Online appendix to ‘Unsecured Credit, Product Variety, and Unemployment Dynamics’

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Appendix A. Additional Figures

Appendix A.1. Number of Firms

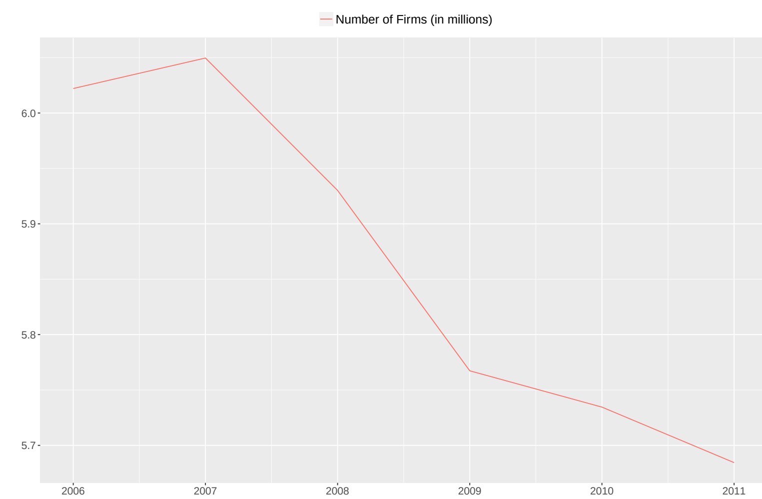

Figure A.1: (Log) Number of Firms from Statistics of U.S. Businesses, Census

Appendix A.2. Relationship between Unemployment and Productivity

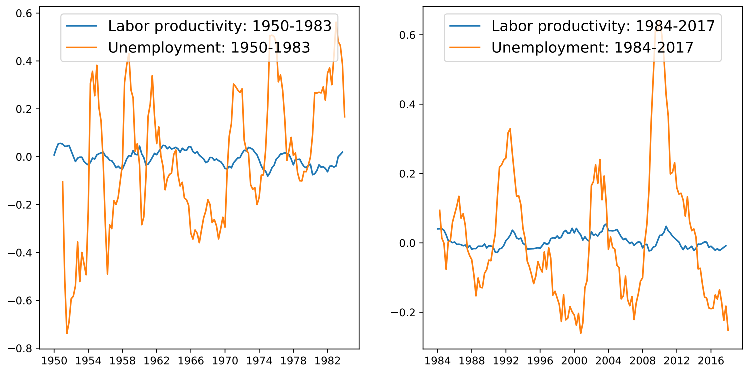

Figure A.2: Labor productivity and unemployment, quarterly frequency. Labor productivity is measured as GDP per worker (FRED code PRS85006163). Both series are logged and filtered via the Hamilton Method. The first plot covers 1950 – 1983 and the second plot spans 1984 – 2017.

Appendix B. Additional derivations

Appendix B.1. Representation of Equilibrium in (θ, u) space

In the spirit of the labor search literature, I represent a steady-state equilibrium as a pair (θ, u) satisfying the Beveridge curve and the following:

$$\frac{r+1-\lambda+\psi}{\psi}[(1-u)z]^{(\rho+\varepsilon-1)/\rho}p(\theta) = A\left[\frac{(1+r)(1-\rho)p}{\gamma(r+\delta)}\right]^{(1-\rho)(1-\varepsilon)/\rho}/(1-\varepsilon) \tag{B.1}$$

I refer to (B.1) as the cost push curve: it describes the equilibrium change in marginal costs associated with a change in unemployment. I first derive the cost push curve and then analyze its properties.

Provided the credit constraint binds, the steady state debt limit is

$$\frac{r+1-\lambda+\psi}{\psi}Spq = AQ^{1-\varepsilon}/(1-\varepsilon) \tag{B.2}$$

We rewrite $Q = S^{1/\rho}q = [(1-u)/n]^{1/\rho}q = \left(\frac{(1-u)z}{q}\right)^{1/\rho} q = [(1-u)z]^{1/\rho}\left[\frac{(1+r)(1-\rho)p}{\gamma(r+\delta)}\right]^{\frac{1-\rho}{\rho}}$. Thus,

$$\frac{r+1-\lambda+\psi}{\psi}Spq = A\left\{[(1-u)z]^{1/\rho}\left[\frac{(1+r)(1-\rho)p}{\gamma(r+\delta)}\right]^{(1-\rho)/\rho}\right\}^{1-\varepsilon}/(1-\varepsilon) \tag{B.3}$$

Furthermore, $Spq{=}\frac{(1-u)z}{q}pq = (1-u)zp$, so that

$$\frac{r+1-\lambda+\psi}{\psi}(1-u)zp = A\left\{[(1-u)z]^{1/\rho}\left[\frac{(1+r)(1-\rho)p}{\gamma(r+\delta)}\right]^{(1-\rho)/\rho}\right\}^{1-\varepsilon}/(1-\varepsilon) \quad \text{(B.4)}$$

This expression can be finally simplified into (B.1). Equation (B.1) implies a positive relationship between θ and u provided that $\rho+\varepsilon > 1$ (which entails $2\rho+\varepsilon-\rho\varepsilon-1 > \rho(1-\varepsilon) > 0$), or if $2\rho+\varepsilon-\rho\varepsilon-1 < 0$. Hence, just in the case $\rho+\varepsilon < 1$ and $2\rho+\varepsilon-\rho\varepsilon-1 > 0$, there is a negative relationship between θ and u.

Lemma 1.

- *The cost-push curve is negatively sloped if and only if* $[(1-\rho)(1-\varepsilon)-\rho]/(\rho+\varepsilon-1) > 0$.
- *A sufficient condition for the cost-push curve to be positively sloped is* $\rho+\varepsilon > 1$. *In the knife-edge case* $\rho+\varepsilon = 1$, *the cost-push curve pins down* θ *uniquely, and the equilibrium is recursive.*
- *If* $\frac{\partial\theta}{\partial u} < 0$, *a sufficient condition for concavity of the cost push curve is* $(\rho+\varepsilon-1)/[(1-\rho)(1-\varepsilon)-\rho] < 1$.

Thus, the cost-push curve is negatively sloped if and only if $\rho+\varepsilon < 1$ and $2\rho+\varepsilon-\rho\varepsilon-1 > 0$.

Appendix B.2. Comparative Statics

Using the steady state representation of equilibrium in (θ, u)-space from Appendix B.1, I show the comparative statics for the two cases: $(I)\rho+\varepsilon > 1$ and $(II)\rho+\varepsilon < 1$ and $2\rho+\varepsilon-\rho\varepsilon-1 > 0$.

