itive in low income countries. For the majority of high income countries it is significantly negative. In addition, the elasticity decreases when the credit-to-GDP ratio is higher. So much so, that in countries with a low credit-to-GDP ratio GDP per capital growth increases the saving rate while in countries with a high credit-to-GDP ratio the opposite is the case.

To explain the empirical findings we build a model in which entrepreneurs are credit constrained and investment projects are indivisible. The credit constraint creates rents for entrepreneurs. The indivisible investment size does not permit all agents to obtain credit to finance entrepreneurial activities. This creates dynamic incentives for entrepreneurs to save more and rely less on external funds. The resulting saving behavior of entrepreneurs generates the relationship between GDP per capita growth, the national saving rate and the credit constraint. We present supporting evidence for our theoretical findings by utilizing cross-country time series data of the number of new businesses registered and the corporate saving rate.

A A Cobb-Douglas Example

Suppose that the production function is Cobb-Douglas, i.e, $f(k) = k^\alpha$ where $\alpha \in (0,1)$. It follows that $w(k) = (1-\alpha)k^\alpha$, $R^+ = (\frac{2}{1-\alpha})^{\frac{1}{\alpha}}$ and $w'(0) = \infty$. This implies that the corner steady state is always locally unstable and there exists either a unqiue interior steady state or an odd number of interior steady states, which solve $\Pi(w, \lambda) = R$ where

$\Pi(w,\lambda) = \frac{w^{-1}(w)}{s(w,\lambda)w}$. If

$$\frac{w\Pi_1(w,\lambda)}{\Pi(w,\lambda)} = \frac{1-\alpha}{\alpha} - \frac{ws_1(w,\lambda)}{s(w,\lambda)} > 0 \tag{17}$$

i.e., if the elasticity of output is small relative to the elasticity of saving ($\alpha < 1/2$ is sufficient), $\Pi(w,\lambda)$ is monotonically increasing in w and thus there exists a unique interior steady state. Let $w^*(R,\lambda)$ denote the unique steady state. Suppose that $w^*(R,\lambda) > 1-\lambda$. If $w_0 < 1-\lambda$, then the saving rate s_t first increases and then decreases as w_t (or y_t) converges to the steady state in the long run.

B Remaining Proofs

We eliminate time subscripts for notational convenience.

Proof of Proposition 1: Let

$$s_1^b = \tfrac{1}{2}\left(1 - \tfrac{\phi-1}{w}\right) \quad \text{and} \quad s_2^b = \tfrac{1-\lambda\phi}{w}. \tag{18}$$

We can easily verify that $s = s_1^b$ solves the unconstrained optimization problem of entrepreneurs

$$U^b = \max_{s\in[0,1]} \left\{ (1-s)\left(\frac{\phi-1}{w} + s\right) \right\}. \tag{19}$$

If $w \geq 1-(2\lambda-1)\phi$, then $s_1^b \geq s_2^b$ and thus entrepreneurs can overcome the credit constraint if their saving rate is s_1^b. In such case, $U^b = \frac{1}{4}(1+\frac{\phi-1}{w})^2$.

If $w \in [1-\lambda\phi, 1-(2\lambda-1)\phi)$, then $s_1^b < s_2^b \leq 1$ and thus entrepreneurs can overcome the credit constraint if their saving rate is s_2^b. In such case, $U^b = (1 - \frac{1-\lambda\pi}{w})\frac{(1-\lambda)\phi}{w}$.

