1}\left(1-l_{t+1}^r\right)^{(1-v)\rho}. \tag{A.9}$$

<u>The first-order condition with respect to l_t^r:</u>

$$\begin{aligned}&(1-v)\left(c_t^w\right)^{v\rho}\left(1-l_t^w\right)^{(1-v)(\rho-1)} = \\ &\beta(1-\tau_t)w_t\left(\omega V^w(a_t^w, b_{t+1}^w) + (1-\omega)V^r(a_t^w, b_{t+1}^w)\right)^{\rho-1}\left(\omega V_1^w(a_t^w, b_{t+1}^w) + (1-\omega)V_1^r\left(a_t^w, b_{t+1}^w\right)\right) + \\ &\beta\mu_{t+1}\nu_t w_t\left(\omega V^w(a_t^w, b_{t+1}^w) + (1-\omega)V^r(a_t^w, b_{t+1}^w)\right)^{\rho-1}\left(\omega V_2^w(a_t^w, b_{t+1}^w) + (1-\omega)V_2^r\left(a_t^w, b_{t+1}^w\right)\right).\end{aligned} \tag{A.10}$$

As in the case of the retiree, it is required to determine the proper valuation of obtaining an additional unit of accrued per-period pension benefits in case the worker remains a worker in period $t+1$, $V_2^w(a_t^w, b_{t+1}^w)$, and in case the worker retires in period $t+1$, $V_2^r\left(a_t^w, b_{t+1}^w\right)$. As in section A.1.1 it holds that $V_2^r\left(a_t^w, b_{t+1}^w\right) = R_{t+1}^r \frac{1}{1+r_{t+1}} V_1^r\left(a_t^w, b_{t+1}^w\right)$, where γ is omitted since an individual who is a worker in period t and retired in period $t+1$ reaps a return on his private financial wealth of $1+r_{t+1}$. Anticipating that the worker consumption function is linear in perceived total lifetime wealth, it holds that $V_2^w\left(a_t^w, b_{t+1}^w\right) = R_{t+1}^w \frac{1}{1+r_{t+1}} V_1^w\left(a_t^w, b_{t+1}^w\right)$. Recall that the annuity factor $R_{t+1}^w = \frac{\mu_{t+2}}{1+r_{t+2}}\left(\frac{\omega}{\Omega_{t+2}} R_{t+2}^w + (1-\frac{\omega}{\Omega_{t+2}}) R_{t+2}^r\right)$ represents the present discounted value to a worker in period $t+1$ of receiving one consumption good each period in which he is retired in the future (corrected for future revaluation μ and the subjective reweighting of transition probabilities Ω).

Using this in (A.10):

$$(1-v)\left(c_t^w\right)^{v\rho}\left(1-l_t^w\right)^{(1-v)(\rho-1)} =$$
$$\beta(1-\tau_t)w_t\left(\omega V^w(a_t^w,b_{t+1}^w)+(1-\omega)V^r(a_t^w,b_{t+1}^w)\right)^{\rho-1}\left(\omega V_1^w(a_t^w,b_{t+1}^w)+(1-\omega)V_1^r\left(a_t^w,b_{t+1}^w\right)\right)+$$
$$\beta\frac{\mu_{t+1}}{1+r_{t+1}}\nu_t w_t\left(\omega V^w(a_t^w,b_{t+1}^w)+(1-\omega)V^r(a_t^w,b_{t+1}^w)\right)^{\rho-1}\left(\omega R_{t+1}^w V_1^w(a_t^w,b_{t+1}^w)+(1-\omega)R_{t+1}^r V_1^r\left(a_t^w,b_{t+}^w\right.\right.$$

We conjecture that the following equivalency holds:

$$\frac{\mu_{t+1}}{1+r_{t+1}}\left(\omega R_{t+1}^w V_1^w(a_t^w,b_{t+1}^w)+(1-\omega)R_{t+1}^r V_1^r\left(a_t^w,b_{t+1}^w\right)\right)=$$
$$R_t^w\left(\omega V_1^w(a_t^w,b_{t+1}^w)+(1-\omega)V_1^r\left(a_t^w,b_{t+1}^w\right)\right). \quad \text{(A.11)}$$

After deriving the consumption and indirect value function of the worker, we will verify that the above equivalency indeed holds. This will ensure that all conjectures add up to consistent solutions across all equations characterising the optimal decisions of retirees and workers. Defining $\tau_t^w = \tau_t - R_t^w\nu_t$ then gives:

$$(1-v)\left(c_t^w\right)^{v\rho}\left(1-l_t^w\right)^{(1-v)(\rho-1)} =$$
$$\beta(1-\tau_t^w)w_t\left(\omega V^w(a_t^w,b_{t+1}^w)+(1-\omega)V^r(a_t^w,b_{t+1}^w)\right)^{\rho-1}\left(\omega V_1^w(a_t^w,b_{t+1}^w)+(1-\omega)V_1^r\left(a_t^w,b_{t+1}^w\right)\right). \quad \text{(A.12)}$$

Combining (A.12) with (A.7):

$$1-l_t^w=\frac{1-v}{v}\frac{c_t^w}{(1-\tau_t^w)w_t}. \quad \text{(A.13)}$$

A.2.2 Writing the Euler equation solely in terms of consumption

We define worker full consumption as $x_t^w \equiv c_t^w + \left(1-l_t^w\right)\left(1-\tau_t^w\right)w_t = \frac{c_t^w}{v}$. Substituting this, the optimal labour supply decisions (A.5) and (A.13), and the envelope conditions (A.8) and (A.9) into (A.7), the first-order condition with respect to c_t^w, gives the worker Euler equation:

$$(x_t^w)^{\rho-1} = \beta(1+r_{t+1}) \left(\frac{(1-\tau_t^w)w_t}{(1-\tau_{t+1}^w)w_{t+1}} \right)^{(1-v)\rho} \left(\omega V^w\left(a_t^w, b_{t+1}^w\right) + (1-\omega)V^r\left(a_t^w, b_{t+1}^w\right)\right)^{\rho-1}$$
$$\left(\omega\left(V^w\left(a_t^w, b_{t+1}^w\right)\right)^{1-\rho}\left(x_{t+1}^w\right)^{\rho-1} + (1-\omega)\left(V^r\left(a_t^w, b_{t+1}^w\right)\right)^{1-\rho}\left(x_{t+1}^r\right)^{\rho-1}\left(\frac{1-\tau_{t+1}^w}{1-\tau_{t+1}^r}\frac{1}{\xi}\right)^{(1-v)\rho}\right).$$

