A Note on the Annuity Role of Estate Tax - ONLINE SUPPLEMENT*

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Abstract

In this paper, we extend the "A Note on the Annuity Role of Estate Tax" (Bishnu and Kumru (2020))by incorporating social security system, consumption tax, and endogenous labor supply. In all these extensions, we show that estate tax cannot play the role of annuities. We also conducted robustness analysis by employing frequently used specifications for bequest motives. We show that the conclusion we reached in the "A Note on the Annuity Role of Estate Tax" is robust to changes in bequest specifications.

Key words: Bequests; Estate Tax; Annuity.
JEL code: D15, E62, H21

1 Introduction

Bishnu and Kumru (2020) show that endogenization of bequest leads to a conclusion that lump sum estate taxation does not have an annuity role anymore. More precisely, they show that estate tax regime can implement the first best allocations by imposing taxes that are contingent on the level and timing of inheritance received by households. In this paper, we extend Bishnu and Kumru (2020)'s main model by incorporating social security, consumption tax, and endogenous labor supply one at a time. We also conduct robustness analysis by

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replacing the bequest motive specification in Bishnu and Kumru (2020) with the other types of bequest motives studied in the literature.

As in Bishnu and Kumru (2020), we use a two-period overlapping generations economy where each agent is subject to a survival risk[1] and can live up to two periods.[2] Agents work and receive labor income in the first period, retire and live on savings in the second period.

To generate a bridge between two distinct sets of literature that deal with the annuity role of estate taxation and social security separately, we extend the main model with a fully funded social security. We show that the social security program has only wealth effect and cannot affect the inter-temporal choice. The only way to rectify the inter-temporal choice is to use the estate tax. Yet, the estate tax still fails to play the annuity role: the only forms of lump-sum and estate tax regime that can restore the Laissez Faire allocations are those that are sensitive to the timing and size of inheritance. We also extend our model by incorporating a consumption tax. We observe that a combination of consumption and estate taxes do not provide meaningful annuities. As in the earlier cases, we show that the optimum level of estate and the lump sum taxes are both dependent on the size and the timing of inheritance received by the agent. Finally we show that endogenizing labor supply does not change our main result.

In order to verify how sensitive our results are to the choice of modeling bequests, a variety of different specification of models that are prominent and frequently used in the literature (four different versions of the model with received inheritance (main model) following Cremer and Pestieau (2006)) have been considered. In particular, we present a comprehensive set of possibilities of intended and accidental bequests for any two consecutive generations.[3] Later a model with dynastic altruism has also been analysed. It is observed that all the setups satisfactorily reconfirm Bishnu and Kumru (2020)'s main findings.

The rest of the paper is structured as follows. Section 2 introduces the model with social security, consumption tax, and endogenous labor supply. Section 3 provides a robustness analysis. Finally, section 4 concludes.

[1]Survival risk and bequests have been extensively studied in the literature. See Yaari (1965), Davies (1981), Abel (1985), Hurd (1989), Bernheim (1991), Cardia and Michel (2004), and Davidoff et al. (2005).

[2]Instead of a two period, a multi-period model can easily be constructed but we do not see any change in the arguments that we have provided above. Also it is straightforward to present our results in a continuous time setup.

[3]In a recent study, Lockwood (2018) showed that bequest motive significantly increases saving and decreases purchases of long term care insurance and annuities.

2 Extensions

In this section, we first extend the Model with Received Inheritance (main model) presented in Bishnu and Kumru (2020) by incorporating social security system, consumption tax, and endogenous labor supply one at a time. Afterwards, we conduct the robustness analysis by considering the frequently used specifications for bequest motives.

2.1 The Presence of Social Security

The reasons that social security is included in this discussion are as follows: 1) social security benefits are like annuity income and 2) estate tax liabilities can implicitly provide annuity income, hence "estate tax is social security for the rich." Caliendo et al. (2014) show that social security has no meaningful annuity role once the inheritance is endogenized in the model. In Bishnu and Kumru (2020), we show that if inheritance is accounted for in the model, the estate tax does not have annuity role as well. This section reinforces the previous results by incorporating the social security into the model. We show that in an environment where private annuity markets are missing and social security is introduced, *neither social security nor estate tax* can provide meaningful annuities.