Proposition 1. *Suppose* $\chi < (1-\delta)(1-s)$ *and (for Case I)* $(2\rho+\varepsilon-\rho\varepsilon-1)[s+\delta(1-s)+1]\frac{\kappa\beta}{z(1-\chi)} > \rho+\varepsilon-1$. *Then Table B.1 provides the comparative statics.*

Table B.1: Comparative statics: pure credit

(a) $\rho + \varepsilon > 1$

Parameter	θ	u	Q	S	d	n	p
A	+	−	+	+	+	−	+
δ	−	+					
s	−	+	−	−	−	−	+
γ	−	+	−	−	−	+	−
r	−	+	−	−	−	+	−
ψ	+	−	+	+	+	−	+
k	−	+	−	−	−	−	+
z	+	−	+	+	+	−	−

(b) $\rho + \varepsilon < 1, 2\rho + \varepsilon - \rho\varepsilon - 1 > 0$

Parameter	θ	u	Q	S	d	n	p
A	+	−	+	+	+	−	+
δ	−	+	−	−	−	+	−
s	−	+	−	−	−	+	−
γ	−	+	−	−	−	+	−
r	−	+	−	−	−	+	−
$\bar{\psi}$	+	−	+	+	+	−	+
k	−	+	−	−	−	+	−
z	+	−	+	+	+	−	+

The comparative statics of A and ψ are identical, and coincide with those of z except possibly for n and p. Productivity shocks, however, give rise to several possibilities. If $\rho + \varepsilon > 1$, then a positive productivity shock involves a net reduction of prices and in firm size (though the total number of firms unambiguously increases). Otherwise, provided that $2\rho + \varepsilon - \rho\varepsilon - 1 > 0$, then prices increase due to the very strong feedback between credit and aggregate demand that raises marginal costs. Similarly, a rise in vacancy posting costs k directly raises marginal costs and prices. In case (b), demand falls sufficiently that prices actually decrease. It helps to rewrite the cost push curve as follows:

$$\left[(1-\varepsilon)\left(\frac{r+1-\lambda+\psi}{\psi A}\right)\right]^{\rho} p(\theta)^{\rho(1-\varepsilon)+\rho+\varepsilon-1}\left[\frac{\gamma(r+\delta)}{(1+r)(1-\rho)}\right]^{1+\rho\varepsilon-\varepsilon-\rho} z^{\rho+\varepsilon-1} = (1-u)^{1-\varepsilon-\rho} \tag{B.5}$$

- Case I: $\rho + \varepsilon > 1$.
  1. Suppose there is a rise in z. This reduces $p(\theta)$ for all θ proportionately greater than the rise in $z^{\rho+\varepsilon-1}$, and thereby shifts the cost-push curve to the left. Hence, u is lower and θ is higher. $zp(\theta)$, which depends only on θ, is higher. Since $zp(\theta)n$ is constant by the free entry condition, n is lower. $S = (1-u)/n$ is higher as well. $d = Spq$ can be rewritten as $d = (1-u)zMC/\rho$, which is higher. Q can be written as $Q = [(1-u)^{\rho} z]/[n^{\frac{1-\rho}{\rho}}]$, which rises since u falls, z increases, and n falls. Notice that $p \propto [(1-u)z]^{\frac{1-\varepsilon-\rho}{\rho(1-\varepsilon)+\rho+\varepsilon-1}}$. Since the exponent is negative, so p falls and q rises.
  2. Now consider an increase in k. This raises $p(\theta)$ and hence shifts the cost push curve to the right, lowering θ and raising u. However, $p(\theta)$ remains higher. Thus,

n is lower. The measure of sellers $S = (1-u)/n$ is lower. To see this, first write $S = \frac{1-u}{\gamma(r+\delta)}(1+r)(1-\rho)zp(\theta)$, so that $S \propto (1-u)zp(\theta)$. From the equilibrium conditions, one can show that $(1-u)zp(\theta) \propto (1-u)^{\rho+\varepsilon}p(\theta)^{2\rho+\varepsilon-\rho\varepsilon}$, or that $[(1-u)p(\theta)]^{1-\rho-\varepsilon} \propto p(\theta)^{\rho(1-\varepsilon)}$. As $p(\theta)$ is higher and $\rho+\varepsilon > 1$, $(1-u)zp(\theta)$ is lower and hence S is lower. $Q = S^{1/\rho}zn$ is lower as well. The debt level $d = (1-u)zp(\theta)$ is lower.

3. Consider an increase in ψ. The shift of the cost push curve depends on $\left(\frac{\psi}{r+\psi}\right)^{\rho}$, which increases with ψ. Thus, the cost push curve shifts to the left, so that θ rises and u falls. $p(\theta)$ rises, so that n falls. From $S = (1-u)/n, S$ clearly rises. $Q = S^{1/\rho}q$ can be rearranged as $(1-u)^{1/\rho}z/(n^{(1-\rho)/\rho})$, which rises. Debt limits $d = (1-u)zp(\theta)$ rise, as $p(\theta)$ is higher and u is lower.
4. Consider an increase in r. From the cost-push curve, at a given u, $p(\theta)$ is lower. Provided $p(\theta)$ weakly increases with r, then θ must decrease, so that the cost-push curve shifts to the right.

   We can show that $\frac{\partial p}{\partial \beta} = \frac{1}{z(1-\chi)}[\chi k\theta - \frac{k(1-\delta)(1-s)}{f(\theta)}] = \frac{k\theta}{z(1-\chi)}[\chi - \frac{(1-\delta)(1-s)}{h(\theta)}]$. As $h(\theta) < 1$, this quantity is bounded above by $\frac{k}{z(1-\chi)}[\chi - (1-\delta)(1-s)]$, which is negative provided that $(1-\delta)(1-s) > \chi$. Hence, $\frac{\partial p(\theta)}{\partial r} = -\frac{\partial p(\theta)}{\partial \beta}\frac{1}{(1+r)^2} > 0$.

   Given that the Beveridge curve is unaffected, θ falls and u rises. As r is higher, it remains to show that equilibrium $MC(\theta)$ is lower.

