Online Appendix for "Simple Mandates, Monetary Rules and Trend-Inflation"

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Online Appendix

A Inflation and Price Dispersion Dynamics

This sub-appendix shows how first order conditions expressed as summations, as in Calvo price or wage contracts, can be expressed as difference equations suitable for coding in Dynare. Then the dynamic form of price or wage dispersion, Δ_t^p is derived.

A.1 A Useful Lemma

In the first order conditions for Calvo contracts and expressions for value functions we are confronted with expected discounted sums of the general form

$$\Omega_t = \mathbb{E}_t \left[\sum_{k=0}^{\infty} \beta^k X_{t,t+k} Y_{t+k} \right] \tag{A.1}$$

where $X_{t,t+k}$ has the property $X_{t,t+k} = X_{t,t+1} X_{t+1,t+k}$ and $X_{t,t} = 1$ (for example an inflation, interest or discount rate over the interval $[t, t+k]$).

Lemma

Ω_t can be expressed as

$$\Omega_t = Y_t + \beta \mathbb{E}_t \left[X_{t,t+1} \Omega_{t+1} \right] \tag{A.2}$$

Proof

$$\begin{aligned}
\Omega_t &= X_{t,t} Y_t + \mathbb{E}_t \left[\sum_{k=1}^{\infty} \beta^k X_{t,t+k} Y_{t+k} \right] \\
&= Y_t + \mathbb{E}_t \left[\sum_{k'=0}^{\infty} \beta^{k'+1} X_{t,t+k'+1} Y_{t+k'+1} \right] \\
&= Y_t + \beta \mathbb{E}_t \left[\sum_{k'=0}^{\infty} \beta^{k'} X_{t,t+1} X_{t+1,t+k'+1} Y_{t+k'+1} \right] \\
&= Y_t + \beta \mathbb{E}_t \left[X_{t,t+1} \Omega_{t+1} \right] \qquad \square
\end{aligned}$$

A.2 Price and Wage Dynamics

Consider the optimal price which can be written as

$$\frac{P_t^O}{P_t} = \frac{J_t^p}{JJ_t^p} \tag{A.3}$$

and summations JJ_t^p and J_t^p are of the form considered in the Lemma above. Applying the Lemma we then have the recursive form:

$$\begin{aligned}
JJ_t^p - \xi_p \mathbb{E}_t[\Lambda_{t,t+1}\Pi_{t+1}^{\zeta_p-1} JJ_{t+1}^p] &= Y_t \\
J_t^p - \xi_p \mathbb{E}_t[\Lambda_{t,t+1}\Pi_{t+1}^{\zeta_p} J_{t+1}^p] &= \frac{1}{1-\frac{1}{\zeta_p}} Y_t MC_t MS_t \\
1 &= \xi_p \Pi_t^{\zeta_p-1} + (1-\xi_p)\left(\frac{J_t^p}{JJ_t^p}\right)^{1-\zeta_p} \\
MC_t &= \frac{P_t^W}{P_t} = \frac{W_t}{P_t F_{H,t}} \qquad \text{(A.4)} \\
\Delta_t^p &\equiv \frac{1}{n}\sum_{j=1}^{n}(P_t(j)/P_t)^{-\zeta_p} = \xi_p \Pi_t^{\zeta_p}\Delta_{t-1} + (1-\xi_p)\left(\frac{J_t^p}{JJ_t^p}\right)^{-\zeta_p}
\end{aligned}$$

as in the main text.

With *indexing* by an amount $\gamma \in [0,1]$, the optimal price-setting first-order condition for a firm j setting a new optimized price $P_t^O(j)$ is now given by

$$P_t^O(j) = \frac{\frac{1}{(1-1/\zeta_p)}\mathbb{E}_t\left[\sum_{k=0}^{\infty}\xi_p^k \Lambda_{t,t+k} P_{t+k} MC_{t+k} MS_{t+k} Y_{t+k}(j)\right]}{\mathbb{E}_t\left[\sum_{k=0}^{\infty}\xi_p^k D_{t,t+k} Y_{t+k}(j)\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\gamma}\right]}$$

Price dynamics are now given by

$$\begin{aligned}
\frac{P_t^O}{P_t} &= \frac{J_t^p}{JJ_t^p} \\
JJ_t^p - \xi_p \beta \mathbb{E}_t[\tilde{\Pi}_{t+1}^{\zeta_p-1} JJ_{t+1}^p] &= Y_t U_{C,t} \\
J_t^p - \xi_p \beta \mathbb{E}_t[\tilde{\Pi}_{t+1}^{\zeta_p} J_{t+1}^p] &= \frac{1}{1-\frac{1}{\zeta_p}} MC_t MS_t Y_t U_{C,t} \\
\tilde{\Pi}_t &\equiv \frac{\Pi_t}{\Pi_{t-1}^{\gamma}}
\end{aligned}$$

Wage dynamics follows similarly.

A.3 Dynamics of Price Dispersion

Price dispersion lowers aggregate output as follows. As with consumption goods, the demand equations for each differentiated good m with price $P_t(m)$ forming aggregate investment and public services takes the form

$$I_t(m) = \left(\frac{P_t(m)}{P_t}\right)^{-\zeta_p} I_t\,; \quad G_t(m) = \left(\frac{P_t(m)}{P_t}\right)^{-\zeta_p} G_t \tag{A.5}$$

Hence equilibrium for good m gives

$$Y_t(m) = A_t H_t(m) \left(\frac{K_t(m)}{Y_t(m)}\right)^{\frac{1-\alpha}{\alpha}} = (C_t + I_t + G_t)\left(\frac{P_t(m)}{P_t}\right)^{-\zeta_p} \tag{A.6}$$

where $Y_t(m)$, $H_t(m)$ and $K_t(m)$ are the quantities of output, hours and capital needed in the wholesale sector to produce good m in the retail sector. Since the capital-labour ratio is constant integrating over m, and using $H_t = \int_0^1 H_t(m)dm$ we obtain

$$Y_t = \frac{F(A_t, H_t, K_t)}{\Delta_t^p} \tag{A.7}$$

as in the main text.

Price dispersion is linked to inflation as follows. Assuming as before that the number of firms is large we obtain the following dynamic relationship:

$$\Delta_t^p = \xi_p \Pi_t^{\zeta_p} \Delta_{t-1} + (1-\xi_p)\left(\frac{J_t^p}{JJ_t^p}\right)^{-\zeta_p} \tag{A.8}$$

Proof

In the next period, ξ_p of these firms will keep their old prices, and $(1-\xi_p)$ will change their prices to P_{t+1}^O. By the law of large numbers, we assume that the distribution of prices among those firms that do not change their prices is the same as the overall distribution in period t. It follows that we may write

$$
\begin{aligned}
\Delta^p_{t+1} &= \xi_p \sum_{j_{no\ change}} \left(\frac{P_t(j)}{P_{t+1}}\right)^{-\zeta_p} + (1-\xi_p)\left(\frac{J^p_{t+1}}{JJ^p_{t+1}}\right)^{-\zeta_p} \\
&= \xi_p \left(\frac{P_t}{P_{t+1}}\right)^{-\zeta_p} \sum_{j_{no\ change}} \left(\frac{P_t(j)}{P_t}\right)^{-\zeta_p} + (1-\xi_p)\left(\frac{J^p_{t+1}}{JJ^p_{t+1}}\right)^{-\zeta_p} \\
&= \xi_p \left(\frac{P_t}{P_{t+1}}\right)^{-\zeta_p} \sum_{j} \left(\frac{P_t(j)}{P_t}\right)^{-\zeta_p} + (1-\xi_p)\left(\frac{J^p_{t+1}}{JJ^p_{t+1}}\right)^{-\zeta_p} \\
&= \xi_p \Pi^{\zeta_p}_{t+1}\Delta_t + (1-\xi_p)\left(\frac{J^p_{t+1}}{JJ^p_{t+1}}\right)^{-\zeta_p} \qquad \square
\end{aligned}
$$

Wage dispersion follows similarly.

B The Stationary Equilibrium

To stationarize the model labour-augmenting technical progress parameter is decomposed into a cyclical component, stationary A_t, and a deterministic trend $\bar{A}_t$:

$$
\begin{aligned}
A_t &= \bar{A}_t A^c_t \\
\bar{A}_t &= (1+g)\bar{A}_{t-1}
\end{aligned}
$$

Then we can define stationarized variables by

$$
\begin{aligned}
\frac{\Omega_t}{\bar{A}^{1-\sigma}_t} &= \frac{U_t}{\bar{A}^{1-\sigma}_t} + \beta E_t \frac{\Omega_{t+1}}{\bar{A}^{1-\sigma}_{t+1}} \left(\frac{\bar{A}_{t+1}}{\bar{A}_t}\right)^{1-\sigma} \\
\frac{U_t}{\bar{A}^{1-\sigma}_t} &= \frac{\left[\frac{C_t}{\bar{A}_t} - \chi \frac{C_{t-1}}{\bar{A}_{t-1}} \frac{\bar{A}_{t-1}}{\bar{A}_t}\right]^{1-\sigma}}{1-\sigma} \exp\left[(\sigma-1)\frac{H^{1+\psi}_t}{1+\psi}\right] \\
\Lambda_{t,t+1} &= \beta \frac{U_{C,t+1}}{U_{C,t}} = \beta(1+g)^{-\sigma)} \frac{U^c_{C,t+1}}{U^c_{C,t}} \equiv \frac{\beta_g}{1+g} \frac{U^c_{C,t+1}}{U^c_{C,t}}
\end{aligned}
$$

where the growth-adjusted discount rate is defined as

$$
\beta_g \equiv \beta(1+g)^{1-\sigma},
$$

the Euler equation is still

$$E_t\left[\Lambda_{t,t+1}R_{t+1}\right]$$

Now stationarize remaining variables by defining cyclical components:

$$\begin{aligned}
\frac{U_{C,t}}{\bar{A}_t^{-\sigma}} &= \frac{(1-\sigma)\frac{U_t}{\bar{A}_t^{1-\sigma}}}{\frac{C_t}{\bar{A}_t}-\chi\frac{C_{t-1}}{\bar{A}_{t-1}}\frac{\bar{A}_{t-1}}{\bar{A}_t}} - \beta\chi\left(\frac{\bar{A}_{t+1}}{\bar{A}_t}\right)^{-\sigma}\frac{(1-\sigma)\frac{U_{t+1}}{\bar{A}_{t+1}^{1-\sigma}}}{\frac{C_{t+1}}{\bar{A}_{t+1}}-\chi\frac{C_t}{\bar{A}_t}\frac{\bar{A}_t}{\bar{A}_{t+1}}} \\
Y_t^c &\equiv \frac{Y_t}{\bar{A}_t} = \frac{(A_tH_t^d)^\alpha\left(\frac{K_{t-1}}{\bar{A}_t}\right)^{1-\alpha}-\frac{F_t}{\bar{A}_t}}{\Delta_t^p} = \frac{(A_tH_t^d)^\alpha\left(\frac{K_{t-1}^c}{(1+g_t)}\right)^{1-\alpha}-F}{\Delta_t^p} \\
K_t^c &\equiv \frac{K_t}{\bar{A}_t} \\
K_t^c &= (1-\delta)\frac{K_{t-1}^c}{1+g_t}+(1-S(X_t^c))I_t^c \\
X_t^c &= (1+g_t)\frac{I_t^c}{I_{t-1}^c} \\
S(X_t^c) &= \phi_X(X_t^c-1-g_t)^2 \\
S'(X_t^c) &= 2\phi_X(X_t^c-1-g_t) \\
C_t^c &\equiv \frac{C_t}{\bar{A}_t} \\
I_t^c &\equiv \frac{I_t}{\bar{A}_t} \\
W_t^c &\equiv \frac{W_t}{\bar{A}_t}
\end{aligned}$$

Rewrite the equilibrium conditions as

Household:

$$\begin{aligned}
\frac{\Omega_t}{\bar{A}_t^{1-\sigma}} &= \frac{U_t}{\bar{A}_t^{1-\sigma}}+\beta E_t\frac{\Omega_{t+1}}{\bar{A}_{t+1}^{1-\sigma}}\left(\frac{\bar{A}_{t+1}}{\bar{A}_t}\right)^{1-\sigma} \\
\frac{U_t}{\bar{A}_t^{1-\sigma}} &= \frac{\left[\frac{C_t}{\bar{A}_t}-\chi\frac{C_{t-1}}{\bar{A}_{t-1}}\frac{\bar{A}_{t-1}}{\bar{A}_t}\right]^{1-\sigma}}{1-\sigma}\exp\left[(\sigma-1)\frac{H_t^{1+\psi}}{1+\psi}\right] \\
\frac{K_t}{\bar{A}_t} &= (1-\delta)\frac{K_{t-1}}{\bar{A}_{t-1}}\frac{\bar{A}_{t-1}}{\bar{A}_t}+(1-S(X_t))\frac{I_t}{\bar{A}_t}IS_t \\
X_t &= \frac{\frac{I_t}{\bar{A}_t}\bar{A}_t}{\frac{I_{t-1}}{\bar{A}_{t-1}}\bar{A}_{t-1}}
\end{aligned}$$

$$S(X_t) = \phi_X(X_t - 1 - g)^2$$

$$S'(X_t) = 2\phi_X(X_t - 1 - g)$$

$$\frac{\lambda_t}{\bar{A}_t^{-\sigma}} = \frac{(1-\sigma)\frac{U_t}{\bar{A}_t^{1-\sigma}}}{\frac{C_t}{\bar{A}_t} - \chi\frac{C_{t-1}}{\bar{A}_{t-1}}\frac{\bar{A}_{t-1}}{\bar{A}_t}} - \beta\chi\left(\frac{\bar{A}_{t+1}}{\bar{A}_t}\right)^{-\sigma}\frac{(1-\sigma)\frac{U_{t+1}}{\bar{A}_{t+1}^{1-\sigma}}}{\frac{C_{t+1}}{\bar{A}_{t+1}} - \chi\frac{C_t}{\bar{A}_t}\frac{\bar{A}_t}{\bar{A}_{t+1}}}$$

$$\frac{W_{h,t}}{\bar{A}_t} = \frac{\left[\frac{C_t}{\bar{A}_t} - \chi\frac{C_{t-1}}{\bar{A}_{t-1}}\frac{\bar{A}_{t-1}}{\bar{A}_t}\right]H_t^{\psi}}{1 - \beta\chi\frac{U_{t+1}/\bar{A}_{t+1}^{1-\sigma}}{U_t/\bar{A}_t^{1-\sigma}}\left(\frac{\bar{A}_{t+1}}{\bar{A}_t}\right)^{-\sigma}\frac{\frac{C_t}{\bar{A}_t}-\chi\frac{C_{t-1}}{\bar{A}_{t-1}}\frac{\bar{A}_{t-1}}{\bar{A}_t}}{\frac{C_{t+1}}{\bar{A}_{t+1}}-\chi\frac{C_t}{\bar{A}_t}\frac{\bar{A}_t}{\bar{A}_{t+1}}}}$$

$$r_t^K = a'(u_t)$$

$$1 = RPS_t\mathbb{E}_t\left[\Lambda_{t,t+1}R_{t+1}\right]$$

$$Q_t = \mathbb{E}_t\left\{\Lambda_{t,t+1}\left[r_{t+1}^K u_{t+1} - a(u_{t+1}) + Q_{t+1}(1-\delta)\right]\right\}$$

$$1 = Q_t\left[1 - S(X_t) - S'(X_t)X_t\right]IS_t$$
$$+ \mathbb{E}_t\left[\Lambda_{t,t+1}Q_{t+1}S'(X_{t+1})X_{t+1}^2 IS_{t+1}\right]$$

$$\Lambda_{t,t+1} = \beta\frac{\frac{\lambda_{t+1}}{\bar{A}_{t+1}^{-\sigma}}\bar{A}_{t+1}^{-\sigma}}{\frac{\lambda_t}{\bar{A}_t^{-\sigma}}\bar{A}_t^{-\sigma}}$$

$$R_t = \left[\frac{R_{n,t-1}}{\Pi_t}\right]$$

$$a(u_t) = \gamma_1(u_t - 1) + \frac{\gamma_2}{1-\gamma_2}\frac{\gamma_1}{2}(u_t - 1)^2$$

$$a'(u_t) = \gamma_1 + \frac{\gamma_2}{1-\gamma_2}\gamma_1(u_t - 1)$$

Wage setting:

$$\Pi_t^w = \frac{\frac{W_t}{\bar{A}_t}}{\frac{W_{t-1}}{\bar{A}_{t-1}}\frac{\bar{A}_{t-1}}{\bar{A}_t}}\Pi_t$$

$$\frac{J_t^w}{\bar{A}_t} = \frac{1}{1-\frac{1}{\zeta_w}}\frac{W_{h,t}}{\bar{A}_t}H_t^d MRSS_t$$
$$+ \xi_w\mathbb{E}_t\Lambda_{t,t+1}\frac{\left(\Pi_{t,t+1}^w\right)^{\zeta_w}}{\left(\Pi_{t-1,t}\right)^{\gamma_w\zeta_w}}\frac{J_{t+1}^w}{\bar{A}_{t+1}}\frac{\bar{A}_{t+1}}{\bar{A}_t}$$

$$JJ_t^w = H_t^d + \xi_w\mathbb{E}_t\Lambda_{t,t+1}\frac{\left(\Pi_{t,t+1}^w\right)^{\zeta_w}}{\left(\Pi_{t-1,t}\right)^{\gamma_w(\zeta_w-1)}\Pi_{t,t+1}}JJ_{t+1}^w$$

$$
\begin{aligned}
\frac{W^O_{n,t}}{W_{n,t}} &= \frac{\frac{J^w_t}{\bar{A}_t}}{\frac{W_t}{\bar{A}_t} JJ^w_t} \\
1 &= \xi_w \left(\frac{\Pi^{\gamma_w}_{t-1}}{\Pi^w_t}\right)^{1-\zeta_w} + (1-\xi_w)\left(\frac{W^O_{n,t}}{W_{n,t}}\right)^{1-\zeta_w} \\
\Delta_{w,t} &= \xi_w \frac{(\Pi^w_t)^{\zeta_w}}{\Pi^{\zeta_w \gamma_w}_{t-1}} \Delta_{w,t-1} + (1-\xi_w)\left(\frac{W^O_{n,t}}{W_{n,t}}\right)^{-\zeta_w}
\end{aligned}
$$

Retail firm:

$$
\begin{aligned}
\frac{Y^W_t}{\bar{A}_t} &= \left(\frac{A_t}{\bar{A}_t} H^d_t\right)^{\alpha} \left(u_t \frac{K_{t-1}}{\bar{A}_{t-1}} \frac{\bar{A}_{t-1}}{\bar{A}_t}\right)^{1-\alpha} - \tilde{F} \frac{Y^W_t}{\bar{A}_t} \\
\frac{W_t}{\bar{A}_t} &= \alpha \frac{P^W_t}{P_t} \frac{\frac{Y^W_t}{\bar{A}_t} + \tilde{F} \frac{Y^W_t}{\bar{A}_t}}{H^d_t} \\
r^K_t &= (1-\alpha) \frac{P^W_t}{P_t} \frac{\frac{Y^W_t}{\bar{A}_t} + \tilde{F} \frac{Y^W_t}{\bar{A}_t}}{u_t \frac{K_{t-1}}{\bar{A}_{t-1}} \frac{\bar{A}_{t-1}}{\bar{A}_t}}
\end{aligned}
$$