Suppose the government still aims at imposing per capita tax revenue R. On top of that, the government runs a balanced budget social security program: agent pays social security tax at the rate τ when young and receives social security benefits b when old. Also, agents pay a lump sum tax T. Once again start with ignoring the inheritance and assuming that annuity is present. The agent now solves the following maximization problem:

$$\max_{c_1,c_2,B_1,B_2,a,k} EU = u(c_1) + (1-p)v(B_1) + pu(c_2) + pv(B_2), \tag{1}$$

subject to

$$c_1 + a + k = y(1-\tau) - T, \;\; B_1 = k, \;\; c_2 + B_2 = \frac{a}{p} + k + b.$$

The self-balanced social security program has a budget constraint $y\tau = pb$ or simply $b = y\tau/p$. By inserting social security benefits b into the budget, the life time budget constraint of the agent can be written as:

$$c_1 + (1-p)B_1 + pc_2 + pB_2 = y - T.$$

This budget constraint is exactly the same as when there is no social security. Hence we confirm the result that when private annuity markets exist, social security does not improve

welfare. Next we assume away the annuity markets, where one may think that social security and estate tax can work together to restore the first best allocations by providing meaningful annuities. With the same objective function, the agent's budget constraints are as follows:

$$c_1 + k = y(1-\tau) - T; \;\; B_1 = k - E; \;\; c_2 + B_2 = k + b.$$

In order to restore the first order allocations, we can implement the following tax regime:

$$E = c^* - b \;\; and \;\; T = (1-\tau)y - 2c^* - B^* + b.$$

Note that with the social security, the estate tax decreases since the private saving, k is reduced to $B^* + c^* - b$. In the absence of social security, $k = B^* + c^*$. On the other hand, the lump sum tax increases to maintain the tax revenue and the budget i.e. $b - \tau y = \frac{y\tau}{p} - \tau y > 0$. The total revenue can thus still meet the balanced budget requirement:

$$\begin{aligned} T + (1-p)E &= y - 2c^* - B^* - \tau y + b + (1-p)(c^* - b) \\ &= y - 2c^* - B^* + (1-p)c^* = y - (1+p)c^* - B^* = R. \end{aligned}$$

The conclusion is that with the combination of social security, estate tax, and lump sum tax, the first best allocations can be implemented when the private annuity markets are missing. This is because the additional tax and transfer program (the social security) increases the government's capabilities of arranging allocations. However, once we incorporate inheritance into the model, we have three observations (I, II and III) below. While I and II have been presented in a setup where both annuities and social security are present, III is under the situation where estate tax replaces annuity. After stating each of the observations we briefly explain the reason behind obtaining that result.

Presence of Annuity:

Observation I: In the model with actuarially fair annuities and social security if there is inequality in inheritance received by agents, there will be inequalities in consumption and bequest left.

Showing this is straightforward. The budgets constraints of an agent who receives inheritance when young (n_1) are:

$$\begin{aligned} c_1(n_1) + a + k(n_1) = y(1-\tau) + H_1(n_1) - T, \;\; B_1(n_1) &= k(n_1) - E, \\ c_2(n_1) + B_2(n_1) &= \frac{a}{p} + k(n_1) + b. \end{aligned}$$

The budget constraints of an agent who receives inheritance when old (n_2) are:

$$c_1(n_2) + a + k(n_2) = y(1-\tau) - T, \;\; B_1(n_2) = k(n_2) - E,$$
$$c_2(n_2) + B_2(n_2) = \frac{a}{p} + k(n_2) + H_2(n_2) + b.$$

Since the two types of agents have different life time budget constraints, they have different levels of consumption and bequest left. Similar to what we established in Proposition 1, there will be inequality in this economy.

Observation II: In the model with annuities and social security, a uniform lump sum tax and estate scheme cannot generate equal allocations.