If $w < 1 - \lambda\phi$, then $s_1^b < 1 < s_2^b$ and thus entrepreneurs cannot overcome the credit constraint even if they save their entire wage. ■

Proof of Proposition 2: If $w \geq 1 - (2\lambda - 1)\phi$, then it follows from (8) that $U_t^b = U^\ell \Leftrightarrow \phi = 1$. Hence, $w \geq 1 - (2\lambda - 1)\phi \Leftrightarrow w \geq 2(1 - \lambda)$. If $w \in [1 - \lambda\phi, 1 - (2\lambda - 1)\phi)$, then it follows from (8) that $U_t^b = U^\ell \Leftrightarrow \phi = \frac{1}{2\lambda}\left(1 - w + \sqrt{1 - 2w + \frac{w^2}{1-\lambda}}\right)$. Hence, $w \in [1 - \lambda\phi, 1 - (2\lambda - 1)\phi) \Leftrightarrow w \in [0, 2(1 - \lambda))$. ■

Lemma 1. *(a) For $\lambda \in (0,1)$, the entrepreneurial rent $\phi(w,\lambda)$ is a continuous and strictly decreasing function on $w \in (0, 2(1-\lambda))$ and satisfies the following boundary properties*

$$\lim_{w\downarrow 0} \phi(w,\lambda) = \frac{1}{\lambda} \quad \textit{and} \quad \lim_{w\uparrow 2(1-\lambda)} \phi(w,\lambda) = 1. \tag{20}$$

(b) For $w \in (0, 2(1-\lambda))$, $\phi(w,\lambda)$ is a strictly decreasing function while $\lambda\phi(w,\lambda)$ is a strictly increasing function on $\lambda \in (0,1)$.

Proof of Lemma 1: If $\lambda \in (0,1)$ and $w < 2(1-\lambda)$, then the entrepreneurial rent is

$$\phi(w,\lambda) = \frac{1-w+\psi(w,\lambda)}{2\lambda} \quad \text{where} \quad \psi(w,\lambda) := \sqrt{1 - 2w + \frac{w^2}{1-\lambda}}. \tag{21}$$

(a) Differentiating both sides of (21) with respect to w and re-arranging terms, we obtain

$$\phi_1(w,\lambda) = \frac{1}{\psi(w,\lambda)}\left(\frac{w}{2(1-\lambda)} - \phi(w,\lambda)\right) < 0 \tag{22}$$

because when $w \in (0, 2(1-\lambda))$, $\frac{w}{2(1-\lambda)} < 1 < \phi(w,\lambda)$ and $\psi(w,\lambda) \in (1, 1/\lambda)$. This implies monotonicity of $w \mapsto \phi(w,\lambda)$. Taking limits of both sides of (21), we obtain the boundary properties of ϕ, which along with $\phi(w,\lambda) \equiv 1$ for $w \geq 2(1-\lambda)$ imply continuity of ϕ.

(b) Differentiating both sides of (21) with respect to λ and re-arranging terms, we obtain

$$\frac{\lambda\phi_2(w,\lambda)}{\phi(w,\lambda)} = \frac{w^2}{4(1-\lambda)^2}\frac{1}{\psi(w,\lambda)\phi(w,\lambda)} - 1 \in (-1,0) \tag{23}$$

because when $w \in (0, 2(1-\lambda))$, $\frac{w}{2(1-\lambda)} < 1 < \phi(w,\lambda)$ and $\psi(w,\lambda) \in (1, 1/\lambda)$. The monotonicity properties of $\lambda \mapsto \phi(w,\lambda)$ and $\lambda \mapsto \lambda\phi(w,\lambda)$ are implied by (23). ■

Lemma 2. *(a) For $\lambda \in (0,1)$, the saving rate of entrepreneurs $s^b(w,\lambda)$ is a strictly decreasing function on $w \in (0, 2(1-\lambda))$ and satisfies the following boundary properties*

$$\lim_{w\downarrow 0} s^b(w,\lambda) = 1 \quad \textit{and} \quad \lim_{w\uparrow 2(1-\lambda)} s^b(w,\lambda) = \tfrac{1}{2}. \tag{24}$$

(b) For $w \in (0, 2(1-\lambda))$, $s^b(w,\lambda)$ is a strictly decreasing function on $\lambda \in (0,1)$.

