Online Appendix for "On Phase Shifts in a New Keynesian Model"

1. Results from bi-variate VAR with output and price level

Consider a bivariate VAR with output and the price level. Formally, we estimate

$$y_t = a_0 + \sum_{i=1}^{5} b_i y_{t-i} + \sum_{i=1}^{5} d_i p_{t-i} + \eta_{yt} \tag{1}$$

$$p_t = a_1 + \sum_{i=1}^{5} f_i y_{t-i} + \sum_{i=1}^{5} g_i p_{t-i} + \eta_{pt} \tag{2}$$

with 5 lags chosen by likelihood ratio test. In this case, we are interested in two hypothesis tests. First, we test whether movements in output Granger cause movements in the price level; hence, $H_{0,y\to p}: f_i = 0 \text{ for } i = 1,2,3,4,5$. The p-value for this test is 0.890. One cannot reject the null hypothesis; that is, movements in output do not Granger cause movements in the price level at normal confidence level. Next, we ask whether movements in the price level Granger cause movements in output; that is, the null hypothesis is $H_{0,p\to y}: d_i = 0 \text{ for } i = 1,2,3,4,5.$ The p-value for $H_{0,p\to y}$ is 0.004.[1] Hence, we can reject the null hypothesis and the data are consistent with the notion that there is a left-phase shift characterizing the relationship between output and the price level at business cycle frequencies.[2]

2. Estimation of structural and distribution parameters

The following tables report the results from applying Bayesian estimation methods to the model parameters. Table 3A reports the model parameters for the full sample. Table 3B reports the parameters for the distributions that appear in the Smets-Wouters model economy based on the full sample. Tables 3C and 3D report the model parameters and the distribution parameters estimated from the 1985-2007 sample.

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[1] In this setup, there is no test for heteroscedasticity in the error terms. Note that the evidence presented by applying the standard F-test for Granger causality is consistent with a Wald test and a heteroskedastic-robust covariance estimator. See Haslag and Hsu (2012) for those results.

[2] In the frequency domain, Haslag and Hsu (2012) present evidence on the spectral densities for the price level and output. They show that the peak of the spectrum for the price level occur before the peak of the spectrum for output. At a glance, such evidence suggests that most of the variation in the price level occurs at a lower frequency for the price level compared with variation in output. By the Wald statistic, however, Haslag and Hsu conclude the differences in the power spectral frequencies are not significantly different from one another.

Table OA1
Prior and Posterior Distribution of Structural Parameters (1958:Q1 - 2017:Q4)

Parameter	Notation	Prior distribution			Posterior distribution			
		Dist.	Mean	Std Dev	Mode	Mean	5 %	95 %
S.S. elasticity of capital adjustment cost function	φ	Normal	4.00	1.50	4.45	4.88	3.35	6.42
Intertemporal elasticity of substitution	σ_c	Normal	1.50	0.37	1.47	1.44	1.19	1.68
Habit formation parameter	h	Beta	0.70	0.10	0.51	0.53	0.45	0.62
Degree of wage stickiness	ξ_w	Beta	0.50	0.10	0.83	0.82	0.76	0.89
Elasticity of labor supply	σ_l	Normal	2.00	0.75	1.24	1.28	0.55	2.02
Degree of price stickiness	ξ_p	Beta	0.50	0.10	0.92	0.92	0.89	0.94
Indexation of wage to past inflation	ι_w	Beta	0.50	0.15	0.55	0.53	0.32	0.74
Indexation of prices to past inflation	ι_p	Beta	0.50	0.15	0.21	0.22	0.10	0.34
A positive function of elasticity of capital utilization adjustment cost function	ψ	Beta	0.50	0.15	0.69	0.68	0.54	0.83
1 + share of fixed cost in production	Φ	Normal	1.25	0.12	1.46	1.47	1.35	1.59
Monetary policy response – inflation	r_π	Normal	1.50	0.25	1.70	1.72	1.43	2.00
Degree of interest rate smoothing	ρ	Beta	0.75	0.10	0.88	0.89	0.86	0.92
Monetary policy response – output gap	r_y	Normal	0.12	0.05	0.12	0.12	0.08	0.17
Monetary policy response – change in output gap	$r_{\Delta y}$	Normal	0.12	0.05	0.22	0.22	0.19	0.26
S.S. inflation rate	$\bar{\pi}$	Gamma	0.62	0.10	0.65	0.67	0.52	0.82
A function of discount factor	$100\left(\beta^{-1}-1\right)$	Gamma	0.25	0.10	0.12	0.14	0.05	0.21
S.S. hours worked	$\bar{l}$	Normal	0.00	2.00	0.09	-0.07	-1.63	1.60
S.S. growth rate of output, consumption, investment, etc.	$\bar{\gamma}$	Normal	0.40	0.10	0.36	0.37	0.33	0.40
Share of capital in production	α	Normal	0.30	0.05	0.19	0.19	0.16	0.21

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm.