We clearly observe from I above that despite the existence of the actuarially fair annuity markets, once inheritance is accounted for and determined in the model, uniform social security and estate tax program cannot deliver equal allocations across agents. We omit a formal prove here since the exact logic presented in the case without social security is also applicable.

Presence of an Estate Tax but no Annuity:

Observation III: In the model without private annuities and social security, uniform estate tax cannot deliver the first best allocations.

Without the ability to buy annuities, whose returns are higher than the storage technology, one would think that social security and estate tax may act like annuities for different reasons. Social security pays benefits as long as the retirees survive and the social security benefits deliver the same return as annuities i.e. return on payroll taxes is the same as return on annuities. As Caliendo et al. (2014) show that once the social security is introduced, private saving would be dampened implying lower inheritance. This wealth effect neutralizes the gain from social security's provision of public annuities. Hence, the social security does not provide meaningful annuities. Kopczuk (2003) shows that combined with lump sum taxes, the estate tax can defer the tax liabilities to the end of one's life-span. By being in favor of the survivors, this delayed tax regime can act like annuities. Therefore, the estate tax can restore the first best allocations. As we showed earlier, when we take inheritance into account, the first best allocations include inequalities across agents. This is because agent choose to leave the equal amount of bequests no matter how long they live due to the strong bequest motive. Even though the amount of bequest left is the same, the timing of inheritance received matters from the off-springs' perspective. Earlier inheritance might be preferred to the later inheritance. The differential timing of the inheritance received thus creates differential wealth effects. These heterogeneous life time wealth in turn imply that agents consume at different levels and leave different levels of bequests. Hence, we can con-

clude that the differential timing and size of inheritance create inequalities despite the same labor income among all agents.

If the government's goal is to neutralize the differential wealth effects and implement equal allocations among all agents, then social security program alone can't accomplish the task. After all, the social security program is egalitarian by taxing agents equally and distributing benefits conditional on survival. The only instrument for equal allocations is thus lump sum and estate taxes. However, it is the redistribution role, not the annuity role, that can restore equal allocations. Notice that the redistribution role is not the scope of this study.

If the government's goal is to implement the Laissez Faire allocations where inequality exists, then similar to our earlier case where social security was not introduced, the government should impose lump sum and estate tax schemes that depend on the size and the timing of the bequest received by the agent. Following the same notations as above, type I agents have the following constraints

$$c_1(n_1) + k(n_1) = y(1-\tau) + H_1(n_1) - T(n_1), \;\; B_1(n_1) = k(n_1) - E(n_1),$$
$$c_2(n_1) + B_2(n_1) = k(n_1) + b.$$

Similarly, type II agents have the following constraints:

$$c_1(n_2) + k(n_2) = y(1-\tau) + T(n_2), \;\; B_1(n_2) = k(n_2) - E(n_2),$$
$$c_2(n_2) + B_2(n_2) = k(n_2) + b + H_2(n_2).$$

It can be shown that the design of the lump-sum and estate tax should be as follows:

$$E(n_1) = c^*(n_1) - b, \;\; T(n_1) = y - 2c^*(n_1) - B^*(n_1) + H_1(n_1) + b - \tau y,$$
$$E(n_2) = c^*(n_2) - b, \;\; T(n_2) = y - 2c^*(n_2) - B^*(n_2) + pH_2(n_2) + b - \tau y.$$

The social security program has only wealth implication and cannot affect the inter-temporal choice or Euler equation. The only way to rectify the inter-temporal choice is using the estate tax. However, this tax scheme has its limitation: although each agent's consumption and bequests are smooth across time, their tax burdens are not smooth. Both the lump sum and estate taxes depend on the timing and size of inheritance received by each agent. Hence unlike the uniform social security regime, estate tax regime is not egalitarian. This system

is not reminiscent to any current estate system. This result is similar to Proposition 1-3 in Bishnu and Kumru (2020).[4]