In section A.1.3 we have shown that $V_t^r = (\Delta_t^r)^{\frac{1}{\rho}}\, v x_t^r \left(\frac{1-v}{v}\frac{1}{(1-\tau_t^r)\xi w_t}\right)^{1-v}$. Conjecture similarly that $V_t^w = (\Delta_t^w)^{\frac{1}{\rho}}\, v x_t^w \left(\frac{1-v}{v}\frac{1}{(1-\tau_t^w)w_t}\right)^{1-v}$. Denote with $\Omega_t = \omega + (1-\omega)\left(\frac{1-\tau_t^w}{1-\tau_t^r}\frac{1}{\xi}\right)^{1-v}\left(\frac{\Delta_t^w}{\Delta_t^r}\right)^{\frac{1}{1-\sigma}}$. Plugging these in the above condition and cancelling out terms:

$$\omega x_{t+1}^w + (1-\omega)x_{t+1}^r \left(\frac{1-\tau_{t+1}^w}{1-\tau_{t+1}^r}\frac{1}{\xi}\right)^{1-v}\left(\frac{\Delta_{t+1}^w}{\Delta_{t+1}^r}\right)^{\frac{\sigma}{1-\sigma}} = x_t^w\left(\beta(1+r_{t+1})\Omega_{t+1}\left(\frac{(1-\tau_t^w)w_t}{(1-\tau_{t+1}^w)w_{t+1}}\right)^{(1-v)\rho}\right)^{\sigma}. \quad \text{(A.14)}$$

We can now show that, using (A.14), our conjecture for the value function implies the following difference equation for Δ^w:

$$V^w(a_{t-1}^w, b_t^w) = \max_{c_t^w, l_t^w, a_t^w, b_{t+1}^w}\left(\left((c_t^w)^v(1-l_t^w)^{1-v}\right)^{\rho} + \beta\left(\omega V^w(a_t^w, b_{t+1}^w) + (1-\omega)V^r(a_t^w, b_{t+1}^w)\right)^{\rho}\right)^{\frac{1}{\rho}} \Leftrightarrow$$

$$\left((\Delta_t^w)^{\frac{1}{\rho}}\, v x_t^w\left(\frac{1-v}{v}\frac{1}{(1-\tau_t^w)w_t}\right)^{1-v}\right)^{\rho} = \left(v x_t^w\left(\frac{1-v}{v}\frac{1}{(1-\tau_t^w)w_t}\right)^{1-v}\right)^{\rho} +$$
$$\beta\left(\omega\left(\Delta_{t+1}^w\right)^{\frac{1}{\rho}} v x_{t+1}^w\left(\frac{1-v}{v}\frac{1}{(1-\tau_{t+1}^w)w_{t+1}}\right)^{1-v} + (1-\omega)\left(\Delta_{t+1}^r\right)^{\frac{1}{\rho}} v x_{t+1}^r\left(\frac{1-v}{v}\frac{1}{(1-\tau_{t+1}^r)\xi w_{t+1}}\right)^{1-v}\right)$$
$$\Delta_t^w = 1 + \beta^{\sigma}\Delta_{t+1}^w\left(\left(1+r_{t+1}\right)\Omega_{t+1}\left(\frac{(1-\tau_t^w)w_t}{(1-\tau_{t+1}^w)w_{t+1}}\right)^{1-v}\right)^{\sigma-1}. \quad \text{(A.15)}$$

A.2.3 Deriving the full consumption function

Using (A.14) we can show that the difference equation for Δ^w given by (A.15) is consistent with the following full consumption function:

$$
\begin{aligned}
x_t^w &= \frac{1}{\Delta_t^w}\left(\left(1+r_t\right)a_{t-1}^w + h_t^w + \mu_t b_t^w R_t^w\right),\\
d_t^w &= (1-\tau_t^w)w_t + f_t^w,\\
h_t^w &= d_t^w + \frac{1}{1+r_{t+1}}\left(\frac{\omega}{\Omega_{t+1}}h_{t+1}^w + (1-\frac{\omega}{\Omega_{t+1}})h_{t+1}^r\right),
\end{aligned}
$$

where h_t^w is the perceived human wealth of a worker and d_t^w worker full income. Substituting the above full consumption function in (A.14) indeed gives the same difference equation for Δ^w:

$$
\begin{aligned}
&\omega\frac{1}{\Delta_{t+1}^w}\left(\left(1+r_{t+1}\right)a_t^w + h_{t+1}^w + \mu_{t+1}b_{t+1}^w R_{t+1}^w\right) + \\
&(1-\omega)\left(\frac{1-\tau_{t+1}^w}{1-\tau_{t+1}^r}\frac{1}{\xi}\right)^{1-v}\left(\frac{\Delta_{t+1}^w}{\Delta_{t+1}^r}\right)^{\frac{\sigma}{1-\sigma}}\frac{1}{\Delta_{t+1}^r}\left(\left(1+r_{t+1}\right)a_t^w + h_{t+1}^r + \mu_{t+1}b_{t+1}^w R_{t+1}^r\right) = \\
&\left(\beta(1+r_{t+1})\Omega_{t+1}\left(\frac{(1-\tau_t^w)w_t}{(1-\tau_{t+1}^w)w_{t+1}}\right)^{(1-v)\rho}\right)^{\sigma}\frac{1}{\Delta_t^w}\left(\left(1+r_t\right)a_{t-1}^w + h_t^w + \mu_t b_t^w R_t^w\right) \Leftrightarrow
\end{aligned}
$$

$$
\Delta_t^w\frac{a_t^w + h_t^w - d_t^w + b_{t+1}^w R_t^w}{\left(1+r_t\right)a_{t-1}^w + h_t^w + \mu_t b_t^w R_t^w} = \beta^\sigma\Delta_{t+1}^w\left((1+r_{t+1})\Omega_{t+1}\left(\frac{(1-\tau_t^w)w_t}{(1-\tau_{t+1}^w)w_{t+1}}\right)^{(1-v)}\right)^{\sigma-1} \Leftrightarrow
$$

$$
\Delta_t^w = 1 + \beta^\sigma\Delta_{t+1}^w\left(\left(1+r_{t+1}\right)\Omega_{t+1}\left(\frac{(1-\tau_t^w)w_t}{(1-\tau_{t+1}^w)w_{t+1}}\right)^{1-v}\right)^{\sigma-1}.
$$