Table OA2
Prior and Posterior Distribution of Shock Processes (1958:Q1 - 2017:Q4)

Parameter	Notation	Prior distribution			Posterior distribution			
		Dist.	Mean	Std Dev	Mode	Mean	5 %	95 %
Std dev – TFP shock	σ_a	Invgamma	0.10	2.00	0.54	0.54	0.49	0.59
Std dev – risk premium shock	σ_b	Invgamma	0.10	2.00	0.09	0.10	0.08	0.12
Std dev – government spending shock	σ_g	Invgamma	0.10	2.00	0.47	0.47	0.44	0.51
Std dev – investment-specific tech shock	σ_I	Invgamma	0.10	2.00	0.36	0.37	0.31	0.42
Std dev – monetary policy shock	σ_r	Invgamma	0.10	2.00	0.22	0.22	0.20	0.24
Std dev – price markup shock	σ_p	Invgamma	0.10	2.00	0.21	0.21	0.19	0.24
Std dev – wage markup shock	σ_w	Invgamma	0.10	2.00	0.39	0.39	0.35	0.42
Autocorrelation – TFP disturbance	ρ_a	Beta	0.50	0.20	0.97	0.97	0.96	0.98
Autocorrelation – risk premium disturbance	ρ_b	Beta	0.50	0.20	0.88	0.86	0.80	0.93
Autocorrelation – government spending disturbance	ρ_g	Beta	0.50	0.20	0.97	0.97	0.96	0.99
Autocorrelation – investment-specific tech disturbance	ρ_I	Beta	0.50	0.20	0.79	0.79	0.70	0.88
Autocorrelation – monetary policy disturbance	ρ_r	Beta	0.50	0.20	0.12	0.14	0.05	0.22
Autocorrelation – price markup disturbance	ρ_p	Beta	0.50	0.20	0.94	0.93	0.89	0.96
Autocorrelation – wage markup disturbance	ρ_w	Beta	0.50	0.20	0.98	0.97	0.96	0.99
MA(1) coefficient – price markup disturbance	μ_p	Beta	0.50	0.20	0.98	0.97	0.96	0.99
MA(1) coefficient – wage markup disturbance	μ_w	Beta	0.50	0.20	0.97	0.95	0.93	0.98
Impact of TFP shock on government spending disturbance	ρ_{ga}	Beta	0.50	0.20	0.54	0.54	0.44	0.63

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm.

Table OA3
Prior and Posterior Distribution of Structural Parameters (1985:Q1 - 2007:Q4)

Parameter	Notation	Prior distribution			Posterior distribution			
		Dist.	Mean	Std Dev	Mode	Mean	5 %	95 %
S.S. elasticity of capital adjustment cost function	φ	Normal	4.00	1.50	7.15	6.31	4.45	8.05
Intertemporal elasticity of substitution	σ_c	Normal	1.50	0.37	1.50	1.11	0.87	1.33
Habit formation parameter	h	Beta	0.70	0.10	0.68	0.53	0.42	0.64
Degree of wage stickiness	ξ_w	Beta	0.50	0.10	0.57	0.64	0.48	0.81
Elasticity of labor supply	σ_l	Normal	2.00	0.75	2.39	2.18	1.16	3.14
Degree of price stickiness	ξ_p	Beta	0.50	0.10	0.75	0.84	0.75	0.92
Indexation of wage to past inflation	ι_w	Beta	0.50	0.15	0.48	0.47	0.22	0.72
Indexation of prices to past inflation	ι_p	Beta	0.50	0.15	0.41	0.35	0.14	0.57
A positive function of elasticity of capital utilization adjustment cost function	ψ	Beta	0.50	0.15	0.78	0.66	0.48	0.83
1 + share of fixed cost in production	Φ	Normal	1.25	0.12	1.55	1.48	1.32	1.63
Monetary policy response – inflation	r_π	Normal	1.50	0.25	1.93	1.95	1.52	2.34
Degree of interest rate smoothing	ρ	Beta	0.75	0.10	0.83	0.86	0.82	0.90
Monetary policy response – output gap	r_y	Normal	0.12	0.05	0.04	0.12	0.04	0.19
Monetary policy response – change in output gap	$r_{\Delta y}$	Normal	0.12	0.05	0.11	0.13	0.09	0.17
S.S. inflation rate	$\bar{\pi}$	Gamma	0.62	0.10	0.60	0.60	0.48	0.73
A function of discount factor	$100\left(\beta^{-1}-1\right)$	Gamma	0.25	0.10	0.10	0.18	0.07	0.28
S.S. hours worked	$\bar{l}$	Normal	0.00	2.00	1.25	1.35	0.15	2.53
S.S. growth rate of output, consumption, investment, etc.	$\bar{\gamma}$	Normal	0.40	0.10	0.42	0.46	0.40	0.51
Share of capital in production	α	Normal	0.30	0.05	0.23	0.18	0.13	0.22

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm.