2.2 Consumption Tax

Here we explore whether a combination of estate and consumption taxes can provide meaningful annuities. We assume that the government imposes lump sum consumption taxes t_1 and t_2 in the first and second periods, respectively. The budget constraints for type I and type II agents are given as follows:

$$c_1(n_1) + t_1 + a + k(n_1) = y + H_1(n_1);\;\; B_1(n_1) = k;\;\; c_2(n_1) + t_2 + B_2(n_1) = \frac{a}{p} + k(n_1).$$
$$c_1(n_2) + t_1 + a + k(n_2) = y;\;\; B_1(n_2) = k;\;\; c_2(n_2) + t_2 + B_2(n_2) = \frac{a}{p} + k + H_2(n_2).$$

The constraints can be rewritten as follows:

$$c_1(n_1) + pc_2(n_1) + (1-p)B_1(n_1) + pB_2(n_1) = y + H_1(n_1) - (t_1 + pt_2),$$
$$c_1(n_2) + pc_2(n_2) + (1-p)B_1(n_2) + pB_2(n_2) = y + pH_2(n_2) - (t_1 + pt_2).$$

Since two different types of agents have different budget constraints, we reach the same conclusion as earlier i.e. a combination of estate and consumption taxes does not provide annuities. Now we can show that a proportional tax on consumption doesn't change the conclusion. Assume that the proportional tax rate on consumption is demoted by t. We can write the budget constraints for type I and type II agents are

$$(1+t)(c_1(n_1) + pc_2(n_1)) + (1-p)B_1(n_1) + pB_2(n_1) = y + H_1(n_1),$$
$$(1+t)(c_1(n_2) + pc_2(n_2)) + (1-p)B_1(n_2) + pB_2(n_2) = y + pH_2(n_2),$$

respectively. Using the same argument above, we can state that the estate tax cannot provide meaningful annuities.

[4]Since our focus is solely on analysing the annuity role of estate taxation, we did not incorporate a few interesting features such as fertility and longevity closely related to social security and annuity. See Cigno (1998), Zhang and Zhang (2007), Boldrin et al. (2015), and Barnett et al. (2018) for fertility and see Philipson and Becker (1998), Andersen (2012), and Cipriani (2014) for longevity related issues. There is a growing literature that emphasizes the study of combining intergenerational transfers, instead of a separate analysis of the instrument (see for e.g., Boldrin and Montes (2005), Bishnu (2013), and Bishnu and Wang (2017)), but they are beyond the scope of this analysis.

2.3 Endogenous Labor Supply

Now we extend our model by incorporating labor supply decision. As we know from our earlier discussion, even when there was no heterogeneity in labor income, inequalities in consumption and bequest can be generated when we take inheritance into account. Variations in timing of inheritance received generates inequalities in the size of bequest and consumption allocations.

We want to emphasize two important channels. First, the differential wealth effects generated by the variations in timing of inheritance can influence labor supply decisions. Those who receive higher inheritance or receive inheritance early would work less. Second, the differences in hours worked affects consumption allocations and bequest left due to differences in income. Hence, one can expect that those who work less and thus receive less labor income tend to consume and bequeath less than otherwise. These two effects reinforce each other and can potentially reduce the level of inequality.

In the first period agents choose how much to work and receive labor income accordingly. In the second period, agents retire and stop working. The retirement decision is exogenous. The labor income when young is $y = wl$, where w is the market wage rate and l is the unit of labor supplied. We assume that the market wage rate is the same for everyone. The social security is fully funded and hence $wl\tau = pb$ for each agent. We follow the setup from the previous section and restrict ourselves to the actuarially fair annuity markets only. The agent chooses saving in annuity and storage technology, labor supply, consumption for both periods, and bequest to maximize the following expected life time utility:

$$\max_{c_1,c_2,l,a,k,B_1,B_2} EU = u(c_1) + h(1-l) + (1-p)v(B_1) + pu(c_2) + pu(B_2),$$

where h is a strictly increasing and strictly concave function. We normalize the time endowment as 1. When the agent receives inheritance early, the budget constraints can be written as follows:

$$\begin{aligned} c_1(n_1) + a + k(n_1) = wl(n_1)(1-\tau) + H_1(n_1) - T; \;\; B_1(n_1) = k; \\ c_2(n_1) + B_2(n_1) = \frac{a}{p} + k(n_1) + b(n_1). \end{aligned}$$