Proof of Lemma 2: (a) In equilibrium $U^b = U^\ell \Leftrightarrow$

$$\left(1 - \tfrac{1}{w} + \tfrac{\lambda\phi(w,\lambda)}{w}\right)\tfrac{(1-\lambda)\phi(w,\lambda)}{w} = \tfrac{1}{4} \quad \Leftrightarrow \quad \tfrac{1-\lambda\phi(w,\lambda)}{w} = 1 - \tfrac{w}{4(1-\lambda)\phi(w,\lambda)}. \tag{25}$$

Monotonicity and boundary properties of $w \mapsto \phi(w,\lambda)$ with (24) imply monotonicity and boundary properties of $w \mapsto s^b(w,\lambda)$.

(b) Monotonicity of $\lambda \mapsto s^b(w,\lambda)$ follows from Lemma 1. ■

Lemma 3. *(a) For $\lambda \in (0,1)$, the national saving rate*

$$s(w,\lambda) \equiv \begin{cases} \frac{1}{w+2\lambda\phi(w,\lambda)} & \text{if } w < 2(1-\lambda) \\ \frac{1}{2} & \text{if } w \geq 2(1-\lambda) \end{cases} \tag{26}$$

first increases and then decreases on $w \in (0, 2(1-\lambda))$ achieving its local maximum at $w = 1-\lambda$ and satisfying the boundary properties

$$\lim_{w\downarrow 0} s(w,\lambda) = \lim_{w\uparrow 2(1-\lambda)} s(w,\lambda) = \tfrac{1}{2} \quad \textit{and} \quad \lim_{w\to 1-\lambda} s(w,\lambda) = \tfrac{1}{\lambda}. \tag{27}$$

(b) For $\lambda \in (0,1)$, the fraction of entrepreneurs $\pi(w,\lambda) = s(w,\lambda)w$ is an increasing function on $w > 0$ satisfying the boundary properties

$$\lim_{w\downarrow 0} \pi(w,\lambda) = \tfrac{1}{2} \quad \textit{and} \quad \lim_{w\uparrow 2(1-\lambda)} \pi(w,\lambda) = \tfrac{\lambda}{2}. \tag{28}$$

(c) For $w \in (0, 2(1-\lambda))$, $s(w,\lambda)$ and $\pi(w,\lambda)$ are both strictly decreasing functions on $\lambda \in (0,1)$.

Proof of Lemma 3: (a) It follows from (21) and (26) that

$$s(w,\lambda) = \frac{1}{w+2\lambda\phi(w,\lambda)} = \frac{1}{1+\psi(w,\lambda)} \tag{29}$$

where ψ is defined in (21). Differentiating both sides of (29) and using the definition of ψ, we obtain

$$\frac{ws_1(w,\lambda)}{s(w,\lambda)} = \frac{[s(w,\lambda)]^2 w}{1-s(w,\lambda)}\left(1-\frac{w}{1-\lambda}\right) \quad \text{and} \quad \frac{\lambda s_2(w,\lambda)}{s(w,\lambda)} = -\frac{\lambda[s(w,\lambda)]^2}{2(1-s(w,\lambda))}\left(\frac{w}{1-\lambda}\right)^2 \tag{30}$$

where $s_1(w,\lambda) := \frac{\partial s(w,\lambda)}{\partial w}$ and $s_2(w,\lambda) := \frac{\partial s(w,\lambda)}{\partial \lambda}$. This implies that $s(w,\lambda)$ is strictly increasing on $w \in (0, 1-\lambda)$ and decreasing on $w \in (1-\lambda, 2(1-\lambda))$. This with the boundary properties of $s(w,\lambda)$ implies that the national saving rate is hump-shaped on $w \in (0, 2(1-\lambda))$ achieving its maximum at $w = 1-\lambda$.