   Using $\gamma(r+\delta) = (1+r)(1-\rho)zp(\theta)n$, n increases, as $MC(\theta)$ is lower and r is higher. $d = (1-u)zp$ thus falls. Further, $S = (1-u)/n$ decreases. $Q = (1-u)^{1/\rho}z/(n^{(1-\rho)/\rho})$ decreases from both the increase in u and rise in n.
5. Consider an increase in γ. The cost push curve shifts to the right, so θ and $p(\theta)$ fall, and u increases. As before, from the entry condition n rises. $d = (1-u)zp$ thus falls, and so does $S = (1-u)/n$. $Q = (1-u)^{1/\rho}z/(n^{(1-\rho)/\rho})$ decreases from both the decline in u and rise in n.
6. Consider an increase in A. The cost push curve shifts to the left, so that θ increases and u decreases. $p(\theta)$ rises, so that n falls. From $S = (1-u)/n, S$ clearly rises. $Q = S^{1/\rho}q$ can be rearranged as $(1-u)^{1/\rho}z/(n^{(1-\rho)/\rho})$, which rises. Debt limits $d = (1-u)zp(\theta)$ rise, as p is higher and u is lower.
7. Consider an increase in s. $p(\theta)$ increases, inducing a rightward shift of the cost push curve. Furthermore, the Beveridge curve shifts to the right. The effect of the shift in the cost push curve is to raise u and lower θ. The effect of the rightward shift of the Beveridge curve is to raise u and θ. Thus, u clearly rises, but θ is ambiguous.

   Through implicit differentiation, we can provide a weak necessary and sufficient condition for θ to fall. Rearranging the equilibrium conditions provides

$$(2\rho + \varepsilon - \rho\varepsilon - 1)\log MC(\theta) = \aleph + (1-\rho-\varepsilon)\log\left(\frac{h(\theta)}{s+\delta(1-s)+h(\theta)}\right) \quad \text{(B.6)}$$

in which $\aleph$ is a function of parameters, not including s. Totally differentiating (and using $\frac{\partial MC}{\partial s} = \frac{k\beta(1-\delta)}{f(\theta)z(1-\chi)}$ yields

$$(2\rho+\varepsilon-1)\left[\frac{MC'(\theta)}{MC(\theta)}\frac{d\theta}{ds} + \frac{k\beta(1-\delta)}{f(\theta)z(1-\chi)}\right] = (1-\rho-\varepsilon)\left(\frac{p'(\theta)}{p(\theta)}\frac{d\theta}{ds} - \frac{p'(\theta)d\theta/ds + 1 - \delta}{s+\delta(1-s)+h(\theta)}\right) \quad \text{(B.7)}$$

which can be further rearranged as

$$\frac{d\theta}{ds} = \frac{(1-\rho-\varepsilon)h(\theta)(1-\delta) + h(\theta)[s+\delta(1-s)+h(\theta)](2\rho+\varepsilon-\rho\varepsilon-1)\frac{k\beta(1-\delta)}{f(\theta)z(1-\chi)}}{(1-\rho-\varepsilon)h'(\theta)[s+\delta(1-s)] - (2\rho+\varepsilon-\rho\varepsilon-1)h(\theta)[s+\delta(1-s)+h(\theta)]MC'(\theta)/MC(\theta)} \quad \text{(B.8)}$$

By presupposition, the sign of the denominator is negative, so the sign of $d\theta/ds$ is the opposite sign of the numerator, which has the same sign as

$$1 - \rho - \varepsilon + [s+\delta(1-s)+h(\theta)](2\rho+\varepsilon-\rho\varepsilon-1)\frac{k\beta}{f(\theta)z(1-\chi)} \quad \text{(B.9)}$$

We can derive a sufficient condition in terms of parameters for which (B.9) > 0. Consider the expression $\frac{s+\delta(1-s)+h(\theta)}{f(\theta)}$. This expression is strictly increasing in θ, and minimized as $\theta \to 0$, in which $h(\theta) \to 0$ and $f(\theta) \to 1$. The expression converges to $s + \delta(1-s) + 1$. Thus, (B.9) is positive provided that $(2\rho+\varepsilon-\rho\varepsilon-1)(s+\delta(1-s)+1)\frac{k\beta}{z(1-\chi)} > \rho + \varepsilon - 1$.

Provided this condition is met, θ decreases. To figure out the net effect on MC, we see that the total derivative has the same sign as $(1-\rho-\varepsilon)[h'(\theta)d\theta/ds[s+\delta(1-s)] - h(\theta)(1-\delta)]$, which is positive if $d\theta/ds < 0$. Hence, MC increases, so n decreases. $Q = (1-u)^{1/\rho}z/(n^{\frac{1-\rho}{\rho}})$, which unambiguously decreases. The measure of firms satisfies $S = (1-u)/n$, which is ambiguous. Thus, we derive the comparative static of $(1-u)/n$. From the equilibrium conditions, $\left(\frac{1-u}{n}\right)^{1-\rho-\varepsilon} n^{\rho(1-\varepsilon)}$ is constant. Since n decreases, this means $(1-u)/n$ decreases. Hence, S decreases. $Q = S^{1/\rho}zn$ thus decreases as well. Writing $d = \frac{1-u}{n}nzp$, which decreases since $1-u$ decreases and np is constant from the free entry condition.

8. Consider an increase in δ. As before, the Beveridge curve shifts to the right, symmetrically as with an increase in s. The cost push curve shifts to the right

even more than in the case of s. This is because a higher δ both raises the marginal costs of existing firms and deters entry. Thus, u clearly rises, θ is ambiguous, and the condition (B.9) is sufficient for θ to fall.

- Case II: $\rho+\varepsilon<1$ and $2\rho+\varepsilon(1-\rho)>1$.

  1. Suppose there is a rise in z. This reduces $p(\theta)$ for all θ, and thereby shifts the cost-push curve upward. Hence, u is lower and θ is higher. The rest is identical to Case I, except for p and q. As $p \propto [(1-u)z]^{\frac{1-\varepsilon-\rho}{\rho(1-\varepsilon)+\rho+\varepsilon-1}}$, and the exponent is positive, p increases and q decreases.
  2. Now consider an increase in k. $p(\theta)$ rises and hence shifts the cost push curve downward, lowering θ and raising u. Note that the RHS of the cost push curve is lower, so that $p(\theta)$ remains lower than initially. Thus, n is higher. The measure of sellers $S=(1-u)/n$ is thus lower. $Q=S^{1/\rho}zn$ can be rewritten as $Q=(1-u)z^{\rho}/(n^{1-\rho})$, which is lower as well. The debt level $d=Spq$ can be rewritten as $d=S\frac{\gamma(r+\delta)}{(1+r)(1-\rho)}$, which is lower since S is lower.
  3. Consider an increase in ψ. The shift of the cost push curve depends on $\left(\frac{\psi}{r+\psi}\right)^{\rho}$, which increases with ψ. Thus, the cost push curve shifts to the left, so that θ rises and u falls. The rest follows as in Case I.
  4. Consider an increase in r. From the cost-push curve, at a given u, $p(\theta)$ is lower. Provided $p(\theta)$ weakly increases with r, then θ must be lower, so that the cost-push curve shifts downward. As Case I, $\chi<(1-\delta)(1-s)$ is a sufficient condition.
     Given that the Beveridge curve is unaffected, θ falls and u rises. Since the RHS of (B.5) falls, so must the left hand side. Hence, $p(\theta)$ is lower. Using $\gamma(r+\delta)=(1+r)(1-\rho)zp(\theta)n$, n increases, as $p(\theta)$ is lower and r is higher. $d=(1-u)zp(\theta)$ thus falls. Further, $S=(1-u)/n$ decreases. $Q=(1-u)^{1/\rho}z/(n^{(1-\rho)/\rho})$ decreases from both the increase in u and rise in n.
  5. Consider an increase in γ. The cost push curve shifts to the right, so θ falls and u increases. As before, from the entry condition n rises. The rest follows identically as in Case I.
  6. Consider an increase in A. The cost push curve shifts to the left, so that θ increases and u decreases. The rest follows identically as in Case I.
  7. Consider an increase in s. $p(\theta)$ increases at a given θ, inducing a downward shift of the cost push curve. Furthermore, the Beveridge curve shifts to the right. The effect of the shift in the cost push curve is to raise u and lower θ. Unlike in Case I, the effect of the rightward shift of the Beveridge curve is to raise u and lower θ. Thus, u clearly rises and θ falls. From $\rho+\varepsilon<1$, the RHS of (B.5) falls.