Hence, the life time budget constraint is as follows:

$$c_1(n_1) + pc_2(n_1) + (1-p)B_1(n_1) + pB_2(n_1) = wl(n_1)(1-\tau) + H_1(n_1) + pb(n_1) - T.$$

Noticing $wl\tau = pb$, the life time budget constraint can be rewritten as

$$c_1(n_1) + pc_2(n_1) + (1-p)B_1(n_1) + pB_2(n_1) = wl(n_1) + H_1(n_1) - T.$$

When the agent receives inheritance late, the budget constraints can be written as follows:

$$c_1(n_2) + a + k(n_2) = wl(1-\tau) - T, \;\; B_1(n_2) = k(n_2),$$
$$c_2(n_2) + B_2(n_2) = \frac{a}{p} + k(n_2) + b(n_2) + H_2(n_2).$$

Since $wl\tau = pb$, the lifetime budget constraint can be written as follows:

$$c_1(n_2) + pc_2(n_2) + (1-p)B_1(n_2) + pB_2(n_2) = wl(n_2)(1-\tau) + pH_2(n_2) + pb(n_2) - T.$$

The first order conditions with respect to consumption, leisure and bequest left are as follows:

$$u'(c_1) = u'(c_2), u'(c_1)w(1-\tau) = h'(1-l) \text{ and } v'(B_1) = v'(B_2).$$

It is clear from above that the same c^*, B^* and l^* can't be optimal for both problems. To implement the first best allocations in the absence of annuity, we should have the following constraints for type I and type II agents:

$$c_1(n_1) + k(n_1) = wl(n_1)(1-\tau) + H_1(n_1); B_1(n_1) = k(n_1) - E; c_2(n_1) + B_2(n_1) = k + b(n_1).$$
$$c_1(n_2) + k = wl(n_2)(1-\tau); B_1(n_2) = k(n_2) - E; c_2(n_2) + B_2(n_2) = k(n_2) + b(n_2) + H_2(n_2),$$

respectively. Then, we need to have a tax scheme as follows:

$$E_1(n_1) = c^*(n_1) - \tau wl^*(n_1) = c^*(n_1) - b(n_1); \;\; T(n_1) = y - 2c^*(n_1) - B^*(n_1) + H_1(n_1).$$
$$E_2(n_2) = c^*(n_2) - \tau wl^*(n_2) = c^*(n_2) - b(n_2); \;\; T(n_2) = y - 2c^*(n_2) - B^*(n_2) + pH_2(n_2).$$

As in the earlier cases, we show once again that the level of estate and the lump sum taxes are both dependent on the size and the timing of inheritance received by the agent. Also a point to note here is that unlike the previous case where labor supply is exogenous, b is not the same for two different types of agents.

3 Robustness

This section conducts sensitivity analysis with respect to the modeling choice of bequests. We employ a variety of frequently used bequest motive specifications presented in Cremer

and Pestieau (2006) and also check out our results in dynastic altruism setting for the completeness. We find that for an exhaustive set of modeling specifications, the main result in Bishnu and Kumru (2020) remains unchanged. The following four bequest motive specifications are exhaustive: 1) intended bequests left by parents and off-springs; 2) accidental bequests left by parents but intended bequests left by off-springs; 3) intended bequests left by parents but accidental bequests left by off-springs; and 4) accidental bequests left by parents and off-springs.

In the main model, there is a cohort and in this cohort agents differ in terms of their life time assets based on inheritance received. We find that the distribution of inheritance is non-stationary with the non-zero lowest level of bequests. In the main model, parents and off-springs are altruistic i.e. intended bequests left by parents and off-springs. That means our main model represents the case (1) and we already presented the results. Hence, we only check out the three remaining cases.