(b) It follows from the definition of the national saving rate that

$$\pi(w,\lambda) = \frac{w}{w + 2\lambda\phi(w,\lambda)} = \frac{1}{1 + \frac{2\lambda\phi(w,\lambda)}{w}}. \tag{31}$$

Monotonicity of $w \mapsto \frac{\phi(w,\lambda)}{w}$ implies that $w \mapsto \pi(w,\lambda)$ is a strictly increasing function. In addition

$$\pi_1(w,\lambda) = s(w,\lambda)\left(1 - \frac{s(w,\lambda)w}{\psi(w,\lambda)}\left(\frac{w}{1-\lambda} - 1\right)\right). \tag{32}$$

This with the boundary properties of $s(w,\lambda)$ implies the boundary properties of $\pi(w,\lambda)$.

(c) Monotonicity of $\lambda \mapsto s(w,\lambda)$ and $\lambda \mapsto \pi(w,\lambda)$ follows from Lemma 1 and from definitions of s and π. ■

C Time Discount and Flexible Investment Size

This section shows that we can relax our assumptions of a zero time discount and a fixed investment size and still obtain essentially the same results. Suppose the agent's lifetime utility is $\ln c_{1t} + \beta \ln c_{2t+1}$ and that capital is produced by the following technology

$$F(i_t) = \begin{cases} 0 & \text{if } i_t < I \\ Ri_t & \text{if } i_t \geq I \end{cases}$$

where i_t is the investment of the final good, $F(i_t)$ is the produced amount of capital, and I is the minimum investment size. The lifetime utility of investors is $\ln U^\ell + \ln(w_t^{1+\beta} r_{t+1}^\beta)$ where $U^\ell = \max_{s\in[0,1]}\{(1-s)s^\beta\}$. This implies that $s^\ell = \frac{\beta}{1+\beta}$ and $U^\ell = \frac{\beta^\beta}{(1+\beta)^{1+\beta}}$. The lifetime utility of entrepreneurs is $\ln U^b(w_t/I, \phi_{t+1}) + \ln(w_t^{1+\beta} r_{t+1}^\beta)$ where

$$U^b(w_t/I, \phi_{t+1}, \lambda) = \max_{s\in[0,1]} \left\{ (1-s)\left(\frac{\phi_{t+1}-1}{w_t/I} + s\right)^\beta \middle| \, s \geq \frac{1-\lambda\phi_{t+1}}{w_t/I} \right\}.$$

This implies that the optimal saving rate of entrepreneurs is

$$s_t^b = \max\left\{ \frac{1}{1+\beta}\left(\beta - \frac{\phi_{t+1}-1}{w_t/I}\right), \frac{1-\lambda\phi_{t+1}}{w_t/I} \right\}$$

and

$$U^b(w/I, \phi, \lambda) = \begin{cases} \frac{\beta^\beta}{(1+\beta)^{1+\beta}}\left(1 + \frac{\phi-1}{w/I}\right)^{1+\beta} & \text{if } \frac{w}{I} \geq 1 - \frac{(1+\beta)\lambda - 1}{\beta}\phi \\ \left(1 - \frac{1-\lambda\phi}{w/I}\right)\left(\frac{(1-\lambda)\phi}{w/I}\right)^\beta & \text{if } \frac{w}{I} \in \left[1 - \lambda\phi, 1 - \frac{(1+\beta)\lambda-1}{\beta}\phi\right). \end{cases}$$

If $\frac{w_t}{I} < 1 - \lambda\phi_{t+1}$, then young agents cannot become an entrepreneur because they cannot overcome the credit constraint even if they save the entire wage. The equilibrium entrepreneurial rent is $\phi_{t+1} = 1$ when $\frac{w_t}{I} \geq \frac{(1+\beta)(1-\lambda)}{\beta}$ and $\phi_{t+1} = \phi(\frac{w_t}{I}, \lambda)$, which solves

$$\left(1 - \frac{1-\lambda\phi_{t+1}}{w_t/I}\right)\left(\frac{(1-\lambda)\phi_{t+1}}{w_t/I}\right)^{\beta} = \frac{\beta^{\beta}}{(1+\beta)^{1+\beta}}$$

when $1 - \lambda\phi_{t+1} \leq \frac{w_t}{I} < \frac{(1+\beta)(1-\lambda)}{\beta}$. There is no closed form solution of the above equation when $\beta \neq 1$. However, we can show that the properties of ϕ demonstrated in Lemma 1 hold for $\beta \neq 1$ as well. The credit market clears when

$$\frac{s_t w_t}{I}\left(I - s_t^b w_t\right) = \left(1 - \frac{s_t w_t}{I}\right)\frac{\beta w_t}{1+\beta}.$$