Since it holds that $1-\frac{1}{\Delta_t^w}=\frac{a_t^w+h_t^w-d_t^w+b_{t+1}^w R_t^w}{(1+r_t)a_{t-1}^w+h_t^w+\mu_t b_t^w R_t^w}$, which can be shown using the worker budget constraint:

$$\begin{aligned}
a_t^w &= (1+r_t)\,a_{t-1}^w + (1-\tau_t)w_t l_t^w + f_t^w - c_t^w \Leftrightarrow \\
a_t^w + h_t^w &= (1+r_t)a_{t-1}^w + h_t^w + d_t^w - x_t^w + (\tau^w - \tau)w_t l_t^w \Leftrightarrow \\
a_t^w + h_t^w - d_t^w &= (1+r_t)a_{t-1}^w + h_t^w - R_t^w\left(b_{t+1}^w - \mu_t b_t^w\right) - x_t^w \Leftrightarrow \\
a_t^w + h_t^w - d_t^w + b_{t+1}^w R_{t+1}^w &= (1+r_t)a_{t-1}^w + h_t^w + \mu_t b_t^w R_t^w - \frac{1}{\Delta_t^w}\left((1+r_t)\,a_{t-1}^w + h_t^w + \mu_t b_t^w R_t^w\right) \Leftrightarrow \\
1-\frac{1}{\Delta_t^w} &= \frac{a_t^w + h_t^w - d_t^w + b_{t+1}^w R_{t+1}^w}{(1+r_t)a_{t-1}^w + h_t^w + \mu_t b_t^w R_t^w}.
\end{aligned}$$

This confirms that our conjectures of the worker full consumption function and the worker indirect value function are mutually consistent and are similar to those of the retiree.

A.2.4 Coming back to the worker first-order condition for labour

Now that we have derived the expressions for the subjective reweighting of transition probabilities Ω_t and the indirect value functions of the worker V_t^w and retiree V_t^r, we show that the assumed equivalency (A.11) indeed holds.

$$\begin{aligned}
&\frac{\mu_{t+1}}{1+r_{t+1}}\left(\omega R_{t+1}^w V_1^w(a_t^w, b_{t+1}^w) + (1-\omega)R_{t+1}^r V_1^r\left(a_t^w, b_{t+1}^w\right)\right) = \\
&\qquad\qquad R_t^w\left(\omega V_1^w(a_t^w, b_{t+1}^w) + (1-\omega)V_1^r\left(a_t^w, b_{t+1}^w\right)\right) \Leftrightarrow
\end{aligned}$$

$$\begin{aligned}
&\omega R_{t+1}^w V_1^w(a_t^w, b_{t+1}^w) + (1-\omega)R_{t+1}^r V_1^r\left(a_t^w, b_{t+1}^w\right) = \\
&\left(\frac{\omega}{\Omega_{t+1}}R_{t+1}^w + (1-\frac{\omega}{\Omega_{t+1}})R_{t+1}^r\right)\left(\omega V_1^w(a_t^w, b_{t+1}^w) + (1-\omega)V_1^r\left(a_t^w, b_{t+1}^w\right)\right) \Leftrightarrow
\end{aligned}$$

$$\omega\left(R_{t+1}^w - R_{t+1}^r\right)V_1^w(a_t^w, b_{t+1}^w) = \frac{\omega}{\Omega_{t+1}}\left(R_{t+1}^w - R_{t+1}^r\right)\left(\omega V_1^w(a_t^w, b_{t+1}^w) + (1-\omega)V_1^r\left(a_t^w, b_{t+1}^w\right)\right) \Leftrightarrow$$

$$\Omega_{t+1} = \omega + (1-\omega)\frac{V_1^r\left(a_t^w, b_{t+1}^w\right)}{V_1^w\left(a_t^w, b_{t+1}^w\right)} \Leftrightarrow$$

$$\Omega_{t+1} = \omega + (1-\omega)\left(\frac{1-\tau_{t+1}^w}{1-\tau_{t+1}^r}\frac{1}{\xi}\right)^{1-v}\left(\frac{\Delta_{t+1}^w}{\Delta_{t+1}^r}\right)^{\frac{1}{1-\sigma}},$$

where in the last line we use that, for an individual who is working in period t and retires in period $t+1$, $V_1^r\left(a_t^w, b_{t+1}^w\right) = (1+r_{t+1})\left(\Delta_t^r\right)^{\frac{1}{\sigma-1}}\left(\frac{1-v}{v}\frac{1}{(1-\tau_t^r)\xi w_t}\right)^{1-v}$, while $V_1^w\left(a_t^w, b_{t+1}^w\right) = (1+r_{t+1})\left(\Delta_t^w\right)^{\frac{1}{\sigma-1}}\left(\frac{1-v}{v}\frac{1}{(1-\tau_t^w)w_t}\right)^{1-v}$. This expression for Ω_{t+1} is identical to how it is defined in section (A.2.2), therefore confirming our conjecture.

B Decision problems of firms and government

B.1 Final goods sector

There is a continuum of retail firms, indexed by $z \in [0,1]$. The perfectly competitive final goods sector assembles the differentiated retail goods according to:

$$Y_t = \left(\int_0^1 \left(Y_{z,t}\right)^{\frac{\epsilon-1}{\epsilon}} dz\right)^{\frac{\epsilon}{\epsilon-1}}, \tag{B.1}$$

where $\epsilon > 1$ is the elasticity of demand for the intermediate goods purchased from different retail firms. Each retail good $Y_{z,t}$ is produced by one retail firm (which is also indexed by z) and sold at the nominal price $P_{z,t}$. The final goods producing sector maximises profits taking all prices (P_t, the nominal price of the final good, and $P_{z,t}$, $\forall z \in [0,1]$) as given:

$$\max_{Y_{z,t}} P_t Y_t - \int_0^1 P_{z,t} Y_{z,t} dz.$$

Using (B.1) and differentiating with respect to a particular $Y_{z,t}$ gives rise to the following demand function for the output of a particular retail good z

producing firm:

$$Y_{z,t} = Y_t \left(\frac{P_{z,t}}{P_t} \right)^{-\epsilon}. \tag{B.2}$$

Imposing zero profits in the final goods sector maximisation problem yields that the price of the final good can be understood as an average of the retail firm prices:

$$P_t = \left(\int_0^1 (P_{z,t})^{1-\epsilon} \, dz \right)^{\frac{1}{1-\epsilon}}. \tag{B.3}$$