3.1 Accidental by Parents but Intended by Off-springs

Here we assume that any transfer from parents to their off-springs are accidental (unintended)[5] but the off-springs leave intended bequests. Since the off-springs are altruistic, they have the utility function as in equation 1 in Bishnu and Kumru (2020). We observe that our main result holds in this setup too. The modified budget constraints of type I agents who receive inheritance when they are young are as follows:

$$c_1(n_1) + a + k(n_1) = y - T + H_1(n_1), \;\; B_1(n_1) = k(n_1), \;\; c_2(n_1) + B_2(n_1) = \frac{a}{p} + k(n_1).$$

In this set up, type I agent is the only type that receives inheritance. That means, type II agent does not receive any inheritance. This is because the inheritance is received only when the parents die early i.e. the accidental bequests left by parents. So the budget constraints for the type II are as follows:

$$c_1(n_2) + a + k(n_2) = y - T, \;\; B_1(n_2) = k(n_2), \;\; c_2(n_2) + B_2(n_2) = \frac{a}{p} + k(n_2).$$

[5] Fuster (1999) shows that the existence of accidental bequests may lead to asymptotic growth and which is impossible in an economy with a perfect annuity market.

Since parents who live two periods do not leave bequests, $H_2(n_2) = 0$ in this case. The life time budget constraints for type I and II agents can be written as

$$c_1(n_1) + pc_2(n_1) + (1-p)B_1(n_1) + pB_2(n_1) = y - T + H_1(n_1),$$
$$c_1(n_2) + pc_2(n_2) + (1-p)B_1(n_2) + pB_2(n_2) = y - T,$$

respectively. It is then easy to verify that a uniform tax rate does not work and hence the main result clearly holds. However, interestingly, an inequality within a cohort is created here not because of differential timing of bequests received, rather because of only a fraction of agents in the economy inherits bequests.

3.2 Intended by Parents but Accidental by Off-springs

When bequests leaving is purely unintended (accidental), we need an upper bound on the amount of annuity purchase. The reason is as follows. When a perfect annuity is available, agents have no incentive to save. As argued in the literature, agents without a bequest motive would annuitize all their wealth unless there is some legal restrictions (see Yaari (1965), Sheshinski and Weiss (1981), Abel (1985), and Pecchenino and Pollard (1997) among others).[6]

Let's denote the upper bound of annuity purchase by $\tilde{a}$ that represents the maximum amount of annuity allowed by the government. Agents save k over and above the annuity purchase $\tilde{a}$ for second period consumption. The amount of accidental bequest left by the off-springs is represented by B_{ac}. Agents do not derive any utility from leaving accidental bequests. Thus the cohort who leaves accidental bequest should have a utility function of the following form:

$$\max_{c_1,c_2,k} u(c_1) + pu(c_2). \tag{2}$$

This can be seen as a modification of the utility specification in equation 1 in Bishnu and Kumru (2020), where $v(B_1)$ and $v(B_2)$ are assumed to be zero. If we continue with the assumption of gross rate of interest equals unity and the agents in the cohort under consideration die early, the saving that they have accumulated, precisely $(1-p)k$, becomes accidental inheritance B_{ac} for their children. The budget constraints for type I and type II

[6] Chakraborty (2004) also considers that entire saving is annuitized in a model where health investment influences the survival probability p.

are

$$c_1(n_1) + \tilde{a} + k(n_1) = y - T + H_1(n_1), \;\; B_{ac}(n_1) = k(n_1), \;\; c_2(n_1) = \frac{\tilde{a}}{p} + k(n_1),$$
$$c_1(n_2) + \tilde{a} + k(n_2) = y - T, \;\; B_{ac}(n_2) = k(n_2), \;\; c_2(n_2) = \frac{\tilde{a}}{p} + k(n_2) + H_2(n_2),$$

respectively. These can be equivalently written as

$$c_1(n_1) + pc_2(n_1) + (1-p)k(n_1) = y - T + H_1(n_1),$$
$$c_1(n_2) + pc_2(n_2) + (1-p)k(n_2) = y - T + pH_2(n_2),$$

for type I and II agents respectively. It is then straightforward to verify that our results hold.