Hence, $p(\theta)$ also falls. Thus, n increases. $S = (1-u)/n$ correspondingly decreases. $d = (1-u)zp(\theta)$ falls as well. $Q = (1-u)^{1/\rho}z/(n^{(1-\rho)/\rho})$ decreases from both the increase in u and rise in n.

8. Consider an increase in δ. As before, the Beveridge curve shifts to the right, symmetrically as with an increase in s. The cost push curve shifts to the right even more than in the case of s. This is because a higher δ both raises the marginal costs of existing firms and deters entry. Thus, u rises, θ decreases, and the condition (B.9) is sufficient for θ to fall. As with a rise in s, $p(\theta)$ falls, since the RHS of (B.5) decreases and $(r+\delta)/(1+r)$ rises in the LHS. The remainder is analogous to a rise in s.

Appendix B.3. Concavity of the Cost Push Curve

We can rearrange (B.1) as

$$p(\theta)^{\frac{(1-\rho)(1-\varepsilon)-\rho}{\rho}} = \aleph_0(1-u)^{\frac{\rho+\varepsilon-1}{\rho}}$$

for the composite parameter $\aleph_0 = (1-\varepsilon)^{\frac{r+1-\lambda+\psi}{\psi}} z^{(\rho+\varepsilon-1)\rho}((1+r)(1-\rho)/(\gamma(r+\delta)))^{-(1-\rho)(1-\varepsilon)-\rho}/A > 0$. Further rearrangement yields

$$p(\theta) = \aleph_1(1-u)^{\frac{\rho+\varepsilon-1}{(1-\rho)(1-\varepsilon)-\rho}} \tag{B.10}$$

for $\aleph_1 = \aleph_0^{[\rho/((1-\rho)(1-\varepsilon)-\rho)]}$.

Implicit differentiation with respect to u yields

$$\frac{\partial\theta}{\partial u} = -\frac{\aleph_1}{p'(\theta)}\frac{\rho+\varepsilon-1}{(1-\rho)(1-\varepsilon)-\rho}(1-u)^{\frac{\rho+\varepsilon-1+\rho-(1-\rho)(1-\varepsilon)}{(1-\rho)(1-\varepsilon)-\rho}} \tag{B.11}$$

Hence $\frac{\partial\theta}{\partial u} < 0$ if and only if $(\rho+\varepsilon-1)/((1-\rho)(1-\varepsilon)-\rho) > 0$. The second derivative can be simplified to

$$\frac{\partial^2\theta}{\partial u^2} = \frac{\aleph_1}{p'(\theta)^2}\frac{\rho+\varepsilon-1}{(1-\rho)(1-\varepsilon)-\rho}(1-u)^{\frac{\rho+\varepsilon-1}{(1-\rho)(1-\varepsilon-\rho)}-2}\left[(1-u)\frac{\partial\theta}{\partial u} + p'(\theta)\left(\frac{\rho+\varepsilon-1}{(1-\rho)(1-\varepsilon)-\rho}-1\right)\right] \tag{B.12}$$

A sufficient condition for concavity, provided $\frac{\partial\theta}{\partial u} < 0$ is $(\rho+\varepsilon-1)/((1-\rho)(1-\varepsilon)-\rho) < 1$. In general, this condition is stronger than necessary, but the necessary condition is a complicated relationship involving u and $\aleph_1$.

Appendix B.4. Proof of Lemma 1

The only nontrivial statement is the convexity of $p(\theta)$. Note that it suffices to show that $g(\theta) = 1/f(\theta)$ is convex.

First, write $g(\theta) = (1+\theta^{\xi})^{1/\xi}$, so that

$$g'(\theta) = (1+\theta^{\xi})^{(1-\xi)/\xi}\theta^{\xi-1} \tag{B.13}$$

and

$$\begin{aligned} g''(\theta) &= (1-\xi)(1+\theta^{\xi})^{(1-2\xi)/\xi}(\theta^{\xi-1})^2 + (1+\theta^{\xi})^{(1-\xi)/xi}(\xi-1)\theta^{\xi-2} \\ &> (1-\xi)\theta^{1-2\xi}\theta^{2\xi-2} + \theta^{1-\xi}(\xi-1)\theta^{\xi-2} \\ &= \frac{1-\xi}{\theta} + \frac{\xi-1}{\theta} \\ &= 0 \end{aligned}$$

where we use $(1+\theta^{\xi})^{1/\xi} > \theta$. Hence, $g(\theta)$ is strictly convex for all $\theta > 0$ and all $\xi > 0$. Consequently, $p(\theta)$ is convex in this range as well.

Appendix C. Parameterized Expectations Algorithm.

The use of a global solution algorithm is important for three reasons. First, **?** show that the labor search model has strong nonlinearities. The job finding rate, which is concave, falls fast in recessions but rises only gradually in expansions. Log linearization thus understates the mean and volatility of unemployment and overstates the volatility of market tightness. Second, the use of a global solution algorithm enforces occasionally binding constraints, which arise from the nonnegativity constraint on vacancies ($V_t \geq 0$), debt satiation ($d_t \leq d_t^*$), and nonnegativity constraint on entrants ($S_t \geq (1-\delta)S_{t-1}$). Third, global solution methods enable us to quantify the extent to which amplification depends on the size of the shock.