B.2 Capital producing sector

At the end of period t, the competitive capital producing sector purchases the remaining stock of capital $(1-\delta)\zeta_t K_{t-1}$ from the intermediate goods producing firms at the real price q_t. This capital is combined with I_t units of investment (in the form of output purchased from final goods producers) to produce next period's beginning of period stock of capital K_t. This stock of capital is then sold to the intermediate goods producing firms at the real price q_t. The capital producing sector faces convex adjustment costs when transforming final goods into capital. Capital evolves as follows:

$$K_t = (1-\delta)\,\zeta_t K_{t-1} + \left(1 - S[\frac{I_t}{I_{t-1}}]\right) I_t, \tag{B.4}$$

with $S[\frac{I_t}{I_{t-1}}] = \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2$. This capital evolution specification contains investment adjustment costs in the sense that investing I_t final goods in period t will only increase tomorrow's capital stock by $\left(1 - S[\frac{I_t}{I_{t-1}}]\right) I_t$. This specification is similar to Fernández-Villaverde & Rubio-Ramírez (2006) and Christiano et al. (2005), and κ (the second derivative of $S[\frac{I_t}{I_{t-1}}]$) represents the severity of the investment adjustment costs. In period t the profits of the capital producing sector are given by $\Pi_t^c = q_t K_t - q_t(1-\delta)\zeta_t K_{t-1} - I_t$. The capital producing sector maximises the present discounted value of profits, where we substitute (B.4) in Π_t^c:

$$\max_{\{I_{t+i}\}_{i=0}^{\infty}} \sum_{i=0}^{\infty} \left(\prod_{s=1}^{i} \frac{1}{1+r_{t+s}} \right) \left(q_{t+i} \left(1 - S[\frac{I_{t+i}}{I_{t+i-1}}] \right) I_{t+i} - I_{t+i} \right).$$

Profits (which can arise outside of the steady state) are redistributed lump sum to the group of workers. Differentiating with respect to investment I_t gives the following condition for the investment path:

$$1 = q_t \left(1 - S[\frac{I_t}{I_{t-1}}] + \frac{I_t}{I_{t-1}} S'[\frac{I_t}{I_{t-1}}] \right) + \frac{q_{t+1}}{1+r_{t+1}} \left(\frac{I_{t+1}}{I_t} \right)^2 S[\frac{I_{t+1}}{I_t}].$$

B.3 Intermediate goods sector

There is a continuum of competitive intermediate good producing firms indexed by $j \in [0,1]$. The intermediate good j is produced by the intermediate good j producer according to:

$$Y_{j,t} = (\zeta_t K_{j,t-1})^{\alpha} (L_{j,t})^{1-\alpha}, \tag{B.5}$$
$$\log(\zeta_t) = \rho_\zeta \log(\zeta_{t-1}) + \varepsilon_t.$$

Capital quality is denoted by ζ_t, follows an AR(1)-process and is subject to the unanticipated shock ε_t. $L_{j,t}$ and $K_{j,t-1}$ denote the employed labour and capital by the intermediate good j producing firm. As previously mentioned, the intermediate good producing firms purchase their employed capital for period $t+1$ from the capital producing sector in period t and therefore capital used for production in period t is indexed by $t-1$. A negative realisation of ε_t decreases the quality of the capital stock such that the effective capital used in production in period t is $\zeta_t K_{j,t-1}$. The intermediate good producing firms produce output $Y_{j,t}$ and hire labour $L_{j,t}$ at a unit cost of w_t. The markets for labour and capital are perfectly competitive and so the intermediate good j

producing firm takes their prices as given. The intermediate good producers sell their output to the retail firms at the real price mc_t. After production, the remaining effective capital stock is sold back to the capital producing sector at the real price q_t. The intermediate good producing firms finance their capital purchases each period by obtaining funds from the households and the pension fund. We assume that there are no frictions in the process of obtaining these funds. The intermediate good producing firms offer the households and the pension fund a perfectly state-contingent security, which is best interpreted as equity.

The period t profits of the intermediate good j producing firm are given by:

$$\Pi^i_{j,t} = mc_t \left(\zeta_t K_{j,t-1}\right)^{\alpha} \left(L_{j,t}\right)^{1-\alpha} + q_t(1-\delta)\zeta_t K_{j,t-1} - w_t L_{j,t} - (1+r_t)q_{t-1}K_{j,t-1},$$

which consists of the sale of output to retail firms $mc_t \left(\zeta_t K_{j,t-1}\right)^{\alpha} \left(L_{j,t}\right)^{1-\alpha}$, the sale of the remaining capital stock to the capital producing sector $q_t(1-\delta)\zeta_t K_{j,t-1}$, the hiring of labour $w_t L_{j,t}$ and the repayment of previous period's borrowed funds $(1+r_t)q_{t-1}K_{j,t-1}$. The intermediate good j producing firm maximises the present discounted value of profits taking all prices as given:

$$\max_{\{K_{j,t+i}, L_{j,t+i}\}_{i=0}^{\infty}} \sum_{i=0}^{\infty} \prod_{s=1}^{i} \left(\frac{1}{1+r_{t+s}}\right) \Pi^i_{j,t+i}.$$

Differentiating with respect to $L_{j,t}$ and $K_{j,t}$ gives the following first-order conditions for labour and capital, respectively:

$$\begin{aligned} w_t &= (1-\alpha)\, mc_t \frac{Y_{j,t}}{L_{j,t}}, \qquad \text{(B.6)} \\ q_t &= \frac{1}{1+r_{t+1}} \left(\alpha mc_{t+1} \frac{Y_{j,t+1}}{K_{j,t}} + q_{t+1}(1-\delta)\zeta_{t+1}\right). \end{aligned}$$

Since the intermediate goods sector is perfectly competitive, per-period profits are zero state-by-state. Using (B.6) in $\Pi^i_{j,t} = 0$ gives the required ex post return on capital the intermediate good producing firms pay out to the households and pension fund, confirming the perfectly state-contingent nature of the traded security:

$$1 + r_t = \frac{\alpha mc_t \frac{Y_{j,t}}{K_{j,t-1}} + q_t(1-\delta)\zeta_t}{q_{t-1}}. \tag{B.7}$$

Rewriting (B.6) and (B.7) gives the factor demands:

$$L_{j,t} = (1-\alpha)\, mc_t \frac{Y_{j,t}}{w_t}, \tag{B.8}$$

$$K_{j,t-1} = \frac{\alpha mc_t Y_{j,t}}{q_{t-1}(1+r_t) - q_t(1-\delta)\zeta_t}. \tag{B.9}$$