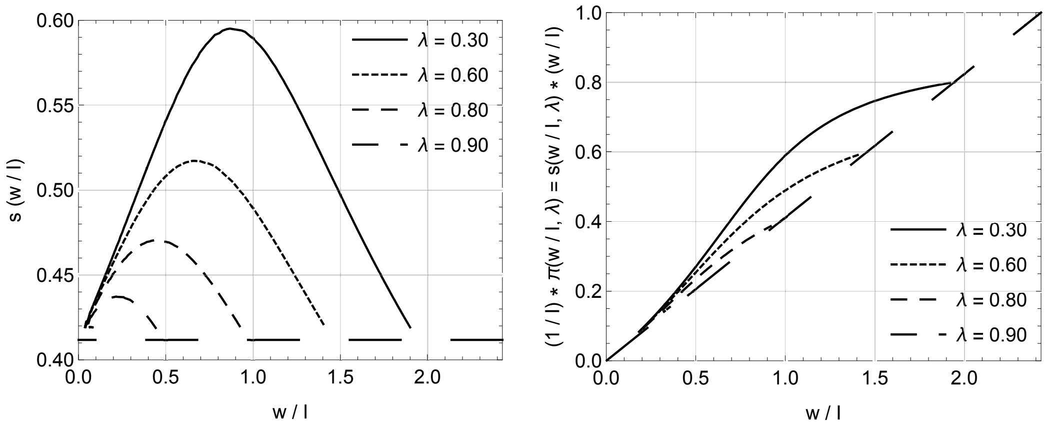
The saving rate of entrepreneurs is

$$s^b\left(\frac{w_t}{I}, \lambda\right) = \begin{cases} \frac{1-\lambda\phi(w_t/I,\lambda)}{w_t/I} & \text{if } \frac{w_t}{I} < \frac{(1+\beta)(1-\lambda)}{\beta} \\ \frac{\beta}{1+\beta} & \text{if } \frac{w_t}{I} \geq \frac{(1+\beta)(1-\lambda)}{\beta}. \end{cases}$$

The fraction of entrepreneurs is

$$s\left(\frac{w_t}{I}, \lambda\right) = \begin{cases} \frac{\beta}{\beta\frac{w_t}{I} + (1+\beta)\lambda\phi(w_t/I,\lambda)} & \text{if } \frac{w_t}{I} < \frac{(1+\beta)(1-\lambda)}{\beta} \\ \frac{\beta}{1+\beta} & \text{if } \frac{w_t}{I} \geq \frac{(1+\beta)(1-\lambda)}{\beta}. \end{cases}$$

Figure 4 shows the national saving rate and the fraction of entrepreneurs when $\beta = 0.70$. The figure indicates that the properties of the saving rate hold under a more general specification of the basic model.



(a) $s\left(\frac{w_t}{I}, \lambda\right)$ (b) $\frac{1}{I}\pi\left(\frac{w_t}{I}, \lambda\right) = s\left(\frac{w_t}{I}, \lambda\right)\frac{w_t}{I}$

Figure 4: The national saving rate and the fraction of entrepreneurs when $\beta = 0.70$.