Table OA4
Prior and Posterior Distribution of Shock Processes (1985:Q1 - 2007:Q4)

Parameter	Notation	Prior distribution			Posterior distribution			
		Dist.	Mean	Std Dev	Mode	Mean	5 %	95 %
Std dev – TFP shock	σ_a	Invgamma	0.10	2.00	0.37	0.40	0.34	0.45
Std dev – risk premium shock	σ_b	Invgamma	0.10	2.00	0.19	0.08	0.05	0.11
Std dev – government spending shock	σ_g	Invgamma	0.10	2.00	0.40	0.40	0.35	0.45
Std dev – investment-specific tech shock	σ_I	Invgamma	0.10	2.00	0.36	0.36	0.27	0.46
Std dev – monetary policy shock	σ_r	Invgamma	0.10	2.00	0.09	0.09	0.08	0.11
Std dev – price markup shock	σ_p	Invgamma	0.10	2.00	0.08	0.07	0.06	0.09
Std dev – wage markup shock	σ_w	Invgamma	0.10	2.00	0.36	0.35	0.26	0.42
Autocorrelation – TFP disturbance	ρ_a	Beta	0.50	0.20	0.94	0.91	0.86	0.96
Autocorrelation – risk premium disturbance	ρ_b	Beta	0.50	0.20	0.12	0.82	0.72	0.94
Autocorrelation – government spending disturbance	ρ_g	Beta	0.50	0.20	0.96	0.96	0.93	0.99
Autocorrelation – investment-specific tech disturbance	ρ_I	Beta	0.50	0.20	0.60	0.58	0.42	0.75
Autocorrelation – monetary policy disturbance	ρ_r	Beta	0.50	0.20	0.47	0.40	0.26	0.53
Autocorrelation – price markup disturbance	ρ_p	Beta	0.50	0.20	0.95	0.85	0.74	0.97
Autocorrelation – wage markup disturbance	ρ_w	Beta	0.50	0.20	0.92	0.87	0.77	0.97
MA(1) coefficient – price markup disturbance	μ_p	Beta	0.50	0.20	0.77	0.69	0.51	0.89
MA(1) coefficient – wage markup disturbance	μ_w	Beta	0.50	0.20	0.74	0.66	0.45	0.88
Impact of TFP shock on government spending disturbance	ρ_{ga}	Beta	0.50	0.20	0.36	0.36	0.20	0.52

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm.

3. Results from Cross-correlation functions

As an added assessment of the goodness-of-fit, we plot the lead-lag cross-correlation function for the model economy for the three candidate shocks—government spending, price markup, and wage markup--deemed sufficient for the contemporaneous correlation coefficients in Figure OA2. In each panel (A through C), the reader can see $\rho\left(p_{t+i}, y_t\right)$ for $i = -5, -4, ..., 4, 5$ in the left-hand column and $\rho\left(\pi_{t+i}, y_t\right)$ for $i = -5, -4, ..., 4, 5$ in the right-hand column. The cross-correlation function for the simulated economy and the actual data, using the H-P filter, are plotted together. The government spending shock in Panel A shows that output increases immediately in the Smets-Wouters model matched by increase in the inflation rate, indicated directly in the right panel and indirectly by the price level being less negative at a quick rate in the left panel. In the NK model, aggregate demand shocks result in more output and the price level response is initially damped by the Calvo clock. In Panel B, the case of a price markup shock permits the price level to increase sharply compared with one period before. It follows that the inflation rate increases.

Overall, the lead-lag cross-correlation functions provide us with an additional insight into the goodness-of-fit for each shock. As you look across the panels, cross-correlation plot for the price markup shock more closely fits the pattern and magnitude of the actual cross-correlation pattern than the other two shocks. The goodness-of-fit is ocular and not formal, but still worth noting as we study shock-by-shock results. In terms of accounting for the phase shift in the postwar period, the price markup shock is promising as both a necessary and sufficient shock.