From this it follows that all intermediate good producing firms employ the same capital-labour ratio:

$$\frac{K_{j,t-1}}{L_{j,t}} = \frac{K_{t-1}}{L_t} = \frac{\alpha}{1-\alpha} \frac{w_t}{q_{t-1}(1+r_t) - q_t(1-\delta)\zeta_t}.$$

Substituting the factor demands into the production function of the intermediate good j producer, we obtain the real intermediate good price mc_t:

$$mc_t = \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{q_{t-1}(1+r_t) - q_t(1-\delta)\zeta_t}{\zeta_t \alpha}\right)^{\alpha}.$$

B.4 Retail sector

After purchasing output from the intermediate good producing firms at the real price mc_t, the retail firms convert the intermediate goods sector output into retail goods which are sold to the final goods sector at the nominal price $P_{z,t}$. The intermediate goods are converted one-to-one into retail goods, which entails that the retailers simply repackage the intermediate goods. We assume that each retail firm produces a differentiated retail good $Y_{z,t}$ such that it operates in a monopolistically competitive market and charges a markup over the input price mc_t. Additionally, we introduce nominal rigidities by means of Calvo (1983)-type pricing frictions. By construction, each period a fraction $1-\theta$ of retail firms can adjust its price (which it will do so in an optimal fashion, taking into account the probability that it might not be able to change its price in future periods) and a fraction θ of firms cannot adjust its price. Denote with $P^*_{z,t}$ the nominal optimal reset price in period t of retail firm z that can change its price. Since the group of workers are assumed to receive the profits of the retail firms, the appropriate pricing kernel used to value profits received in i periods is $\beta^i \frac{\Lambda_{t+i}}{\Lambda_t}$ with $\Lambda_t = v\left(\Delta^w_t\right)^{\frac{\rho+1}{\rho}}\left(\frac{1-v}{v}\frac{1}{(1-\tau^w_t)w_t}\right)^{1-v}$ being the marginal value to a worker of receiving one additional unit of lifetime wealth in period t.

When retail firm z is allowed to change its price in period t, it solves the following optimisation problem:

$$\max_{P^*_{z,t}} \sum_{i=0}^{\infty} (\beta\theta)^i \frac{\Lambda_{t+i}}{\Lambda_t}\left(\frac{P^*_{z,t}}{P_{t+i}} - mc_{t+i}\right) Y_{z,t+i} \text{ , s.t. } Y_{z,t+i} = Y_{t+i}\left(\frac{P^*_{z,t}}{P_{t+i}}\right)^{-\epsilon}.$$

Profit maximisation yields the following first-order condition:

$$\sum_{i=0}^{\infty} (\beta\theta)^i \Lambda_{t+i}\left((1-\epsilon)\frac{P^*_{z,t}}{P_t}\left(\prod_{s=1}^{i}\frac{1}{\Pi_{t+s}}\right)^{1-\epsilon} + \epsilon mc_{t+i}\left(\prod_{s=1}^{i}\frac{1}{\Pi_{t+s}}\right)^{-\epsilon}\right) Y_{t+i} = 0,$$

where $\Pi_{t+s} = \frac{P_{t+s}}{P_{t+s-1}}$. Reorganising and realising that the symmetric nature of the economic environment implies that all price adjusting firms will choose the same price, i.e. $P^*_t = P^*_{z,t}\ \forall z$, yields the following condition characterising the optimal real reset price $\Pi^*_t = \frac{P^*_t}{P_t}$:

$$\Pi^*_t = \frac{\epsilon}{\epsilon-1}\frac{\sum_{i=0}^{\infty}(\beta\theta)^i \Lambda_{t+i} mc_{t+i}\left(\frac{P_{t+i}}{P_t}\right)^{\epsilon} Y_{t+i}}{\sum_{i=0}^{\infty}(\beta\theta)^i \Lambda_{t+i}\left(\frac{P_{t+i}}{P_t}\right)^{\epsilon-1} Y_{t+i}}. \tag{B.10}$$

To express the first-order condition (B.10) recursively, we write it as $\Pi^*_t =$

$\frac{\epsilon}{\epsilon-1}\frac{g_t^1}{g_t^2}$ with:

$$g_t^1 = \Lambda_t mc_t Y_t + \beta\theta \left(\Pi_{t+1}\right)^{\epsilon} g_{t+1}^1,$$
$$g_t^2 = \Lambda_t Y_t + \beta\theta \left(\Pi_{t+1}\right)^{\epsilon-1} g_{t+1}^2.$$

Because of the Calvo-pricing rigidity a share $1-\theta$ of retail firms can adjust its price and sets it to $P_{z,t} = P_t^*$ and a share θ of retail firms cannot adjust its price and has to set it to $P_{z,t} = P_{z,t-1}$. This gives in (B.3) the evolution of the aggregate price level as a geometric average of the past aggregate price level and the current optimal price:

$$1 = \theta \left(\Pi_t\right)^{\epsilon-1} + (1-\theta) \left(\Pi_t^*\right)^{1-\epsilon}.$$

B.5 Government and central bank

Since the government is non-Ricardian in this model, we elect to minimise the role of the fiscal authority so as to not distort our research findings regarding the macroeconomic implications of pension fund restoration policy. As such, we rule out government purchases. We suppose that the central bank follows a Taylor rule with interest rate smoothing. The monetary authority responds to deviations of inflation from the target inflation rate $\bar{\Pi}$ and to deviations of output from steady state output $\bar{Y}$:

$$\frac{1+i_t}{1+\bar{i}} = \left(\frac{1+i_{t-1}}{1+\bar{i}}\right)^{\eta_i} \left(\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\eta_\Pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\eta_Y}\right)^{1-\eta_i},$$

where $\bar{i}$ is the steady-state nominal interest rate, $\eta_i \in (0,1)$ the interest rate smoothing parameter, η_Π the inflation coefficient and η_Y the output coefficient. Additionally, the Fisher relation holds:

$$1 + i_t = \Pi_{t+1} \left(1 + r_{t+1}\right).$$

B.6 Aggregation

For the output markets to clear it is required that $\int_0^1 Y_{z,t}dz = \int_0^1 Y_{j,t}dj = Y_t \int_0^1 \left(\frac{P_{z,t}}{P_t}\right)^{-\epsilon} dz$, for the labour market to clear it is required that $\int_0^1 L_{j,t}dj = L_t$ and for the capital market to clear it is required that $\int_0^1 K_{j,t}dj = K_t$. Integrating the factor demand conditions (B.8) and (B.9) over j gives the aggregate factor demand conditions:

$$L_t = (1-\alpha) mc_t \frac{Y_t v_t^p}{w_t}, \tag{B.11}$$

$$K_{t-1} = \frac{\alpha mc_t Y_t v_t^p}{q_{t-1}(1+r_t) - q_t(1-\delta)\zeta_t}, \tag{B.12}$$

where $v_t^p = \int_0^1 \left(\frac{P_{z,t}}{P_t}\right)^{-\epsilon} dz$ is a measure of price dispersion. Because of the Calvo-pricing rigidity a share $1-\theta$ of retail firms can adjust its price and sets it to $P_{z,t} = P_t^*$ and a share θ of retail firms cannot adjust its price and has to set it to $P_{z,t} = P_{z,t-1}$. This allows us to express v_t^p recursively:

$$v_t^p = (1-\theta)\left(\Pi_t^*\right)^{-\epsilon} + \theta \left(\Pi_t\right)^{\epsilon} v_{t-1}^p. \tag{B.13}$$

Aggregate supply is obtained through integrating (B.5) over j and using that $\frac{K_{j,t-1}}{L_{j,t}} = \frac{K_{t-1}}{L_t}$, $\forall j$ and that $\int_0^1 L_{j,t}dj = L_t$:

$$Y_t v_t^p = \left(\zeta_t K_{t-1}\right)^{\alpha} \left(L_t\right)^{1-\alpha},$$

$$Y_t = C_t + I_t.$$

Savings market clearing requires that the total value of savings (which is the sum of the private savings of workers and retirees and the end-of-period assets of the pension fund) equates the total value of the capital stock:

$$A_t^w + A_t^r + \frac{A_{t+1}^f}{1+r_{t+1}} = q_t K_t.$$

Aggregate profits (comprised of those of the retail sector and the capital goods sector) are given by:

$$F_t = (1 - mc_t v_t^p)\, Y_t + q_t \left(1 - S\left[\frac{I_t}{I_{t-1}}\right]\right) I_t - I_t. \qquad \text{(B.14)}$$

C Equilibrium conditions

C.1 Pension fund

Private annuity factors of retirees and workers:

$$\begin{aligned} R_t^r &= 1 + \gamma \frac{\mu_{t+1}}{(\Pi_{t+1})^{acc}\,(1 + r_{t+1})} R_{t+1}^r \\ R_t^w &= \frac{\mu_{t+1}}{(\Pi_{t+1})^{acc}\,(1 + r_{t+1})} \left(\frac{\omega}{\Omega_{t+1}} R_{t+1}^w + (1 - \frac{\omega}{\Omega_{t+1}}) R_{t+1}^r\right) \end{aligned}$$

Pension fund annuity factors of retirees and workers:

$$\begin{aligned} R_t^{r,f} &= 1 + \frac{\gamma}{(\Pi_{t+1})^{acc}\,(1 + r_{t+1})} R_{t+1}^{r,f} \\ R_t^{w,f} &= \frac{1}{(\Pi_{t+1})^{acc}\,(1 + r_{t+1})} \left(\omega R_{t+1}^{w,f} + (1 - \omega) R_{t+1}^{r,f}\right) \end{aligned}$$

Aggregate per-period pension benefits of retirees and workers:

$$\begin{aligned} (\Pi_t)^{acc}\, B_t^r &= \gamma \left(\mu_{t-1} B_{t-1}^r + \nu_{t-1} \xi w_{t-1} L_{t-1}^r\right) + (1 - \omega) \left(\mu_{t-1} B_{t-1}^w + \nu_{t-1} w_{t-1} L_{t-1}^w\right) \\ (\Pi_t)^{acc}\, B_t^w &= \omega \left(\mu_{t-1} B_{t-1}^w + \nu_{t-1} w_{t-1} L_{t-1}^w\right) \end{aligned}$$

Pension fund assets and liabilities:

$$\begin{aligned} A_t^f &= (1 + r_t) \left(A_{t-1}^f + \tau_{t-1} w_{t-1} L_{t-1} - \mu_{t-1} B_{t-1}^r\right) \\ L_t^f &= R_t^{r,f} B_t^r + R_t^{w,f} B_t^w \end{aligned}$$

Pension fund restoration policy is set such that the following condition is satisfied:

$$\underbrace{\frac{1+r_{t+1}-\upsilon}{1+r_{t+1}}}_{\text{closure fraction}}\underbrace{\left(A_t^f-\bar{f}rL_t^f\right)}_{\text{funding gap}}=$$

$$\bar{f}r\left(\underbrace{\frac{1-\bar{f}r}{\bar{f}r}\mu_t B_t^r+(\mu_t-1)L_t^f}_{\text{revaluation}}+\underbrace{\nu_t w_t\left(\left(R_t^{r,f}-1\right)\xi_t L_t^r+R_t^{w,f}L_t^w\right)}_{\text{accrual}}\right)-\underbrace{\tau_t w_t L_t}_{\text{contribution}}$$

This gives the following pension fund policy in the DB case (with $\bar{\nu}$ exogenously given):

$$\mu_t = 1$$

$$\nu_t = \bar{\nu}$$

$$\frac{1+r_{t+1}-\upsilon}{1+r_{t+1}}\left(A_t^f-\bar{f}rL_t^f\right)=\bar{f}r\left(\frac{1-\bar{f}r}{\bar{f}r}B_t^r+\bar{\nu}w_t\left(\left(R_t^{r,f}-1\right)\xi_t L_t^r+R_t^{w,f}L_t^w\right)\right)-\tau_t w_t L_t$$

This gives the following pension fund policy in the DC case (with $\bar{\tau}$ and $\bar{\nu}$ exogenously given):

$$\tau_t = \bar{\tau}$$

$$\nu_t = \bar{\nu}$$

$$\frac{1+r_{t+1}-\upsilon}{1+r_{t+1}}\left(A_t^f-\bar{f}rL_t^f\right)=$$

$$\bar{f}r\left(\frac{1-\bar{f}r}{\bar{f}r}\mu_t B_t^r+(\mu_t-1)L_t^f+\bar{\nu}w_t\left(\left(R_t^{r,f}-1\right)\xi_t L_t^r+R_t^{w,f}L_t^w\right)\right)-\bar{\tau}w_t L_t$$