3.3 Accidental by Parents and Off-Springs

Under this scenario, both parents and off-springs are not altruistic and therefore any bequest received and left is purely unintended. As in case (3), we assume that the annuity purchase is restricted and it is denoted by $\tilde{a}$. Similarly, the accidental bequest is represented by B_{ac}.

The cohort who leaves accidental bequests has the same utility function as in above subsection.

$$\max_{c_1, c_2, k} u(c_1) + pu(c_2). \tag{3}$$

Type I agents who receive inheritances due to sudden death of their parents now face the following budget constraints:

$$c_1(n_1) + \tilde{a} + k(n_1) = y - T + H_1(n_1); \;\; c_2(n_1) = \frac{\tilde{a}}{p} + k(n_1).$$

Type II agents do not receive any bequests since their parents do not die early. Their budget constraints are as follows:

$$c_1(n_2) + \tilde{a} + k(n_2) = y - T; \;\; B_{ac}(n_2) = k(n_2); \;\; c_2(n_2) = \frac{\tilde{a}}{p} + k(n_2).$$

It is then straightforward to verify that the life time budget constraints for type I and type II agents are as follows:

$$c_1(n_1) + pc_2(n_1) + (1-p)k(n_1) = y - T + H_1(n_1); \; c_1(n_2) + pc_2(n_2) + (1-p)k(n_2) = y - T,$$

respectively. This confirms that our results hold in this framework too where both parents

and off-springs leave accidental bequests. To make our analysis complete, we now verify our results in dynastic altruism framework.

3.4 Dynastic Altruism

A typical generation-t agent cares about life-time welfare of their future generations. Hence, we can write maximization problem as follows:[7]

$$\max U_t = u_t + \beta U_{t+1},$$

where $u_t = u(c_1^t) + pu(c_2^t)$. Consumption by the t-th generation agents in the first and second periods are represented by c_1^t and c_2^t, respectively. The survival probability is denoted by p and the strength of altruism denoted by β. Notice that parents and off-springs have same p and β. Assuming there are no inter-vivos transfers, agents can choose bequests level B_1^t and B_2^t to indirectly affect the children's consumption decisions, while they receive bequest B_1^{t-1} or B_2^{t-1} from their own parents. The budget constraints are given as follows:

$$c_1^t + a^t + B_1^t = y^t - T^t + B_1^{t-1}; \;\; c_2^t + B_2^t = \frac{a^t}{p} + B_2^{t-1} + B_1^t.$$

The life cycle budget constraint can be written as follows:

$$c_1^t + (1-p)B_1^t + pc_2^t + pB_2^t = y^t - T^t + B_1^{t-1} + pB_2^{t-1}. \tag{4}$$

We can clearly observe that there are still two types of t-th generation parent: those who receive bequest early and late. Interestingly, independent of whether the bequest motive is operative or not, our results hold.

When bequest motive is operative for all the generations, two income groups can be easily created based on the timing of parents' death. More precisely, type I agents who receive early inheritance have $B_2^{t-1} = 0$ in their budget constraint (4). On the other hand, Type II agents have the budget equation (4) with $B_1^{t-1} = 0$.

When bequest motive is inoperative for all, neither inheritance is received nor bequest is left. This particular case is not of our interest in this study. However, if we assume that some parents do not survive for both periods, there will be some agents who receive inheritance and some who do not. This in turn generates an inequality. That means, individuals whose parents accidentally leave bequests have different life time income compared to those who do not receive any bequests at all. More importantly, even if the bequests motives switch from

[7] See Barro (1974), Becker and Tomes (1979), and Becker and Tomes (1986).

operative (inoperative) to inoperative (operative) from one generation to other, the same logic stated above can be applied.

4 Conclusion

Bishnu and Kumru (2020) show that once the bequest income is determined by a uniform estate tax, it no longer bears the annuity role. The presence of social security which is often believed to play the role of annuity, cannot change this outcome. We also show that Bishnu and Kumru (2020)'s main result still holds when we incorporate consumption tax and endogenous labor supply. Finally, we show that the results presented in Bishnu and Kumru (2020) are robust to many different bequest specifications.

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