I use the parameterized expectations algorithm, which was introduced by Den Haan and Marcet (1990). The conditional expectations, which here arise in the job creation condition and in the debt limit, are a function of Θ. The strategy is to approximate these conditional expectations by a polynomial function of the state variables.[1] Specifically, we represent the log of the conditional expectation with a polynomial in logs. Solving for the unknown conditional expectations function thus simplifies to calculating the polynomial coefficients. I start with a initial guess of coefficients for these functions, generate data from the model, and update the conditional expectations using nonlinear least squares. Consequently, the algorithm is interpretable as least squares learning.

[1] The rationale for using polynomial functions comes from the Weierstrass theorem, which asserts that for a continuous function $f(x)$ on $[a,b]$, there is a polynomial function $p_n(x)$ arbitrarily close to f: $\forall \varepsilon > 0\, \forall x \in [a,b], |f(x) - p_n(x)| < \varepsilon$.

Using moving bounds, proposed by Maliar and Maliar (2003), stabilizes the algorithm with respect to the initial choice of coefficients. These initial bounds are close to the steady state levels and are gradually removed until they no longer bind in subsequent iterations.

It turns out that the non-negativity constraint on vacancies binds less than 5% of the time, whereas the zero lower bound on firm entry binds over 44% of the time. I detail the solution procedure below.

1. The state space $\Theta_t = (N_{t-1}, S_{t-1}, z_t, \psi_t)$.
2. Consider the job creation condition:

$$\frac{k}{f(\theta_t)} - \lambda_t^V = \rho z_t p_t - w_t + \beta(1-\delta)(1-s)\mathbb{E}_t\left\{\frac{k}{f(\theta_{t+1})} - \lambda_{t+1}^V\right\} \tag{C.1}$$

   where λ_t^V is the Lagrangian multiplier on the nonnegativity constraint on vacancies. Rearrange the job creation condition as a function of prices and market tightness using the wage equation:

$$\rho z_t p_t(1-\chi) = (1-\chi)(b+l) + \frac{k}{f(\theta_t)} - \lambda_t^V + [\chi\beta h(\theta_t) - \beta(1-\delta)(1-s)]\mathbb{E}\left[\frac{k}{f(\theta_{t+1})} - \lambda_{t+1}^V\right] \tag{C.2}$$

   The debt level equals $d_t = S_t p_t q_t = (1-u_t)z_t p_t$. So, $p_t = d_t/[(1-u_t)z_t]$, which, using the law of motion for employment, becomes

$$\frac{\rho d_t(1-\chi)}{(1-s)(1-\delta)(1-u_{t-1}) + h(\theta_t)u_{t-1}} = (1-\chi)(b+l) + \frac{k}{f(\theta_t)} - \lambda_t^V \tag{C.3}$$

$$+ [\chi\beta h(\theta_t) - \beta(1-\delta)(1-s)]\mathbb{E}\left[\frac{k}{f(\theta_{t+1})} - \lambda_{t+1}^V\right] \tag{C.4}$$

3. Consider the debt limit equation:

$$\overline{d}_t/\psi_t = \beta\mathbb{E}\left\{A(S_{t+1}^{1/\rho}q_{t+1})^{1-\varepsilon}/(1-\varepsilon) - S_{t+1}p_{t+1}q_{t+1} + \lambda\overline{d}_{t+1}/\psi_{t+1}\right\} \tag{C.5}$$

   Write the conditional expectations as functions of the state variables and the coefficients:

$$G(\Theta_t, \beta_1) \equiv \mathbb{E}\left[\frac{k}{f(\theta_{t+1})} - \lambda_{t+1}^V\right] = \exp(X_t'\beta_1)$$
$$H(\Theta_1, \beta_2) \equiv \beta\mathbb{E}\left\{\Omega_{t+1} + \lambda\overline{d}_{t+1}/\psi_{t+1}\right\} = \exp(X_t'\beta_2)$$

   The following fact is useful in solving (C.3) for θ. Define $F_1(\theta) = \rho d_t(1-\chi)/[(1-s)(1-\delta)(1-u_{t-1}) + h(\theta_t)]$ and define $F_2(\theta) = (1-\chi)(b+l) + k/f(\theta_t) + [\chi\beta h(\theta_t) - \beta(1-\delta)(1-s)]G(\theta_t, \beta_2)$. Note that F_1 is the left hand side of (C.3) and F_2 is the right-hand side of (C.3) without the Lagrangian multiplier. Observe that F_1 is decreasing, with an upper bound $F_1(0) = \rho d_t(1-\chi)/[(1-s)(1-\delta)(1-u_{t-1})]$ and a greatest lower bound at $\rho d_t(1-\chi)/[(1-s)(1-\delta)(1-u_{t-1})+u_{t-1}]$. Moreover, F_2 is increasing with a lower bound at $F_2(0) = (1-\chi)(b+l)+k-\beta(1-\delta)(1-s)G_t$ and approaches infinite as $\theta \to \infty$. Under these conditions, the equation $F_1(\theta) = F_2(\theta)$ has a unique solution $\theta > 0$ if and only if

$$(1-\chi)(b+l)+k-\beta(1-\delta)(1-s)G_t < \frac{\rho d_t(1-\chi)}{(1-s)(1-\delta)(1-u_{t-1})} \tag{C.6}$$

Otherwise, from complementary slackness, $\theta_t = 0$ and λ_t^v can be recovered from (C.3) evaluated at $\theta = 0$.

4. Start at iteration $i = 1$.
5. Simulate a time series of length $T = 30,000 - 200$ as follows (burn-in of 200). Draw a length T of z_t.
   (a) Initialize S_1 and N_1 at steady-state values.
   (b) Initializing $H(\Theta_t, \beta_2)$, we obtain the debt limit $\overline{d}_t$. The polynomial generating vector X_t consists of the constant one, the first-order terms, the quadratic terms, and the six two-way cross-products of terms, for a total of 15 terms.
   (c) Conjecture that $d_t^0 = \overline{d}_t$, $\lambda_t^V = 0$, and recover $d_t = H_t\psi_t$
   (d) Restrict G_t and d_t to satisfy moving bounds: $\underline{G}_i \leq G_t \leq \overline{G}_i$ and $\underline{d}_i \leq d_t \leq \overline{G}_i$, where

$$\begin{aligned}
\underline{G}_i &= \frac{k}{f(\theta_{ss})}\exp(-ai) \\
\overline{G}_i &= \frac{k}{f(\theta_{ss})}\exp(2-\exp(ai)) \\
\underline{d}_i &= d_{ss}\exp(-ai) \\
\overline{d}_i &= d_{ss}(2-\exp(-ai))
\end{aligned}$$

   The parameter a controls the speed of moving the bounds and d_{ss} and θ_{ss} are the steady state values of debt and market tightness.
   (e) Compute θ_t and λ_t^V using (C.3) and complementary slackness.
   (f) Define $V_t = \theta_t u_{t-1}$.
   (g) Compute p_t from equation (C.2)
   (h) Calculate sellers: $S_t = \max\{(1+r)(1-\rho)d_t/(\gamma(r+\delta)), (1-\delta)S_{t-1}\}$.
   (i) Update unemployment: $u_t = [1-h(\theta_t)]u_{t-1} + [1-(1-s)(1-\delta)](1-u_{t-1})$
   (j) Let $q_t = (1-u_t)z_t/S_t$.