Figure OA1

Cross-Correlation Function for Smets-Wouters Model Economies (Only Price Markup Shock Off) (1958:Q1 - 2017:Q4)

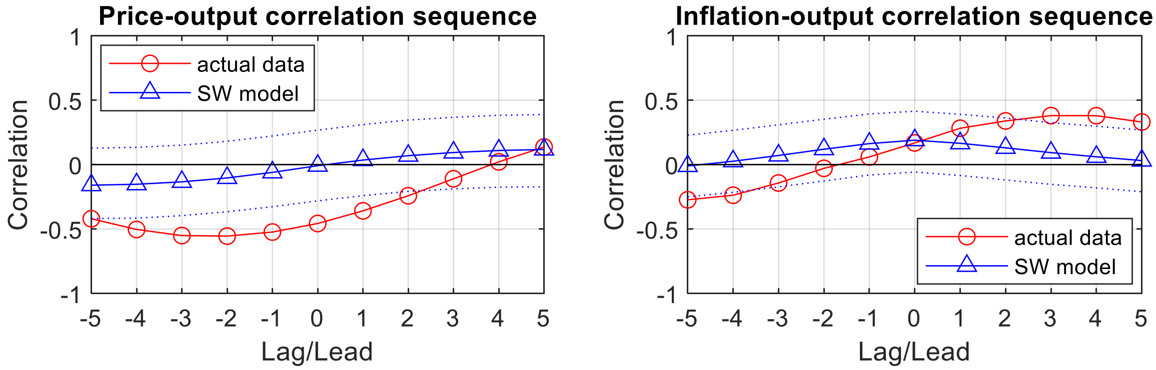
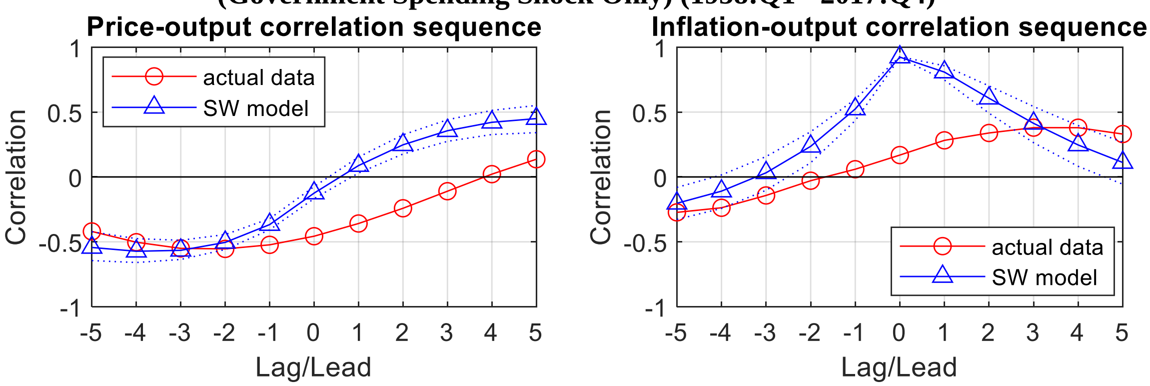
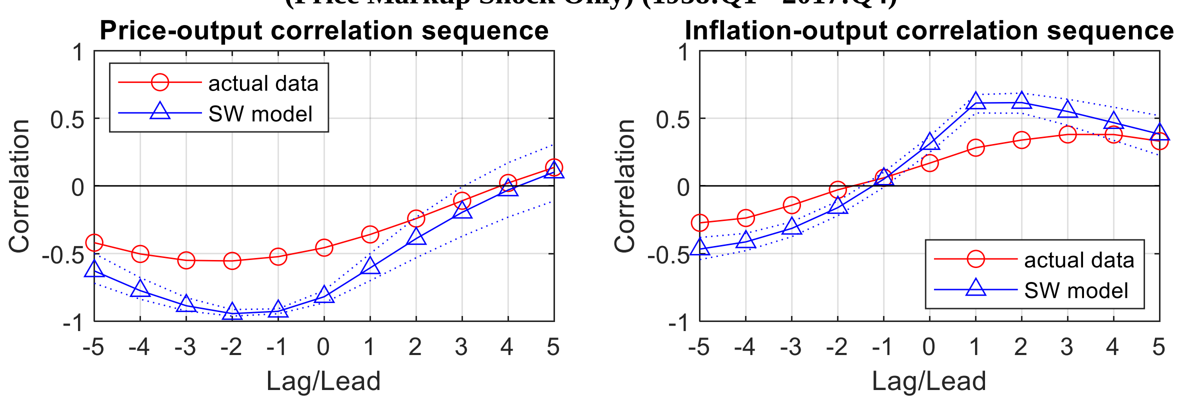


Figure OA2

Panel A: Cross-Correlation Function for Smets-Wouters Model Economies (Government Spending Shock Only) (1958:Q1 - 2017:Q4)



Panel B: Cross-Correlation Function for Smets-Wouters Model Economies (Price Markup Shock Only) (1958:Q1 - 2017:Q4)



Panel C: Cross-Correlation Function for Smets-Wouters Model Economies (Wage Markup Shock Only) (1958:Q1 - 2017:Q4)