Price setting:

$$
\begin{aligned}
MC_t &= \frac{P^W_t}{P_t} \\
\frac{J^p_t}{\bar{A}_t} &= \frac{1}{1-\frac{1}{\zeta_p}} \frac{Y_t}{\bar{A}_t} MC_t MCS_t \\
&\quad + \xi_p \mathbb{E}_t \Lambda_{t,t+1} \frac{(\Pi_{t,t+1})^{\zeta_p}}{(\Pi_{t-1,t})^{\gamma_p \zeta_p}} \frac{J^p_{t+1}}{\bar{A}_{t+1}} \frac{\bar{A}_{t+1}}{\bar{A}_t} \\
\frac{JJ^p_t}{\bar{A}_t} &= \frac{Y_t}{\bar{A}_t} + \xi_p \mathbb{E}_t \Lambda_{t,t+1} \frac{(\Pi_{t,t+1})^{\zeta_p - 1}}{(\Pi_{t-1,t})^{\gamma_p(\zeta_p - 1)}} \frac{JJ^p_{t+1}}{\bar{A}_{t+1}} \frac{\bar{A}_{t+1}}{\bar{A}_t} \\
\frac{P^0_t}{P_t} &= \frac{\frac{J^p_t}{\bar{A}_t}}{\frac{JJ^p_t}{\bar{A}_t}} \\
1 &= \xi_p \left(\frac{\Pi^{\gamma_p}_{t-1}}{\Pi_t}\right)^{1-\zeta_p} + (1-\xi_p)\left(\frac{P^0_t}{P_t}\right)^{1-\zeta_p} \\
\Delta_{p,t} &= \xi_p \frac{\Pi^{\zeta_p}_t}{\Pi^{\zeta_p \gamma_p}_{t-1}} \Delta_{p,t-1} + (1-\xi_p)\left(\frac{P^0_t}{P_t}\right)^{-\zeta_p}
\end{aligned}
$$

Monetary policy:

$$
\begin{aligned}
\log\left(\frac{R_{n,t}}{R_n}\right) &= \rho_r \log\left(\frac{R_{n,t-1}}{R_n}\right) \\
&+ (1-\rho_r)\left(\theta_\pi \log\left(\frac{\Pi_t}{\Pi}\right) + \theta_y \log\left(\frac{Y_t}{Y}\right) + \theta_{dy}\log\left(\frac{Y_t}{Y_{t-1}}\right)\right) \\
&+ \log MPS_t
\end{aligned}
$$

Aggregation:

$$
\begin{aligned}
\frac{Y_t}{\bar{A}_t} &= \frac{C_t}{\bar{A}_t} + \frac{G_t}{\bar{A}_t} + \frac{I_t}{\bar{A}_t} + \frac{a(u_t)}{IS_t}\frac{K_{t-1}}{\bar{A}_{t-1}}\frac{\bar{A}_{t-1}}{\bar{A}_t} \\
H_t &= \Delta_{w,t} H_t^d \\
\frac{Y_t^W}{\bar{A}_t} &= \Delta_{p,t}\frac{Y_t}{\bar{A}_t} \\
R_t^K &= \frac{r_t^K u_t - a(u_t) + Q_t(1-\delta)}{Q_{t-1}}
\end{aligned}
$$

Shock processes:

$$
\begin{aligned}
\log A_t - \log A &= \rho_A(\log A_{t-1} - \log A) + \epsilon_{A,t} \\
\log G_t - \log G &= \rho_G(\log G_{t-1} - \log G) + \epsilon_{G,t} \\
\log MCS_t - \log MCS &= \rho_{MCS}(\log MCS_{t-1} - \log MCS) + \epsilon_{MCS,t} \\
\log MRSS_t - \log MRSS &= \rho_{MRSS}(\log MRSS_{t-1} - \log MRSS) + \epsilon_{MRSS,t} \\
\log IS_t - \log IS &= \rho_{IS}(\log IS_{t-1} - \log IS) + \epsilon_{IS,t} \\
\log MPS_t - \log MPS &= \rho_{MPS}(\log MPS_{t-1} - \log MPS) + \epsilon_{MPS,t} \\
\log RPS_t - \log RPS &= \rho_{RPS}(\log RPS_{t-1} - \log RPS) + \epsilon_{RPS,t}
\end{aligned}
$$

B.1 Summary of the Dynamic Equilibrium

Use this change of variables and dropping the superscript c in trended variables such Ω_t^c, U_t^c, C_t^c etc to arrive to the following stationarized equilibrium conditions:

Household:

$$\Omega_t = U_t + \beta(1+g)^{1-\sigma} E_t \Omega_{t+1} \tag{B.1}$$

$$U_t = \frac{\left[C_t - \chi \frac{C_{t-1}}{1+g}\right]^{1-\sigma}}{1-\sigma} \exp\left[(\sigma-1)\frac{H_t^{1+\psi}}{1+\psi}\right] \tag{B.2}$$

$$K_t = (1-\delta)\frac{K_{t-1}}{1+g} + (1 - S(X_t)) I_t IS_t \tag{B.3}$$

$$X_t = \frac{I_t}{I_{t-1}}(1+g) \tag{B.4}$$

$$S(X_t) = \phi_X (X_t - 1 - g)^2 \tag{B.5}$$

$$S'(X_t) = 2\phi_X (X_t - 1 - g) \tag{B.6}$$

$$\lambda_t = \frac{(1-\sigma)U_t}{C_t - \chi \frac{C_{t-1}}{1+g}} - \beta\chi(1+g)^{-\sigma} \frac{(1-\sigma)U_{t+1}}{C_{t+1} - \chi \frac{C_t}{1+g}} \tag{B.7}$$

$$W_{h,t} = \frac{\left[C_t - \chi \frac{C_{t-1}}{1+g}\right] H_t^{\psi}}{1 - \beta\chi(1+g)^{-\sigma} \frac{U_{t+1}}{U_t} \frac{C_t - \chi \frac{C_{t-1}}{1+g}}{C_{t+1} - \chi \frac{C_t}{1+g}}} \tag{B.8}$$

$$r_t^K = a'(u_t) \tag{B.9}$$

$$1 = RPS_t \mathbb{E}_t \left[\Lambda_{t,t+1} R_{t+1}\right] \tag{B.10}$$

$$Q_t = \mathbb{E}_t \left\{\Lambda_{t,t+1} \left[r_{t+1}^K u_{t+1} - a(u_{t+1}) + Q_{t+1}(1-\delta)\right]\right\} \tag{B.11}$$

$$\begin{aligned} 1 &= Q_t \left[1 - S(X_t) - S'(X_t) X_t\right] IS_t \\ &\quad + \mathbb{E}_t \left[\Lambda_{t,t+1} Q_{t+1} S'(X_{t+1}) X_{t+1}^2 IS_{t+1}\right] \end{aligned} \tag{B.12}$$

$$\Lambda_{t,t+1} = \beta(1+g)^{-\sigma} \frac{\lambda_{t+1}}{\lambda_t} \tag{B.13}$$

$$R_t = \left[\frac{R_{n,t-1}}{\Pi_t}\right] \tag{B.14}$$

$$a(u_t) = \gamma_1 (u_t - 1) + \frac{\gamma_2}{1-\gamma_2} \frac{\gamma_1}{2} (u_t - 1)^2 \tag{B.15}$$

$$a'(u_t) = \gamma_1 + \frac{\gamma_2}{1-\gamma_2} \gamma_1 (u_t - 1) \tag{B.16}$$

Wage setting:

$$\Pi_t^w = (1+g)\frac{W_t}{W_{t-1}} \Pi_t \tag{B.17}$$

$$J^w_t = \frac{1}{1-\frac{1}{\zeta_w}} W_{h,t} H^d_t MRSS_t + \xi_w (1+g) \mathbb{E}_t \Lambda_{t,t+1} \frac{\left(\Pi^w_{t,t+1}\right)^{\zeta_w}}{\left(\Pi_{t-1,t}\right)^{\gamma_w \zeta_w}} J^w_{t+1} \tag{B.18}$$

$$JJ^w_t = H^d_t + \xi_w \mathbb{E}_t \Lambda_{t,t+1} \frac{\left(\Pi^w_{t,t+1}\right)^{\zeta_w}}{\left(\Pi_{t-1,t}\right)^{\gamma_w(\zeta_w-1)} \Pi_{t,t+1}} JJ^w_{t+1} \tag{B.19}$$

$$\frac{W^O_{n,t}}{W_{n,t}} = \frac{J^w_t}{W_t JJ^w_t} \tag{B.20}$$

$$1 = \xi_w \left(\frac{\Pi^{\gamma_w}_{t-1}}{\Pi^w_t}\right)^{1-\zeta_w} + (1-\xi_w)\left(\frac{W^O_{n,t}}{W_{n,t}}\right)^{1-\zeta_w} \tag{B.21}$$

$$\Delta_{w,t} = \xi_w \frac{\left(\Pi^w_t\right)^{\zeta_w}}{\Pi^{\zeta_w \gamma_w}_{t-1}} \Delta_{w,t-1} + (1-\xi_w)\left(\frac{W^O_{n,t}}{W_{n,t}}\right)^{-\zeta_w} \tag{B.22}$$

Retail firm:

$$Y^W_t = \left(A_t H^d_t\right)^\alpha \left(u_t \frac{K_{t-1}}{1+g}\right)^{1-\alpha} - \tilde{F} Y^W \tag{B.23}$$

$$W_t = \alpha \frac{P^W_t}{P_t} \frac{Y^W_t + \tilde{F} Y^W}{H^d_t} \tag{B.24}$$

$$r^K_t = (1-\alpha) \frac{P^W_t}{P_t} \frac{Y^W_t + \tilde{F} Y^W}{u_t \frac{K_{t-1}}{1+g}} \tag{B.25}$$