6. After simulating a series of length T, compute

$$
\begin{aligned}
y_t^1 &= \frac{k}{f(\theta_{t+1})} - \lambda_{t+1}^V \\
y_t^2 &= \beta \left\{ A(S_{t+1}^{1/\rho} q_{t+1})^{1-\varepsilon}/(1-\varepsilon) - S_{t+1} p_{t+1} q_{t+1} + \lambda \bar{d}_{t+1}/\psi_{t+1} \right\}
\end{aligned}
$$

7. Recover β_1^{new} and β_2^{new} using nonlinear least squares:

$$
\begin{aligned}
\beta_1^{new} &= \arg\min \frac{1}{T} \sum_{t=0}^{T} |y_t^1 - \exp(X_t' \beta_1)|^2 \\
\beta_2^{new} &= \arg\min \frac{1}{T} \sum_{t=0}^{T} |y_t^2 - \exp(X_t' \beta_2)|^2
\end{aligned}
$$

8. Update β_i as $\Gamma \beta_i^{new} + (1-\Gamma)\beta_i$ for $i = 1, 2$, with $\Gamma = 0.9$.
9. Update iteration i until $||\beta_1^{new} - \beta_1||/||\beta_1|| + ||\beta_2^{new} - \beta_2||/||\beta_2|| < 1e-6$.

After convergence, I check the conjecture that the first best is never attained. After the burn-in period, $d_t \leq d_t^*$ holds for each period of the simulation. The remaining variables are straightforward to recover.

Appendix D. Histograms

I simulate 30,000 draws of data and plot the histogram for the price level, unemployment, market tightness, the measure of sellers, real wages, debt, consumption, debt relative to consumption, and aggregate profits.

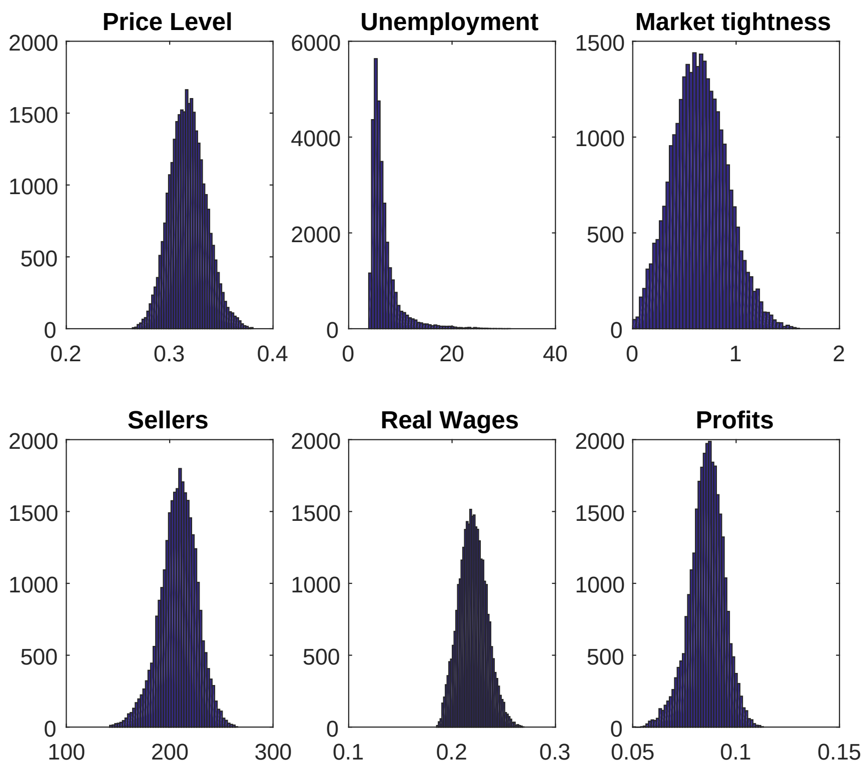

Figure D.3: Histograms of simulations

For this model, the lower bound on firm entry turns out to be much more important than the non-negativity constraint on vacancies. Figure D.4 indicates that the lower bound on entry binds 44% of the time, whereas the zero lower bound on hiring binds less than 5% of the time. Moreover,

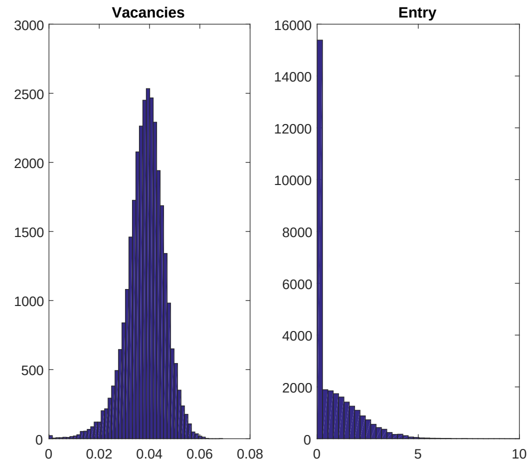

Figure D.4: Firm entry and vacancies

Figure D.5 shows that whenever the non-negativity constraint on vacancies does bind, there is no entry. Consequently, the theoretical concern with dispersion of firm size is not quantitatively relevant.

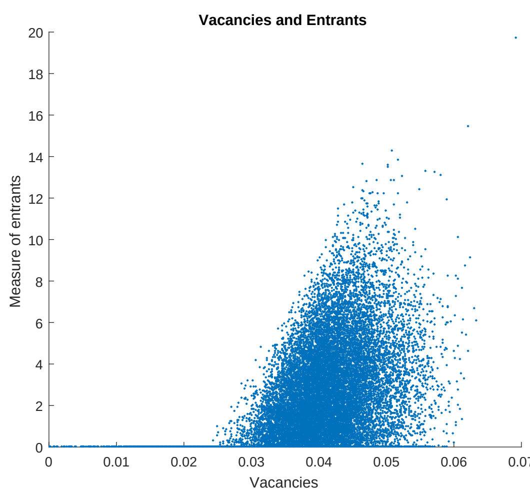

Figure D.5: Scatterplot: vacancies and number of entrants.