C.2 Workers and retirees

Inverse MPCW of retirees and workers:

$$\Delta_t^r = 1 + \gamma\beta^\sigma \Delta_{t+1}^r \left((1 + r_{t+1}) \left(\frac{(1-\tau_t^r)w_t}{(1-\tau_{t+1}^r)w_{t+1}} \right)^{1-v} \right)^{\sigma-1}$$

$$\Delta_t^w = 1 + \beta^\sigma \Delta_{t+1}^w \left((1 + r_{t+1})\, \Omega_{t+1} \left(\frac{(1-\tau_t^w)w_t}{(1-\tau_{t+1}^w)w_{t+1}} \right)^{1-v} \right)^{\sigma-1}$$

Subjective reweighting of transition probabilities:

$$\Omega_t = \omega + (1-\omega) \left(\frac{1-\tau_t^w}{1-\tau_t^r} \frac{1}{\xi} \right)^{1-v} \left(\frac{\Delta_t^w}{\Delta_t^r} \right)^{\frac{1}{1-\sigma}}$$

Effective contribution rates on labour:

$$\tau_t^r = \tau_t - (R_t^r - 1)\, \nu_t$$

$$\tau_t^w = \tau_t - R_t^w \nu_t$$

Aggregate full consumption of retirees and workers:

$$X_t^z = \frac{1}{\Delta_t^z} \left((1 + r_t) A_{t-1}^z + H_t^z + \mu_t B_t^z R_t^z \right), \quad z \in \{w, r\}.$$

Aggregate human wealth of retirees and workers:

$$H_t^r = D_t^r + \frac{\gamma}{1 + r_{t+1}} H_{t+1}^r$$

$$H_t^w = D_t^w + \frac{1}{1 + r_{t+1}} \left(\frac{\omega}{\Omega_{t+1}} H_{t+1}^w + (1 - \frac{\omega}{\Omega_{t+1}}) \frac{1}{\psi} H_{t+1}^r \right)$$

Aggregate full income of retirees and workers:

$$D_t^r = N^r (1 - \tau_t^r) \xi w_t$$

$$D_t^w = N^w (1 - \tau_t^w) w_t + F_t$$

Aggregate consumption of retirees, workers and total population:

$$C_t^z = vX_t^z, \quad z \in \{w, r\},$$
$$C_t = C_t^r + C_t^w$$

Aggregate labour supply of retirees, workers and total population, where $w_t^r = \xi w_t$ and $w_t^w = w_t$:

$$L_t^z = N^z - \frac{(1-v)X_t^z}{(1-\tau_t^w)w_t^z}, \quad z \in \{w, r\},$$
$$L_t = L_t^w + \xi L_t^r.$$

Aggregate private savings of retirees and workers:

$$A_t^r = (1+r_t)A_{t-1}^r + \mu_t B_t^r + (1-\tau_t)\xi w_t L_t^r - C_t^r + \frac{1-\omega}{\omega}A_t^w$$
$$A_t^w = \omega\left((1+r_t)A_{t-1}^w + (1-\tau_t)w_t L_t^w + F_t - C_t^w\right)$$

C.3 Firms and government

Production function:

$$Y_t v_t^p = (\zeta_t K_{t-1})^\alpha (L_t)^{1-\alpha}$$

Aggregate resource constraint:

$$Y_t = C_t + I_t$$

Marginal cost:

$$mc_t = \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{q_{t-1}(1+r_t) - q_t(1-\delta)\zeta_t}{\zeta_t \alpha}\right)^\alpha$$

Real interest rate:

$$1 + r_t = \frac{\alpha mc_t v_t^p \frac{Y_t}{K_{t-1}} + q_t(1-\delta)\zeta_t}{q_{t-1}}$$

Capital stock law of motion:

$$K_t = (1-\delta)\zeta_t K_{t-1} + \left(1 - S[\frac{I_t}{I_{t-1}}]\right) I_t$$

Adjustment costs percentage:

$$S[\frac{I_t}{I_{t-1}}] = \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$$

Investment:

$$1 = q_t \left(1 - S[\frac{I_t}{I_{t-1}}] + \frac{I_t}{I_{t-1}} S'[\frac{I_t}{I_{t-1}}]\right) + \frac{q_{t+1}}{1+r_{t+1}} \left(\frac{I_{t+1}}{I_t}\right)^2 S[\frac{I_{t+1}}{I_t}]$$

Market clearing for savings:

$$A_t^w + A_t^r + \frac{A_{t+1}^f}{1+r_{t+1}} = q_t K_t$$

Optimal real reset price:

$$\Pi_t^* = \frac{\epsilon}{\epsilon - 1}\frac{g_t^1}{g_t^2}$$

$$g_t^1 = \Lambda_t mc_t Y_t + \beta\theta \left(\Pi_{t+1}\right)^{\epsilon} g_{t+1}^1$$

$$g_t^2 = \Lambda_t Y_t + \beta\theta \left(\Pi_{t+1}\right)^{\epsilon-1} g_{t+1}^2$$

Pricing kernel of intermediate goods producing firms:

$$\Lambda_t = v \left(\Delta_t^w\right)^{\frac{\rho+1}{\rho}} \left(\frac{1-v}{v}\frac{1}{(1-\tau_t^w)w_t}\right)^{1-v}$$

Evolution of aggregate price level:

$$1 = \theta(\Pi_t)^{\epsilon-1} + (1-\theta)(\Pi_t^*)^{1-\epsilon}$$

Price dispersion:

$$v_t^p = (1-\theta)\left(\Pi_t^*\right)^{-\epsilon} + \theta\left(\Pi_t\right)^{\epsilon} v_{t-1}^p$$

Profits:

$$F_t = (1 - mc_t v_t^p)\, Y_t + q_t \left(1 - S[\frac{I_t}{I_{t-1}}]\right) I_t - I_t$$

Fisher relation:

$$1 + i_t = \Pi_{t+1}\,(1 + r_{t+1})$$

Monetary policy rule:

$$\frac{1+i_t}{1+\bar{i}} = \left(\frac{1+i_{t-1}}{1+\bar{i}}\right)^{\eta_i} \left(\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\eta_\Pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\eta_Y}\right)^{1-\eta_i}$$