Price setting:

$$MC_t = \frac{P^W_t}{P_t} \tag{B.26}$$

$$J^p_t = \frac{1}{1-\frac{1}{\zeta_p}} Y_t MC_t MCS_t + \xi_p (1+g) \mathbb{E}_t \Lambda_{t,t+1} \frac{\left(\Pi_{t,t+1}\right)^{\zeta_p}}{\left(\Pi_{t-1,t}\right)^{\gamma_p \zeta_p}} J^p_{t+1} \tag{B.27}$$

$$JJ^p_t = Y_t + \xi_p (1+g) \mathbb{E}_t \Lambda_{t,t+1} \frac{\left(\Pi_{t,t+1}\right)^{\zeta_p - 1}}{\left(\Pi_{t-1,t}\right)^{\gamma_p(\zeta_p-1)}} JJ^p_{t+1} \tag{B.28}$$

$$\frac{P^0_t}{P_t} = \frac{J^p_t}{JJ^p_t} \tag{B.29}$$

$$1 = \xi_p \left(\frac{\Pi^{\gamma_p}_{t-1}}{\Pi_t}\right)^{1-\zeta_p} + (1-\xi_p)\left(\frac{P^0_t}{P_t}\right)^{1-\zeta_p} \tag{B.30}$$

$$\Delta_{p,t} = \xi_p \frac{\Pi_t^{\zeta_p}}{\Pi_{t-1}^{\zeta_p \gamma_p}} \Delta_{p,t-1} + (1 - \xi_p) \left(\frac{P_t^0}{P_t} \right)^{-\zeta_p} \tag{B.31}$$

Monetary policy:

$$\begin{aligned} \log \left(\frac{R_{n,t}}{R_n} \right) &= \rho_r \log \left(\frac{R_{n,t-1}}{R_n} \right) \\ &+ (1 - \rho_r) \left(\theta_\pi \log \left(\frac{\Pi_t}{\Pi} \right) + \theta_y \log \left(\frac{Y_t}{Y} \right) + \theta_{dy} \log \left(\frac{Y_t}{Y_{t-1}} \right) \right) \\ &+ \log MPS_t \end{aligned} \tag{B.32}$$

Aggregation:

$$Y_t = C_t + G_t + I_t + a(u_t) \frac{K_{t-1}}{1+g} \tag{B.33}$$

$$H_t = \Delta_{w,t} H_t^d \tag{B.34}$$

$$Y_t^W = \Delta_{p,t} Y_t \tag{B.35}$$

$$R_t^K = \frac{r_t^K u_t - a(u_t) + Q_t(1-\delta)}{Q_{t-1}} \tag{B.36}$$

Shock processes:

$$\log A_t - \log A = \rho_A (\log A_{t-1} - \log A) + \epsilon_{A,t} \tag{B.37}$$

$$\log G_t - \log G = \rho_G (\log G_{t-1} - \log G) + \epsilon_{G,t} \tag{B.38}$$

$$\log MCS_t - \log MCS = \rho_{MCS} (\log MCS_{t-1} - \log MCS) + \epsilon_{MCS,t} \tag{B.39}$$

$$\log MRSS_t - \log MRSS = \rho_{MRSS} (\log MRSS_{t-1} - \log MRSS) + \epsilon_{MRSS,t} \tag{B.40}$$

$$\log IS_t - \log IS = \rho_{IS} (\log IS_{t-1} - \log IS) + \epsilon_{IS,t} \tag{B.41}$$

$$\log MPS_t - \log MPS = \rho_{MPS} (\log MPS_{t-1} - \log MPS) + \epsilon_{MPS,t} \tag{B.42}$$

$$\log RPS_t - \log RPS = \rho_{RPS} (\log RPS_{t-1} - \log RPS) + \epsilon_{RPS,t} \tag{B.43}$$

This is a system of 43 equation in the following 43 macroeconomic variables (in order of appearance): V, U, C, H, K, $S(X)$, X, I, IS, $S'(X)$, λ, W_h, r^K, $a'(u)$, RPS, Λ, R, Q, u, $a(u)$,

R_n, Π, Π^w, W, J^w, H^d, $MRSS$, JJ^w, $\frac{W_n^O}{W_n}$, Δ_w, Y^W, A, $\frac{P^W}{P}$, MC, J^p, Y, MCS, JJ^p, $\frac{P^0}{P}$, Δ_p, MPS, G, R^K plus 7 AR1 Shock Processes.

Finally we define a consumption equivalent welfare measure CE_t as the inter-temporal increase in welfare resulting from a permanent 1% increase in the equilibrium path of consumption as

$$\begin{aligned} CE_t &= \mathbb{E}_t\left[\sum_{t=s}^{\infty}\beta^s U(1.01C_{t+s}, 1.01C_{t-1+s}, H_{t+s})\right] \\ &- \mathbb{E}_t\left[\sum_{t=s}^{\infty}\beta^s U(C_{t+s}, C_{t-1+s}, H_{t+s})\right] \\ &= \frac{[1.01C_t - \chi 1.01C_{t-1}]^{1-\sigma}}{1-\sigma}\exp\left[(\sigma-1)\frac{H_t^{1+\psi}}{1+\psi}\right] - U(C_t, C_{t-1}, H_t) \\ &+ \beta\mathbb{E}_t CE_{t+1} \\ &= (1.01^{1-\sigma} - 1)U_t + \beta\mathbb{E}_t CE_{t+1} \end{aligned} \tag{B.44}$$

The stationary version is then

$$CE_t = (1.01^{1-\sigma} - 1)U_t + \beta(1+g)^{1-\sigma}\mathbb{E}_t CE_{t+1} \tag{B.45}$$

In our results we compute consumption equivalent differences using the stationary steady state CE.

B.2 The Balanced-Growth Deterministic Steady State

Having stationarized the model we now drop the superscript c. The exogenous variables have steady states $A = MCS = MRSS = IS = MPS = RPS = 1$, $G = g_y Y$. Moreover, $u = 1$ in steady state. Given the steady state inflation rate Π and hours H, the steady state values of the other variables can be computed in stationary form as

$$\begin{aligned} S(X) &= 0 \\ S'(X) &= 0 \\ \Pi^w &= (1+g)\Pi \\ Q &= 1 \end{aligned}$$

$$\Lambda = \beta(1+g)^{-\sigma}$$

$$r^K = \frac{1}{\Lambda} - (1-\delta)$$

$$a(u) = 0$$

$$a'(u) = \gamma_1$$

$$r^K = \gamma_1 \Rightarrow \gamma_1 = \frac{1}{\beta(1+g)^{-\sigma}} - (1-\delta)$$

$$\frac{P^0}{P} = \left(\frac{1-\xi_p \Pi^{(1-\gamma_p)(\zeta_p-1)}}{1-\xi_p}\right)^{\frac{1}{1-\zeta_p}}$$

$$\Delta_p = \frac{1-\xi_p}{1-\xi_p \Pi^{\zeta_p(1-\gamma_p)}} \left(\frac{P^0}{P}\right)^{-\zeta_p}$$

$$MC = \left(1-\frac{1}{\zeta_p}\right) \frac{1-\xi_p(1+g)\Lambda\Pi^{\zeta_p(1-\gamma_p)}}{1-\xi_p(1+g)\Lambda\Pi^{(\zeta_p-1)(1-\gamma_p)}} \frac{P^0}{P}$$

$$\frac{P^W}{P} = MC$$

$$\frac{W_n^O}{W_n} = \left(\frac{1-\xi_w \Pi^{\gamma_w(1-\zeta_w)}(\Pi^w)^{\zeta_w-1}}{1-\xi_w}\right)^{\frac{1}{1-\zeta_w}}$$

$$\Delta_w = \frac{1-\xi_w}{1-\xi_w \frac{(\Pi^w)^{\zeta_w}}{\Pi^{\zeta_w\gamma_w}}} \left(\frac{W_n^O}{W_n}\right)^{-\zeta_w}$$

$$H^d = \frac{H}{\Delta_w}$$

$$\frac{K}{Y^W} = \frac{(1-\alpha)(1+g)(1+\tilde{F})}{ur^K} \frac{P^W}{P}$$

$$Y^W = \frac{H^d}{(1+\tilde{F})^{\frac{1}{\alpha}}} \left(\frac{\frac{K}{Y^W}}{1+g}\right)^{\frac{1-\alpha}{\alpha}}$$

$$K = Y^W \frac{K}{Y^W}$$

$$Y = \frac{Y^W}{\Delta_p}$$

$$I = \frac{K}{1} \frac{g+\delta}{1+g}$$

$$G = g_y Y$$

$$C = Y - G - I$$

$$JJ^w = \frac{H^d}{1-\xi_w \Lambda \left(\Pi^w\right)^{\zeta_w} \Pi^{\gamma_w(1-\zeta_w)-1}}$$

$$W = \alpha \frac{P^W}{P} \frac{Y^W + F}{H^d}$$

$$
\begin{aligned}
J^w &= \frac{W_n^O}{W_n} W J J^w \\
\frac{W_h}{W} &= \frac{\left(1 - \xi_w (1+g) \Lambda \frac{(\Pi^w)^{\zeta_w}}{\Pi^{\gamma_w \zeta_w}}\right)\left(1 - \frac{1}{\zeta_w}\right) J^w}{W H^d} \\
&= \frac{\left(1 - \xi_w (1+g) \Lambda \frac{(\Pi^w)^{\zeta_w}}{\Pi^{\gamma_w \zeta_w}}\right)\left(1 - \frac{1}{\zeta_w}\right) \frac{W_n^O}{W_n}}{1 - \xi_w \Lambda \left(\Pi^w\right)^{\zeta_w} \Pi^{\gamma_w (1-\zeta_w)-1}}
\end{aligned}
$$