Appendix E. Monte Carlo Procedure for Generalized Impulse Responses

1. Fix the length of a series $T = 200$ and number of Monte Carlo iterations $N = 150$
2. Draw matrix E of normal random variables with standard deviation σ_z of size $T \times N$.
3. For each sequence $j \in \{1, \ldots, N\}$:
   (a) Let $z_1 = \omega + E_{1,j}$ and $\widehat{z}_1 = E_{1,j}$ for impulse ω
   (b) for $t = 2 : T$, define $z_t^j = \rho_z z_{t-1}^j + E_{t,j}$ and $\widehat{z}_t^j = \rho_z \widehat{z}_{t-1}^j + E_{t,j}$
   (c) Let $Y(z^j)$ be the simulated series under sequence z^j and $Y(\widehat{z}^j)$ be the simulated series under sequence $\widehat{z}^j$.
   (d) Compute the proportional deviation $y_j = (Y(z^j) - Y(\widehat{z}^j))/Y_0(\widehat{z}^j)$ where $Y_0(\widehat{z}^j)$ is the first element of the sequence $Y(\widehat{z}^j)$.
4. Compute the mean response $IRF = \frac{1}{N}\sum_{j=1}^{N} y_j$.

Appendix F. Nonlinear Effects of the Amplification Mechanism

We examine nonlinear effects of the amplification mechanism from two angles. The first checks if doubling the shock more than doubles the responses (percentage-wise). The second checks for differences in magnitudes of a negative versus positive productivity shock. Figure F.6 compares the impulse responses of a two-standard deviation credit shock to the high and low-product diversity economies. The percentage amplification is about 69%, compared to about 62% under a one-standard deviation shock. Therefore, the amplification mechanism of the model increases in importance with the size of the shock, which is consistent with **?**. Thus, the effects studied here are especially relevant for large shocks.

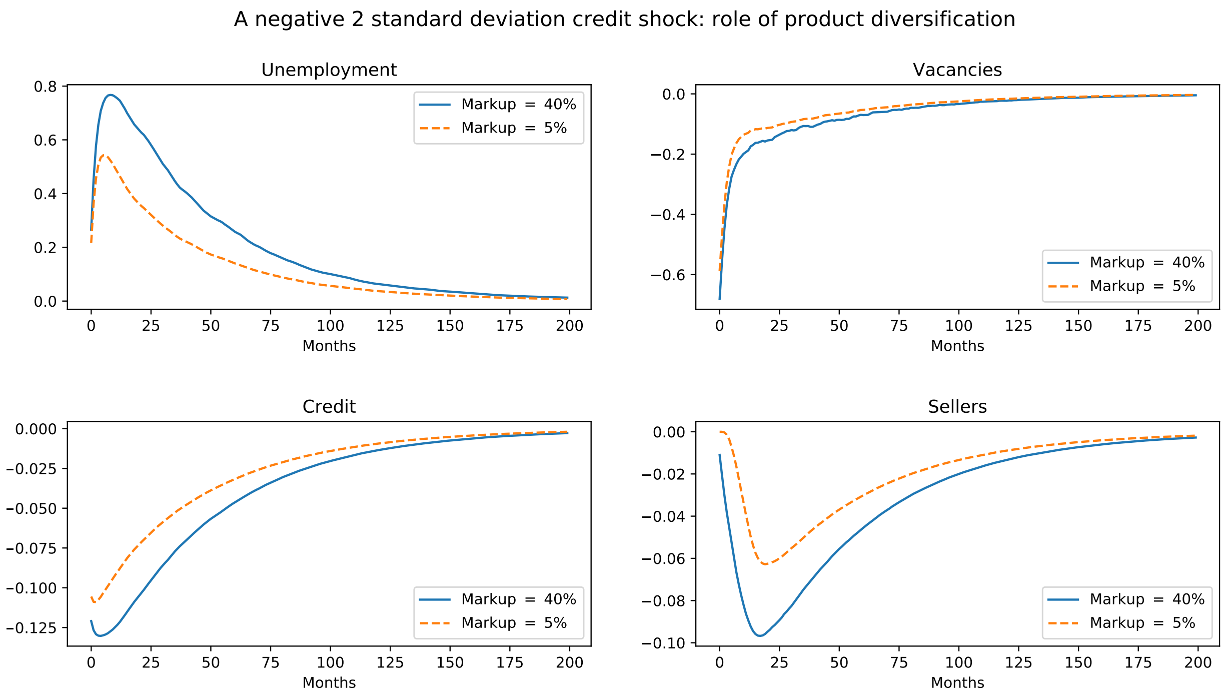

Figure F.6: Response of a two-standard deviation credit shock: markup comparison. Each variable is expressed in proportional deviations.

I also consider the effects of a one-standard deviation *positive* productivity shock. From the congestion effects of labor market matching, it is difficult to reduce unemployment significantly once it is already low. The concavity of the job finding rate implies that the unemployment rate changes asymmetrically to uniform movements in market tightness. Accordingly, we expect a positive productivity shock to have more modest effects on unemployment. However, unlike Bethune et al. (2015), the rise in aggregate demand depends on the change in product diversity, not of the matching rate in the goods market. Figure shows the (generalized) impulse responses in this case.