Capital quality:

$$\log(\zeta_t) = \rho_\zeta \log(\zeta_{t-1}) + \varepsilon_t$$

D Sensitivity analyses

Within the literature of adapted Gertler (1999)-models the calibrated values of the intertemporal elasticity of substitution range between $\frac{1}{4}$ and $\frac{1}{2}$, and we report the welfare effects for these two values in table 1. We adjust the accrual and contribution rates such that the size of the pension fund remains $\frac{A^f}{4Y} = 0.88$ in the steady state. In the real accounting framework retirees more strongly prefer a DB pension fund for higher levels of σ because the funding gap is larger after the adverse capital quality shock materialises. The workers who are alive at $t = 1$ also more strongly prefer a DB pension fund, because at a higher level of σ the subjective reweighting of transition probabilities variable Ω is higher, implying that they are more eager to have the value of their previously accumulated pension wealth guaranteed. The workers born after $t = 1$, on the other hand, do not have previously accumulated pension wealth and are negatively affected by their distorted labour supply for higher levels of σ. In the nominal accounting framework the effects are the opposite. For lower values of σ the funding surplus is larger due to a higher inflation path. Retirees then more strongly prefer a DC pension fund, while the opposite holds for all workers who more strongly prefer the cheap accrual of new pension wealth to a revaluation of previously accumulated pension wealth.

We consider both a smaller pension fund (with pension fund assets equal to 50% of yearly output, the OECD average in 2016) and a larger one (with pension fund assets equal to 125% of yearly output, the weighted OECD average in 2016). Table 1 indicates that qualitatively the reported results for the default calibration are maintained and that the stakes are simply scaled up. The only exception comes from the welfare of the future generations in a nominal accounting framework, who have a less pronounced preference for the DB pension fund when it manages more assets. This stems from the fact that the funding gap is larger for the smaller pension fund due to a higher path for inflation. In the DB system the effective contribution rate on labour income is therefore lower (in terms of relative deviation from its steady state value) for the smaller pension fund compared to the larger pension fund.

Lastly, we consider slower recoveries with a half-life of two and four years. When the pension fund postpones the closure of its funding gap in the real accounting framework, retirees in the meantime receive a pension that more closely matches what was promised to them before the adverse capital quality shock materialised. As such, the retiree preference for either type of pension fund diminishes. The workers alive at $t = 1$ have a similar preference, because with a longer half-life labour supply is distorted comparatively less in the first periods after the adverse capital quality shock and more in future periods. The workers born after $t = 1$ are on the receiving end of these distortions and therefore more strongly prefer a DC pension fund as the closure speed becomes lower. In the nominal accounting framework, the individuals alive in period $t = 1$ have a stronger preference for their preferred pension system when the recovery speed is higher because then the funding surplus is distributed more quickly. The future generations, however, more strongly prefer a DB pension fund with a longer recovery as they then capture a larger portion of the cheap accrual of new pension wealth.

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Equivalent variations	**Real business cycle** Retirees alive at $t = 1$	Workers alive at $t = 1$	Workers born after $t = 1$	**Total**
$\sigma = \frac{1}{4}$	-0.35%	-0.08%	$+0.06\%$	-0.37%
$\sigma = \frac{1}{3}$	-0.44%	-0.14%	$+0.07\%$	-0.51%
$\sigma = \frac{1}{2}$	-0.50%	-0.28%	$+0.08\%$	-0.70%
$\frac{A^f}{4Y} = 0.50$	-0.22%	-0.07%	$+0.04\%$	-0.25%
$\frac{A^f}{4Y} = 0.88$	-0.44%	-0.14%	$+0.07\%$	-0.51%
$\frac{A^f}{4Y} = 1.25$	-0.72%	-0.22%	$+0.10\%$	-0.84%
Half-life $= 1$ year	-0.44%	-0.14%	$+0.07\%$	-0.51%
Half-life $= 2$ years	-0.41%	-0.25%	$+0.09\%$	-0.57%
Half-life $= 4$ years	-0.37%	-0.39%	$+0.12\%$	-0.64%

Equivalent variations	**New-Keynesian, real framework** Retirees alive at $t = 1$	Workers alive at $t = 1$	Workers born after $t = 1$	**Total**
$\sigma = \frac{1}{4}$	-0.31%	$+0.16\%$	$+0.08\%$	-0.07%
$\sigma = \frac{1}{3}$	-0.41%	$+0.11\%$	$+0.13\%$	-0.17%
$\sigma = \frac{1}{2}$	-0.47%	$+0.00\%$	$+0.16\%$	-0.31%
$\frac{A^f}{4Y} = 0.50$	-0.21%	$+0.08\%$	$+0.07\%$	-0.06%
$\frac{A^f}{4Y} = 0.88$	-0.41%	$+0.11\%$	$+0.13\%$	-0.17%
$\frac{A^f}{4Y} = 1.25$	-0.67%	$+0.11\%$	$+0.21\%$	-0.35%
Half-life $= 1$ year	-0.41%	$+0.11\%$	$+0.13\%$	-0.17%
Half-life $= 2$ years	-0.37%	$+0.01\%$	$+0.17\%$	-0.19%
Half-life $= 4$ years	-0.32%	-0.14%	$+0.22\%$	-0.23%

Equivalent variations	**New-Keynesian, nominal framework** Retirees alive at $t = 1$	Workers alive at $t = 1$	Workers born after $t = 1$	**Total**
$\sigma = \frac{1}{4}$	$+1.90\%$	-0.52%	-0.56%	$+0.82\%$
$\sigma = \frac{1}{3}$	$+1.45\%$	-0.36%	-0.36%	$+0.73\%$
$\sigma = \frac{1}{2}$	$+0.97\%$	-0.06%	-0.21%	$+0.70\%$
$\frac{A^f}{4Y} = 0.50$	$+1.11\%$	-0.01%	-0.43%	$+0.67\%$
$\frac{A^f}{4Y} = 0.88$	$+1.45\%$	-0.36%	-0.36%	$+0.73\%$
$\frac{A^f}{4Y} = 1.25$	$+1.52\%$	-0.66%	-0.24%	$+0.62\%$
Half-life $= 1$ year	$+1.45\%$	-0.36%	-0.36%	$+0.73\%$
Half-life $= 2$ years	$+1.31\%$	-0.01%	-0.46%	$+0.84\%$
Half-life $= 4$ years	$+1.13\%$	$+0.46\%$	-0.58%	$+1.01\%$

Table 1: Welfare effects of switching from a DB pension fund to a DC pension fund for various parameter changes to the baseline calibration. Measured

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