To examine the impact of trend inflation Π on the steady state further we consider the zero growth case $g = 0$ for which wage and price inflation are equal ($\Pi^w = \Pi$). Then we have for price-setting:

$$
\begin{aligned}
\frac{P^0}{P} &= \left(\frac{1 - \xi_p \Pi^{(1-\gamma_p)(\zeta_p - 1)}}{1 - \xi_p}\right)^{\frac{1}{1-\zeta_p}} \\
\Delta_p &= \frac{1 - \xi_p}{1 - \xi_p \Pi^{\zeta_p (1-\gamma_p)}} \left(\frac{P^0}{P}\right)^{-\zeta_p} \\
MC &= \left(1 - \frac{1}{\zeta_p}\right) \frac{1 - \xi_p \Lambda \Pi^{\zeta_p (1-\gamma_p)}}{1 - \xi_p (1+g) \Lambda \Pi^{(\zeta_p - 1)(1-\gamma_p)}} \frac{P^0}{P}
\end{aligned}
$$

and for wage-setting:

$$
\begin{aligned}
\frac{W_n^O}{W_n} &= \left(\frac{1 - \xi_w \Pi^{(1-\gamma_w)(\zeta_w - 1)}}{1 - \xi_w}\right)^{\frac{1}{1-\zeta_w}} \\
\Delta_w &= \frac{1 - \xi_w}{1 - \xi_w \Pi^{(1-\gamma_w)\zeta_w}} \left(\frac{W_n^O}{W_n}\right)^{-\zeta_w} \\
\frac{W_h}{W} &= \frac{\left(1 - \xi_w \Lambda \Pi^{(1-\gamma_w)\zeta_w}\right)\left(1 - \frac{1}{\zeta_w}\right) \frac{W_n^O}{W_n}}{1 - \xi_w \Lambda \Pi^{(1-\gamma_w)(\zeta_w - 1)}}.
\end{aligned}
$$

Thus for $\zeta_p > 1$, both the optimized price $\frac{P^0}{P}$ and price dispersion Δ_p *increase* with the trend inflation rate Π. However noting that the price mark-up is the inverse of the real marginal cost, i.e. equal to $= 1/MC$, we can see that the price response to the re-optimized price *decreases* with Π. Analogous results for $\zeta_w > 1$ hold for the optimized nominal wage, wage dispersion and the wage mark-up which is the inverse of $\frac{W_h}{W}$.

B.3 Solution of the Deterministic Steady State

We solve for the steady state as follows:

1. We guess the value of H.
2. We solve for the steady state of the model given our guess.
3. We use the foc on hours

$$W_{h,t} = \frac{\left[C_t - \chi \frac{C_{t-1}}{1+g}\right] H_t^{\psi}}{1 - \beta\chi(1+g)^{-\sigma} \frac{U_{t+1}}{U_t} \frac{C_t - \chi \frac{C_{t-1}}{1+g}}{C_{t+1} - \chi \frac{C_t}{1+g}}}$$

   to evaluate our guess. Note that the above equation in steady state simplifies to

$$W_h = \frac{\left[C - \chi \frac{C}{1+g}\right] H^{\psi}}{1 - \beta\chi(1+g)^{-\sigma}}$$

   which eliminates the need to compute the steady state value for utility.

The rest of the variables can be computed as

$$U = \frac{\left[C - \chi \frac{C}{1+g}\right]^{1-\sigma}}{1-\sigma} \exp\left[(\sigma - 1)\frac{H^{1+\psi}}{1+\psi}\right]$$

$$V = \frac{U}{1 - \beta(1+g)^{1-\sigma}}$$

$$X = 1 + g$$

$$\lambda = \frac{(1-\sigma)U}{C - \chi \frac{C}{1+g}} - \beta\chi(1+g)^{-\sigma} \frac{(1-\sigma)U}{C - \chi \frac{C}{1+g}}$$

$$R = \frac{1}{\Lambda}$$

$$R_n = R\Pi$$

$$J^p = \frac{YMCMCS}{\left(1 - \frac{1}{\zeta_p}\right)\left(1 - \xi_p(1+g)\Lambda\Pi^{\zeta_p(1-\gamma_p)}\right)}$$

$$JJ^p = \frac{J^p}{\frac{P^0}{P}}$$

$$R^K = r^K + 1 - \delta$$

$$CE = \frac{(1.01^{1-\sigma} - 1)U}{1 - \beta(1+g)^{1-\sigma}}$$

C Calibrated and Estimated Parameters

From our non-zero-inflation-growth steady state we impose the restrictions

$$R_n = \frac{\Pi}{\beta(1+g)^{-\sigma}} \tag{C.46}$$

on β. This implies that β **can** be calibrated as

$$\beta = \frac{\Pi}{R_n(1+g)^{-\sigma}} \tag{C.47}$$

However, in order to evaluate welfare ranking with a consistent form of the objective function, we set β given (C.47) with $\bar{\Pi}$ and g both estimated directly as the trend of the data with σ imposed at the prior given by 1.5. For our US data and estimation period, this gives $\beta = 0.9995$ which is then imposed on the rest of the estimation and used for the optimized rules.

The first-order condition for capital utilisation is

$$r_t^K = a'(u_t) \tag{C.48}$$

which has the linear approximation

$$\hat{r}_t^K = \frac{\gamma_2}{\gamma_1}\hat{u}_t \tag{C.49}$$

Smets and Wouters write the above equation as (see equation (6) in their paper)

$$z_t = z_1 r_t^k \tag{C.50}$$

where $z_1 = \frac{1-\psi}{\psi}$ and they estimate ψ. Consequently, $z_1 = \frac{\gamma_1}{\gamma_2}$.

Recall that the capital utilisation adjustment function is

$$a(u_t) = \gamma_1(u_t - 1) + \frac{\gamma_2}{2}(u_t - 1)^2 \tag{C.51}$$

which can be rewritten as

$$
\begin{aligned}
a(u_t) &= \gamma_1(u_t - 1) + \frac{\gamma_2}{\gamma_1}\frac{\gamma_1}{2}(u_t - 1)^2 \\
&= \gamma_1(u_t - 1) + \frac{1}{z_1}\frac{\gamma_1}{2}(u_t - 1)^2 \\
&= \gamma_1(u_t - 1) + \frac{\psi}{1-\psi}\frac{\gamma_1}{2}(u_t - 1)^2
\end{aligned}
\tag{C.52}
$$

Its derivative is

$$
a'(u_t) = \gamma_1 + \frac{\psi}{1-\psi}\gamma_1(u_t - 1) \tag{C.53}
$$

The production function (equation (5) in the paper) is given by

$$
y_t = \phi_p(\alpha k_t^s + (1-\alpha)l_t + \varepsilon_t^a) \tag{C.54}
$$

where $\phi_p = \frac{y_* + \Phi}{y_*}$ is one plus the share of fixed costs in production.(*) They use the prior $\phi_p \sim \mathcal{N}(1.25, 0.25)$ for the parameter (may be missing from the paper altogether), which implies that $\frac{\Phi}{y^*} \sim \mathcal{N}(0.25, 0.25)$. Hence we need to rewrite the equilibrium condition (**??**) as

$$
Y_t^W = \left(A_t H_t^d\right)^\alpha (u_t K_{t-1})^{1-\alpha} - \tilde{F} Y^W \tag{C.57}
$$

and define the prior on $\tilde{F} = \frac{F}{\frac{Y^W}{A_t}}$.

(*)In the technical appendix the production function is given by

$$
y_t(i) = Z_t k_t(i)^\alpha L_t(i)^{1-\alpha} - \Phi \tag{C.55}
$$

which becomes

$$
\hat{y}_t = \alpha \frac{y_* + \Phi}{y_*}\hat{k}_t + (1-\alpha)\frac{y_* + \Phi}{y_*}\hat{L}_t + \frac{y_* + \Phi}{y_*}\hat{Z}_t \tag{C.56}
$$

when loglinearized.

D Identification, MCMC Convergence in Estimation and a Model Specification Test

This section describes and provides results for a range of identification tests, MCMC convergence a model specification test using an estimated DSGE-VAR.

D.1 Identification

Following Iskrev and Ratto (2010), we provide the identification (locally) analysis of the our tool model here.

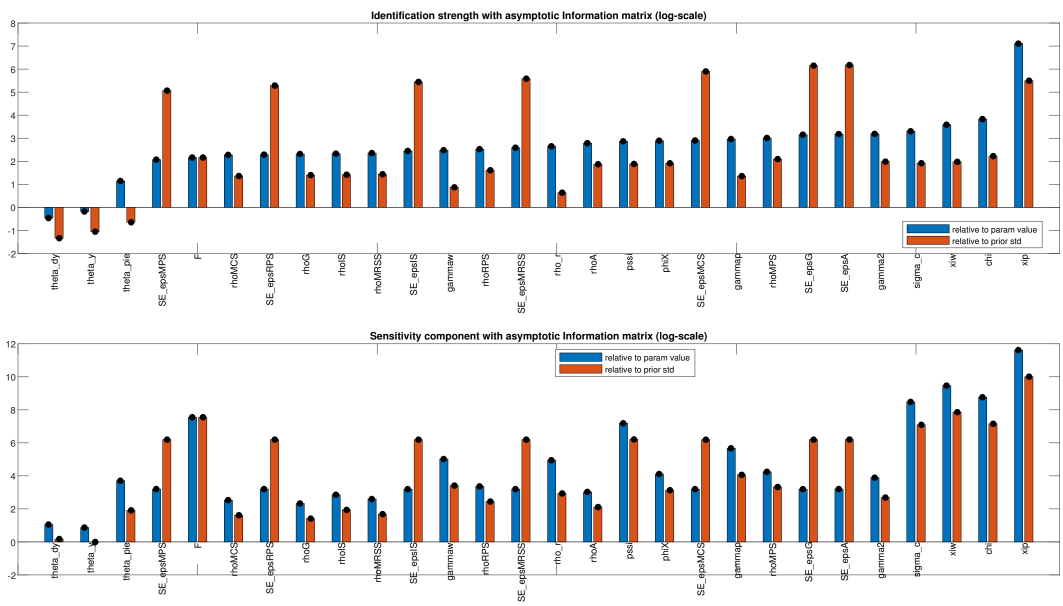

Figure 1: Identification Strength in the tool Model

In the upper panel of the figure the bars depict the identification strength of the parameters based in the Fisher information matrix normalized by either the parameter at the prior mean (blue bars) or by the standard deviation at the prior mean (red bars). Intuitively, the bars represent the normalized curvature of the log likelihood function at the prior mean in the direction of the parameter. If the strength is 0 (for both bars) the parameter is not identified as the likelihood function is flat in this direction. The larger the absolute value if the bars, the stronger the identification. Hence, it is clear that all parameters are identified in the model. However, the feedback parameters, θ_y, θ_{dy}, on the real economic activities are weekly identified.