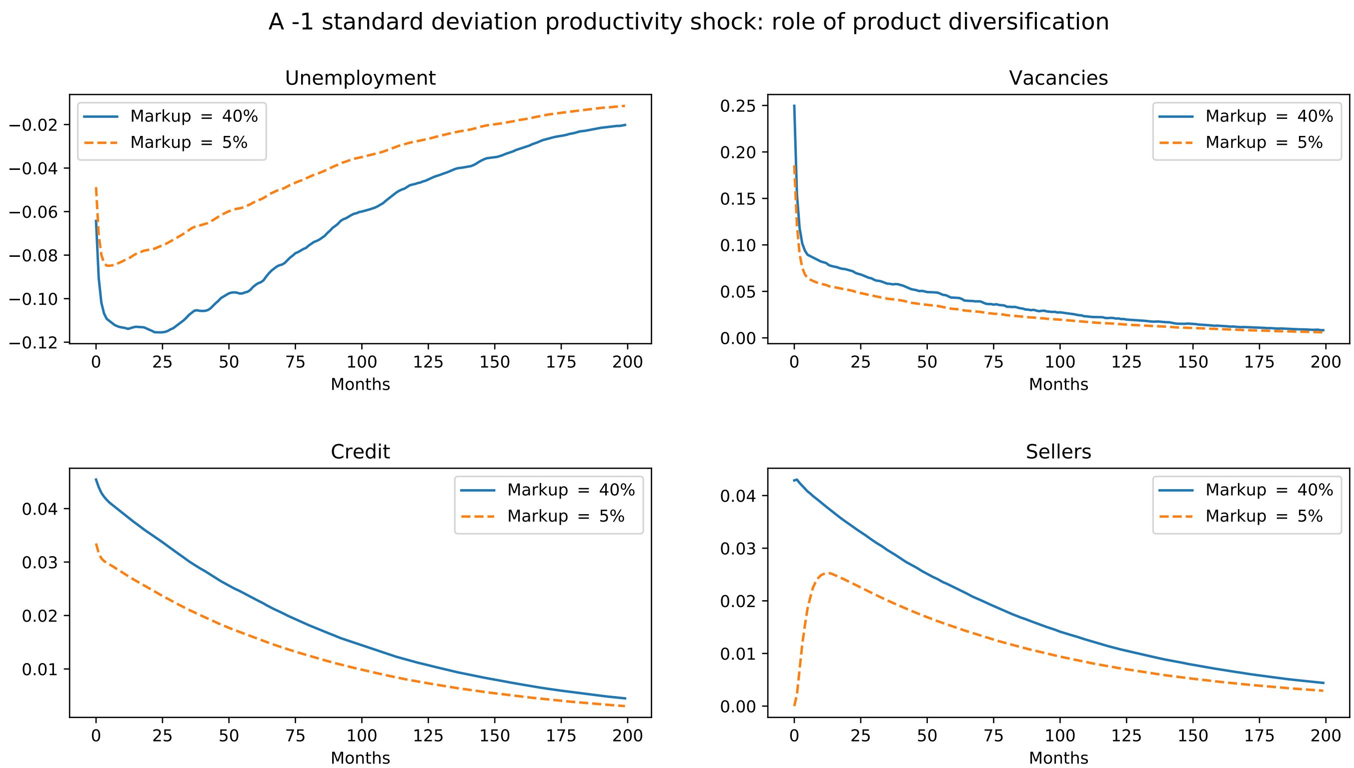

Figure F.7: Response to a positive unit standard deviation productivity shock: markup comparison. Each variable is expressed in proportional deviations.

Conditional on the markup, the effect on unemployment is smaller than with a negative productivity shock. Under 40% markups, there is a peak fall of 12%, compared to a peak rise of 16%. Under 5% markups, there is a peak fall of slightly over 8%, compared to a peak rise slightly over 10%. However, the general equilibrium effect of credit and product diversity remains substantial even with a positive shock. Comparing the cumulative responses, there is about 60% amplification. By comparison, Bethune et al. (2015) consider a rise in productivity as high as 6% and still find a negligible difference in unemployment between the partial equilibrium response holding credit fixed and the general equilibrium response.

Appendix G. Robustness Checks

Appendix G.1. Debt limit parameter $\overline{\psi}$

I consider the change on the simulated model moments of changing the target ζ to 0.3 from 0.1, which corresponds to $\overline{\psi} = 0.094$. I compare in two different ways. First, I take the same financial shocks as in the baseline case but solve the business cycle model under the new parameterization. Second, I both update the shocks and solve the model under the new values.

x	SD	Cor(x,u)	Cor(x,d)	Cor(x,x_{-1})	Cor(x,x_{-2})
u	0.391	1.0	-0.806	0.978	0.946
d	0.0793	-0.806	1.0	0.96	0.919
S	0.0712	-0.755	0.962	0.97	0.933
V	0.203	-0.586	0.849	0.808	0.688
z	0.0252	-0.328	0.415	0.957	0.914
ψ	0.0837	-0.619	0.9	0.957	0.914

Table G.2: Moments of key variables simulated from model with $\overline{\psi} = 0.094$, monthly. Data is transformed by proportional deviations and filtered by the Hamilton Method. The financial shocks are the same as the baseline specification.

From Table G.3, we see that unemployment and credit are even more volatile than the baseline parameterization, though product variety is somewhat less. Vacancies are also slightly more volatile.

I now consider the moments consistent with the updated financial shocks.

x	SD	Cor(x,u)	Cor(x,d)	Cor(x,x_{-1})	Cor(x,x_{-2})
u	0.368	1.0	-0.811	0.978	0.946
d	0.0762	-0.811	1.0	0.96	0.919
S	0.069	-0.766	0.964	0.97	0.933
V	0.194	-0.597	0.856	0.81	0.691
z	0.0252	-0.343	0.429	0.957	0.914
ψ	0.0799	-0.626	0.896	0.957	0.913

Table G.3: Moments of key variables simulated from model, monthly. Data is transformed by proportional deviations and filtered by the Hamilton Method. The financial shocks are constructed using the new parameterization.

Under the new financial shocks, the moments generally lie in between the baseline values and those solved under the new parameterization with the original financial shocks. For instance, the volatility of unemployment is 0.37, compared to 0.32 in the baseline and 0.39 under first robustness scenario. Similarity , the volatility of credit is 0.0762, compared to 0.062 in the baseline and 0.079 under the first robustness scenario. Similarly, the correlation between unemployment and credit is -0.811, compared to -0.823 in the baseline and -0.806 the first robustness scenario. The same holds for other moments. The new financial shocks have a slightly smaller standard deviation than in the baseline scenario.

Appendix G.2. Entry cost parameter γ

I report the results for calibrating the entry cost parameter γ at 10% of the target value; to satisfy the other targets, which leads to $\gamma = 0.0366$, and the demand parameter A drops to 0.0745. I keep the financial shocks the same as the baseline.

x	SD	Cor(x,u)	Cor(x,d)	Cor(x,x_{-1})	Cor(x,x_{-2})
u	0.332	1.0	-0.824	0.978	0.946
d	0.0672	-0.824	1.0	0.96	0.919
S	0.0618	-0.786	0.972	0.969	0.932
V	0.17	-0.604	0.847	0.804	0.682
z	0.0252	-0.386	0.492	0.957	0.914
ψ	0.0837	-0.601	0.858	0.957	0.914

Table G.4: Moments of key variables simulated from model with γ set to equal one tenth of the target (35.7% of annualized profits), monthly. Data is transformed by proportional deviations and filtered by the Hamilton Method.

The main difference is that the variables are slightly more volatile than in the baseline model, but the difference is insubstantial. The non-negativity constraint on vacancies binds slightly more often (at 5.96%), and the second-order autocorrelation of vacancies is 0.682 compared to 0.695 in the baseline.

Appendix H. References

Bethune, Z., Rocheteau, G., Rupert, P., 2015. Aggregate unemployment and household unsecured debt. Review of Economic Dynamics 18 (1), 77–100.

Den Haan, W. J., Marcet, A., 1990. Solving the stochastic growth model by parameterizing expectations. Journal of Business & Economic Statistics 8 (1), 31–34.

Maliar, L., Maliar, S., 2003. Parameterized expectations algorithm and the moving bounds. Journal of Business & Economic Statistics 21 (1), 88–92.