D Tables

Country	GDP p.c. (y) [in Thousands]	Credit/GDP (λ)	Country	GDP p.c. (y) [in Thousands]	Credit/GDP (λ)
Albania	3.171	0.092	Indonesia	3.142	0.319
Algeria	2.912	0.333	Iran	4.22	0.227
Angola	3.053	0.049	Ireland	13.204	0.604
Argentina	6.986	0.185	Israel	10.699	0.566
Armenia	5.062	0.079	Italy	13.643	0.644
Australia	14.364	0.501	Jamaica	4.439	0.238
Austria	14.83	0.738	Japan	14.311	1.517
Azerbaijan	4.316	0.064	Mauritius	9.942	0.428
Bahrain	14.837	0.442	Mexico	5.268	0.224
Bangladesh	1.268	0.175	Mongolia	1.87	0.154
Barbados	13.165	0.496	Morocco	2.468	0.261
Belarus	13.087	0.127	Mozambique	1.299	0.131
Belize	5.672	0.363	Nepal	0.897	0.129
Benin	0.746	0.156	Netherlands	14.625	0.863
Bolivia	1.915	0.258	New Zealand	11.354	0.552
Bosnia & Herz.	4.445	0.426	Nicaragua	1.601	0.25
Brazil	4.604	0.423	Niger	0.588	0.094
Bulgaria	6.262	0.368	Nigeria	0.849	0.11
Burkina Faso	0.663	0.108	Norway	16.985	0.484
Burundi	0.506	0.097	Oman	10.919	0.249
Cambodia	1.819	0.074	Pakistan	1.499	0.245
Cameroon	1.491	0.167	Panama	3.426	0.606
Canada	15.04	0.817	Papua New Guinea	1.459	0.185
Central Afr. Rep.	0.651	0.104	Paraguay	2.791	0.198
Chad	0.874	0.079	Peru	2.984	0.168
Chile	6.164	0.442	Philippines	2.084	0.271
China	2.624	0.874	Poland	8.417	0.276
Colombia	3.42	0.283	Portugal	8.506	0.749
Congo, Rep. of	1.421	0.145	Qatar	30.938	0.299
Costa Rica	5.111	0.234	Romania	6.685	0.149
Croatia	9.394	0.401	Russia	8.37	0.187
Cyprus	12.216	1.238	Rwanda	0.794	0.063
Czech Republic	15.303	0.487	Samoa	3.888	0.243
Denmark	14.335	0.642	Senegal	1.144	0.22
Djibouti	3.651	0.354	Sierra Leone	1.326	0.048
Dom. Republic	3.527	0.231	Singapore	14.802	0.744
Ecuador	3.029	0.217	Slovenia	16.958	0.382
Egypt	2.282	0.28	South Africa	5.125	0.902
El Salvador	2.88	0.303	Spain	11.61	0.79
Eq. Guinea	5.549	0.096	Tajikistan	2.2	0.173
Estonia	11.733	0.494	Tanzania	0.626	0.087
Ethiopia	0.746	0.151	Thailand	3.498	0.657
Fiji	2.983	0.286	Togo	0.684	0.185
Finland	13.084	0.565	Trinidad &Tobago	7.552	0.336
France	13.184	0.806	Tunisia	3.919	0.514
Gabon	4.584	0.147	Turkey	3.179	0.182
Gambia, The	0.823	0.134	Turkmenistan	6.589	0.017
Georgia	4.983	0.105	Uganda	0.58	0.066
Germany	17.353	0.913	Ukraine	6.208	0.191
Ghana	0.925	0.071	United Arab Emir.	32.473	0.29
Greece	10.546	0.354	United Kingdom	13.117	0.737
Guatemala	3.043	0.167	United States	18.805	1.219
Guinea-Bissau	0.641	0.088	Uruguay	5.732	0.323
Guyana	1.562	0.333	Venezuela	5.365	0.29
Haiti	1.431	0.138	Vietnam	2.432	0.43
Honduras	1.898	0.294	Yemen	0.928	0.055
Hungary	10.36	0.399	Zambia	1.006	0.119
Iceland	15.676	0.628	Zimbabwe	2.238	0.298
India	1.307	0.213			

Table 5: List of countries