We also use dynare **?** to examine the identification exercise of the model with different criteria. For instance, Komunjer and Ng (2011) provide a difference route to the local identification of a linear state space, they examine directly the relationship between the coefficients of the state-space representation of the DSGE model and the parameter vector θ. In addition, the setup also accounts for the condition of left-invertablity (or the number of structural shocks is different from the that of the observables). However, in our setup, we always have the "squared matrix", so the full-rank condition on the coefficients matrix and on the Jacobian matrix as in Ratto (2008) is sufficient for local identification.

Qu and Tkachenko (2012) work in the frequency domain, i.e. whether the mean and spectrum of observables is uniquely determined by the deep parameters at all frequencies? Using a frequency domain approximation of the likelihood function and utilizing the information matrix equality, they express the Hessian as the outer product of the Jacobian matrix of derivatives of the spectral density with respect to the set of estimated parameters denoted θ. However, this approach has to be implemented numerically. For each conjectured θ_0 we have to compute the rank of the spectral density matrixy. Because in a typical implementation the computation of the matrix relies on numerical differentiation (and integration) over the subset frequency domains, there might arise discordant results in the matrix rank. For instance, if two parameters jointly enter the model and play a very similar role in the model after linearization (i.e. stickiness level of price parameter and the rate of substitution jointly determine the speed of adjustment of prices through the Calvo probability), thus they are separately unidentifiable. Qu and Tkachenko (2012) procedure tests the identification over a subset of estimated parameters, so the model fails to pass the test over each subset of parameters on the persistence of shocks. The usual procedure to bypass this problem is to fix one of the subset parameters.

D.2 MCMC Convergence

The convergence property is represented in figure (2). The appended (Interval) shows the Brooks and Gelman's convergence diagnostics for the 80% interval. The blue line shows the 80%

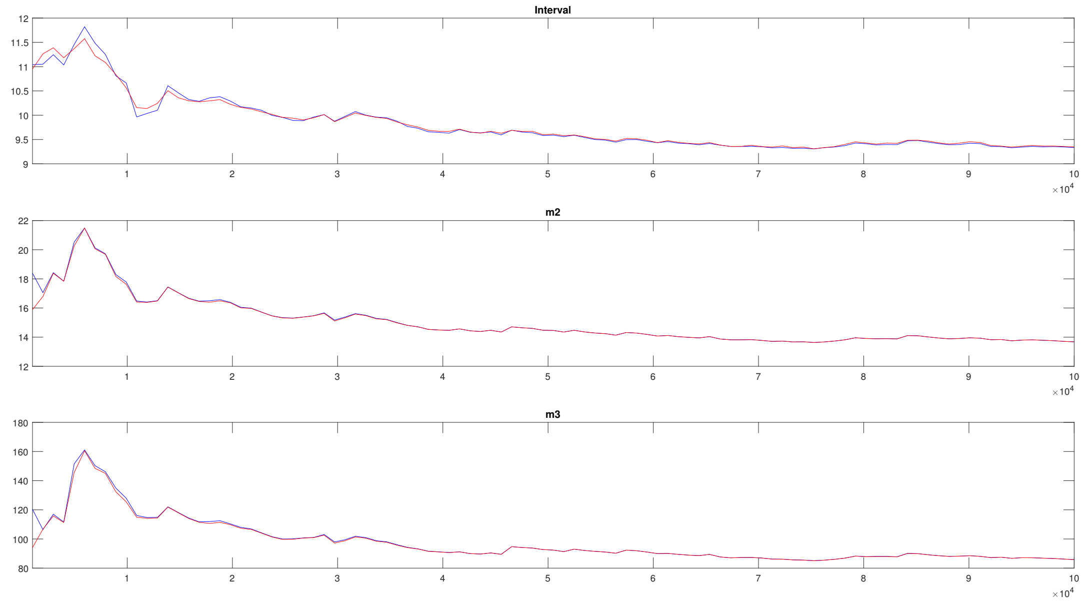

Figure 2: Multivariate convergence diagnostic

interval/quantile range based on the pooled draws from all sequences, while the red line shows the mean interval range based on the draws of the individual sequences. The appended (m2) and (m3) show an estimate of the same statistics for the second and third central moments, i.e. the squared and cubed absolute deviations from the pooled and the within-sample mean, respectively. All statistics are based on the range of the posterior likelihood function. The posterior kernel is used to aggregate the parameters. Convergence is indicated by the two lines stabilizing and being close to each other.

The figures from (3) to (6) indicate the prior-posterior plots. The grey line shows the prior density, while the black line shows the density of the posterior distribution. The green horizontal line indicates the posterior mode. If the posterior looks like the prior, either your prior was a very accurate reflection of the information in the data or the parameter under consideration is only weakly identified and the data does not provide much information to update the prior.

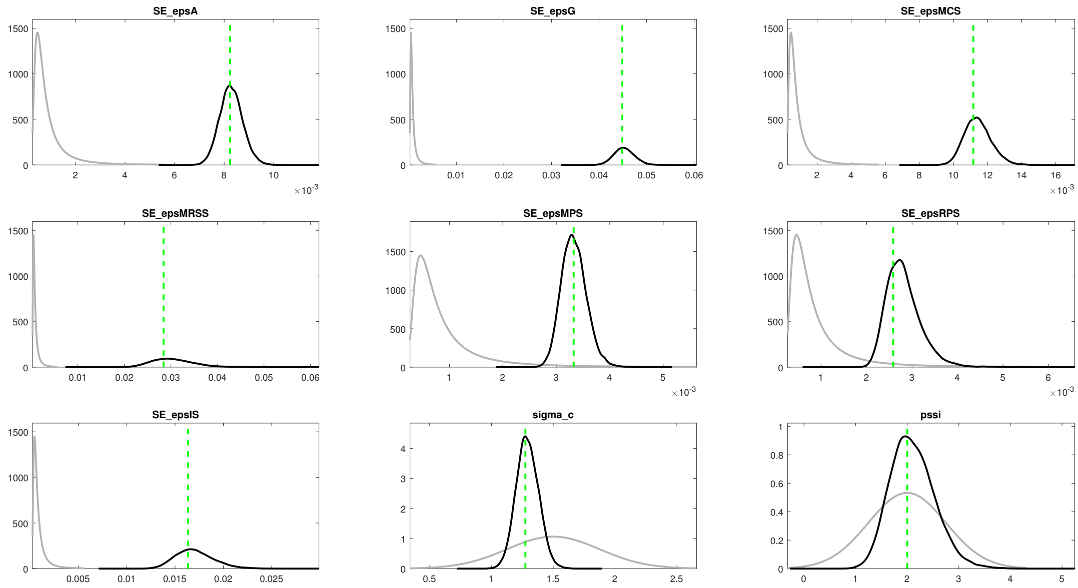

Figure 3: Priors and Posteriors for 100000 MCMC draws

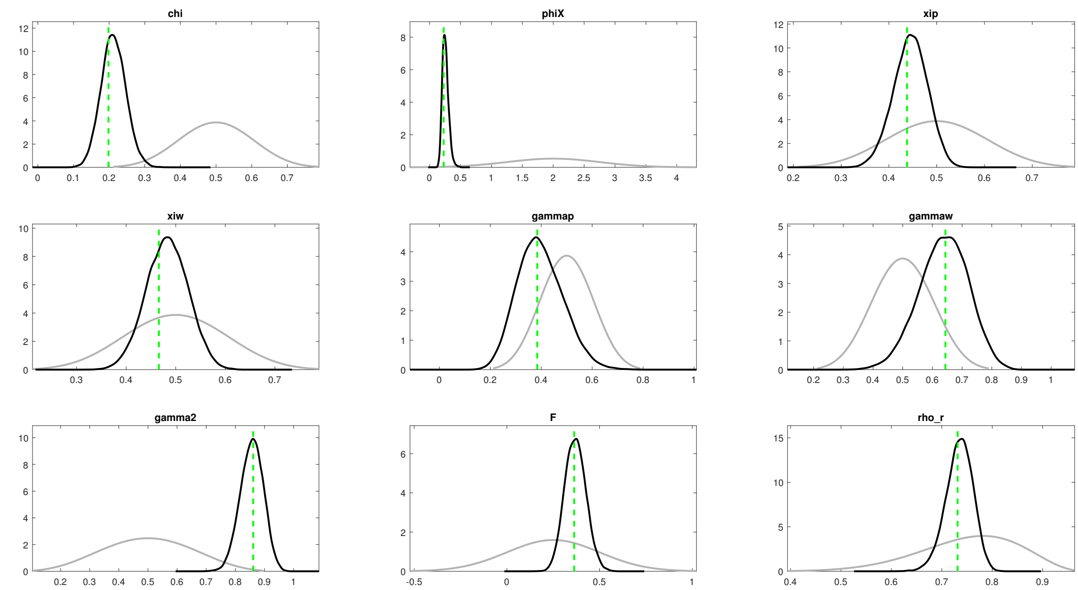

Figure 4: Priors and Posteriors for 100000 MCMC draws

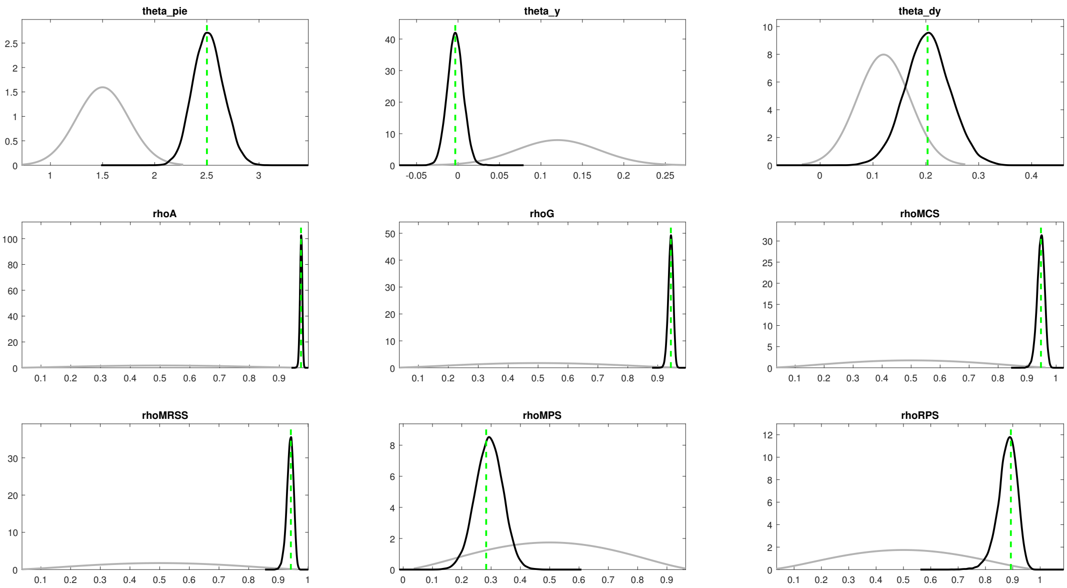

Figure 5: Priors and Posteriors for 100000 MCMC draws

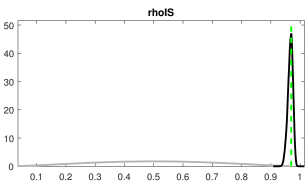

Figure 6: Priors and Posteriors for 100000 MCMC draws

D.3 DSGE-VAR

Following An and Schorfheide (2007), we also examine the estimated DSGE model's Impulse-Response functions (IRFs) to shocks with an estimated DSGE-VAR. The marginal posterior density is from a sample of λT observations generated from the DSGE model, T generated by the VAR and $(\lambda + 1)T$ generated by the DSGE-VAR where T is the sample size. λ is a hyper-parameter that scales the prior covariance matrix. The prior is diffuse for small values of λ and shifts its mass closer to the DSGE model restrictions as $\lambda \to \infty$.

Overall, the sign and magnitude of the DSGE and DSGE-VAR impulse responses are quite similar. Especially, regarding the IRFs to technology shock (figure (7)), the IRFs are almost identical. However, along some dimensions, such as the impact of preference and investment shocks (figures (12) and (13)) on policy rate, there is substantial uncertainty about how it propagates through the system, but still shows a almost-close initial reaction to shocks. Nevertheless, the model dynamics can be broadly described using the estimated impulse responses.

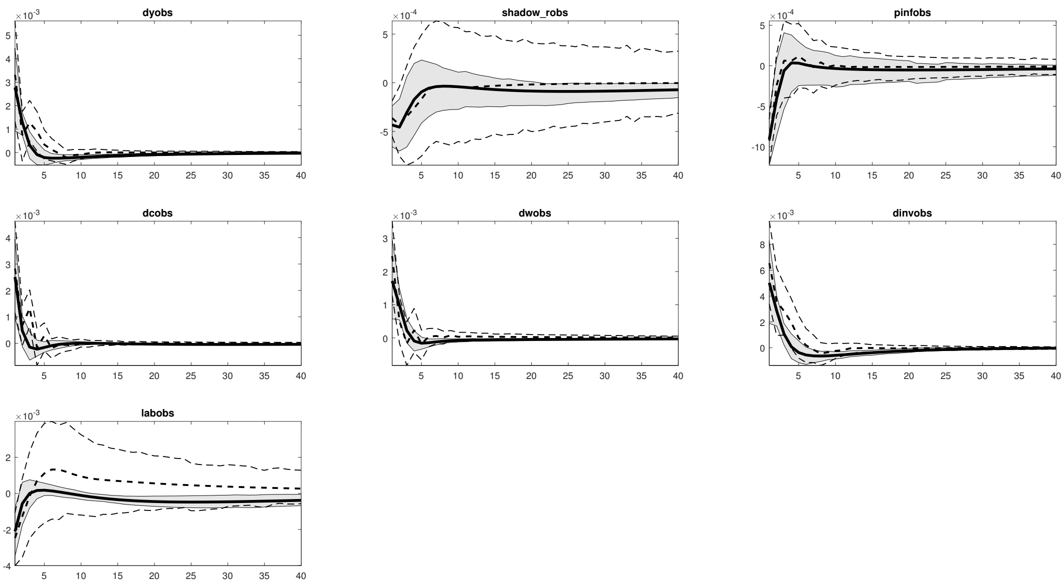

Figure 7: We compare the IRFs to technology shock, ϵ_A of the DSGE_VAR with the estimated DSGE. **The Dashed lines are the first, fifth and ninth posterior deciles of the DSGE-VAR's IRFs, the bold dark curve is theposterior mean of the DSGE's IRfs and the shaded surface covers the space between the first and ninth posterior deciles of the DSGE's IRFs.**

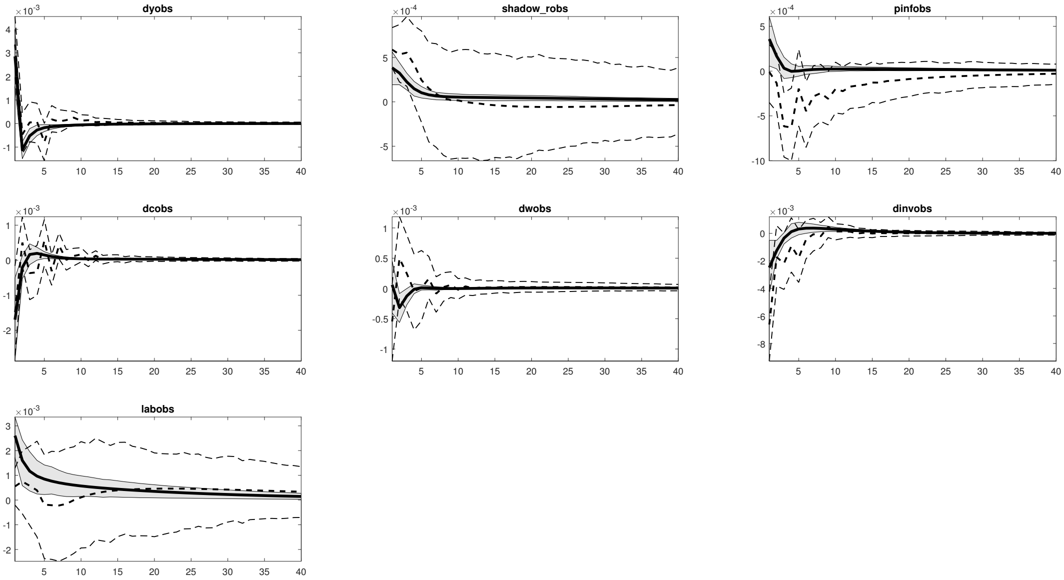

Figure 8: We compare the IRFs to government spending shock, ϵ_G of the DSGE_VAR with the estimated DSGE.

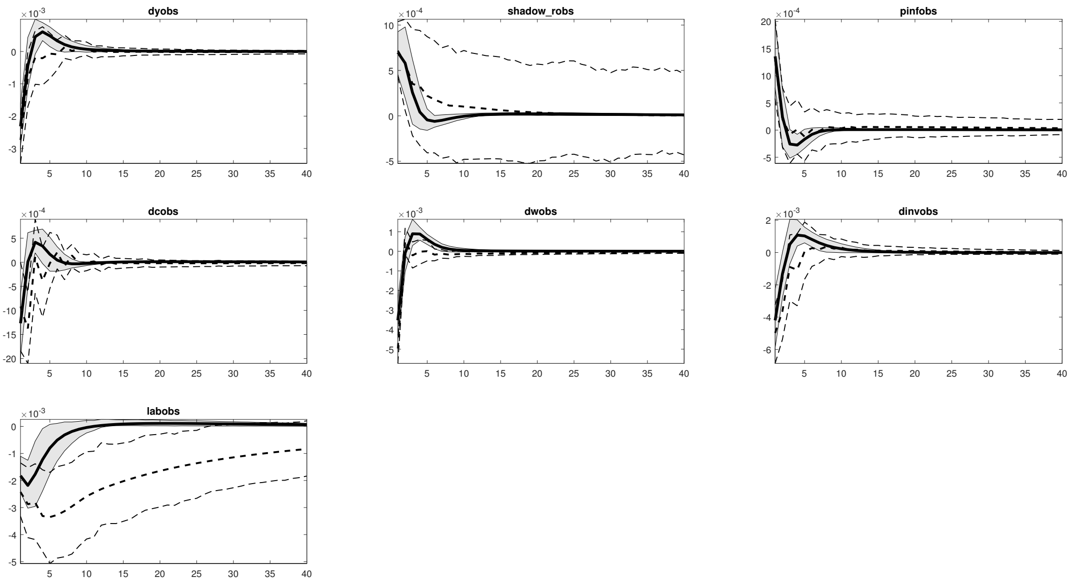

Figure 9: We compare the IRFs to Price Markup shock, ϵ_{MCS} of the DSGE_VAR with the estimated DSGE.

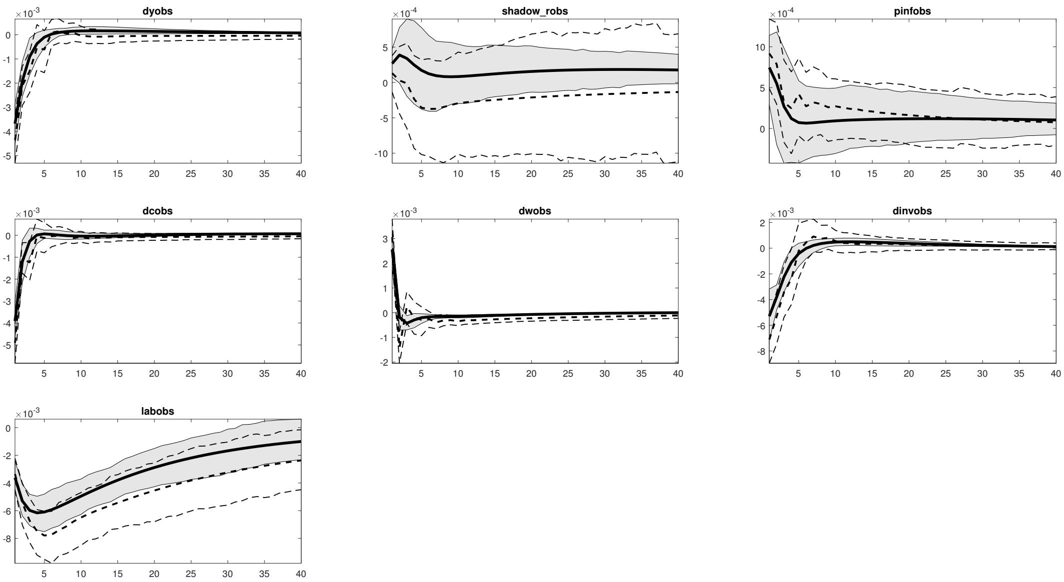

Figure 10: We compare the IRFs to Wage Markup shock, ϵ_{MRSS} of the DSGE_VAR with the estimated DSGE.

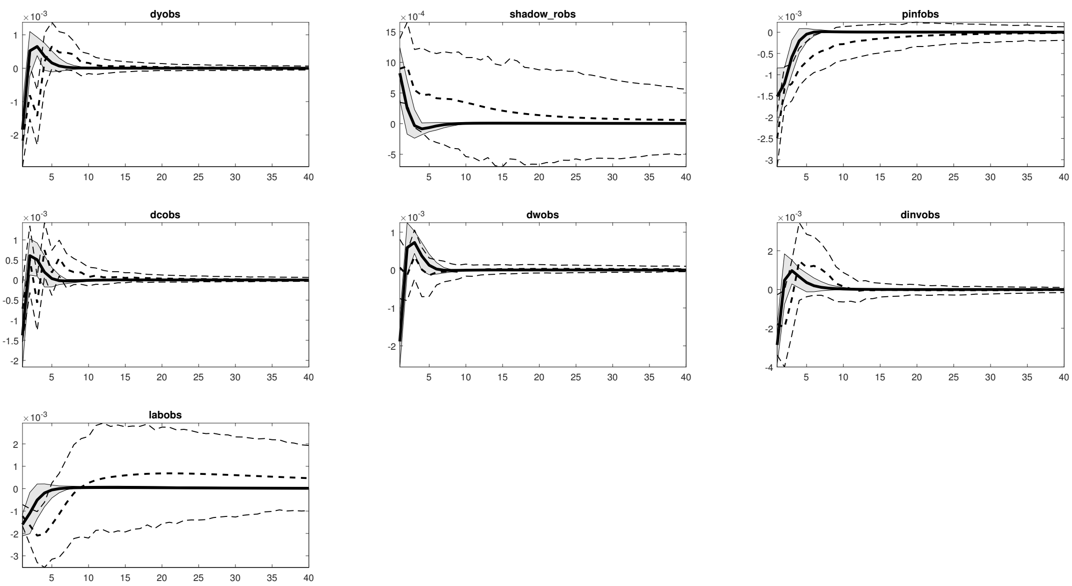

Figure 11: We compare the IRFs to monetary policy shock, ϵ_{MPS} of the DSGE_VAR with the estimated DSGE.

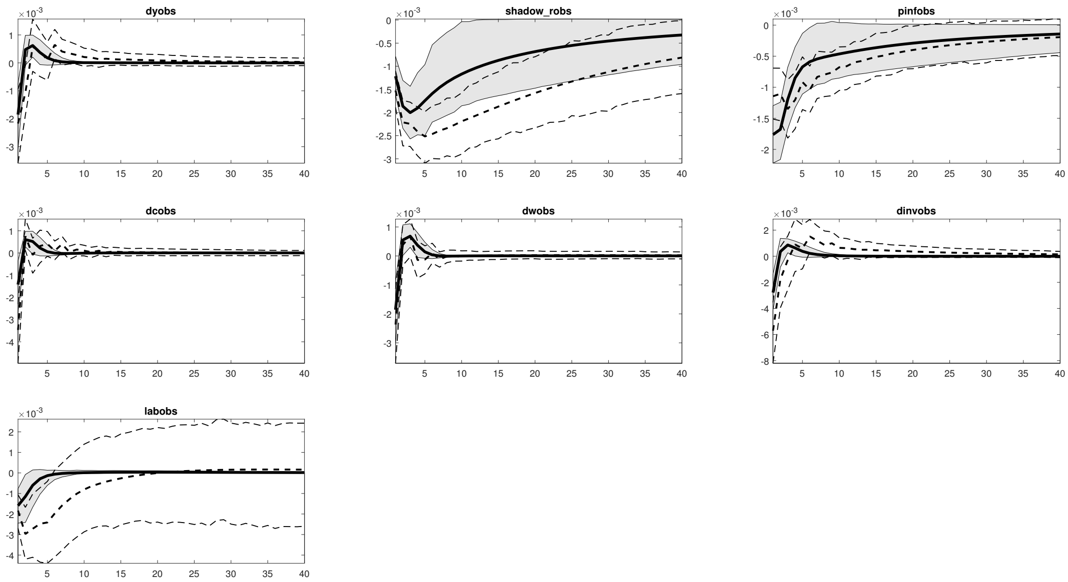

Figure 12: We compare the IRFs to preference shock, ϵ_{RPS} of the DSGE_VAR with the estimated DSGE.

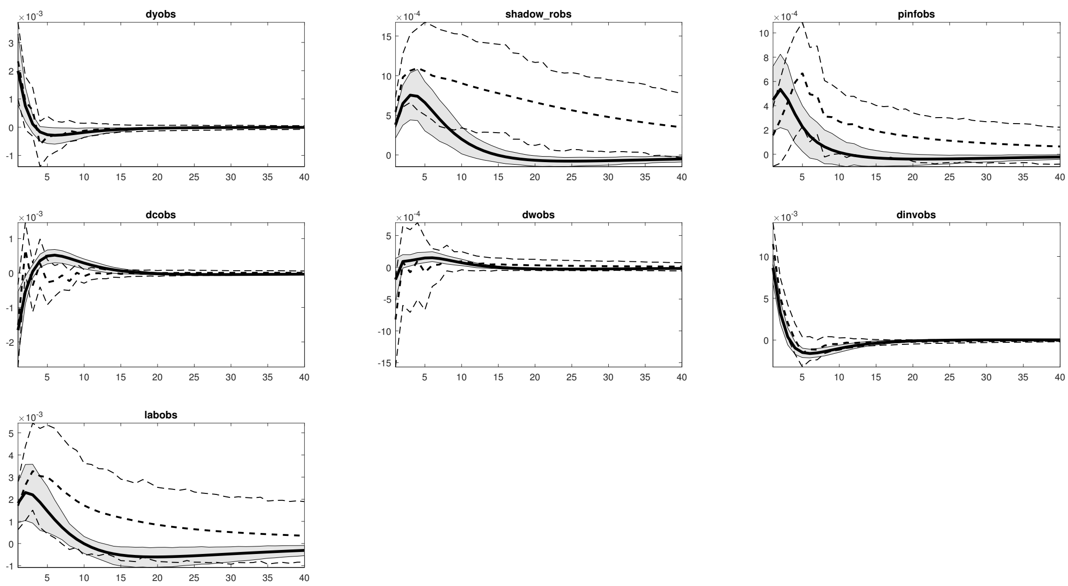

Figure 13: We compare the IRFs to investment shock, ϵ_{IS} of the DSGE_VAR with the estimated DSGE.

Table 1: Estimated results (posterior mean with a number of draws **equal to 100000**) of the pure DSGE and DSGE-VAR

Parameters	Notations	**Prior**			**DSGE-VAR**		**DSGE**	
		pdf	Mean	Std	Post. Mean	Sdt.	Post. Mean	Sdt.
Technology shock	ϵ_A	**IG**	0.001	0.02	0.005	0.0006	0.0083	0.0005
Government spending shock	ϵ_G	**IG**	0.001	0.02	0.0238	0.0023	0.0452	0.0021
Markup shock	ϵ_{MCS}	**IG**	0.001	0.02	0.0105	0.0007	0.0114	0.0007
Wage Markup shock	ϵ_{MRSS}	**IG**	0.001	0.02	0.0169	0.0032	0.0308	0.0040
Monetary shock	ϵ_{MPS}	**IG**	0.001	0.02	0.0019	0.0003	0.0033	0.0002
Preference shock	ϵ_{RPS}	**IG**	0.001	0.02	0.0017	0.0001	0.0028	0.0003
Investment shock	ϵ_{IS}	**IG**	0.001	0.02	0.0093	0.0017	0.0168	0.0018
AR1 technology shock	ρ_A	**B**	0.50	0.20	0.8962	0.0274	0.9730	0.0039
AR1 gov. spending shock	ρ_G	**B**	0.50	0.20	0.8713	0.0343	0.9427	0.0080
AR1 mark-up shock	ρ_{MCS}	**B**	0.50	0.20	0.5076	0.1184	0.9469	0.0127
AR1 Wage Markup shock	ρ_{MRSS}	**B**	0.50	0.20	0.9657	0.0096	0.9388	0.0109
AR1 Monetary shock	ρ_{MPS}	**B**	0.50	0.20	0.2208	0.0609	0.2944	0.0478
AR1 Preference shock	ρ_{RPS}	**B**	0.50	0.20	0.9022	0.0111	0.8769	0.0329
AR1 Investment shock	ρ_{IS}	**B**	0.50	0.20	0.7811	0.0040	0.9651	0.0086
Investment adj cost	ϕ_X	**N**	2	0.75	0.5306	0.0406	0.2531	0.0449
Inverse intertemporal EOS	σ	**N**	1.5	0.375	0.9207	0.1213	1.2945	0.0991
Internal Habit	χ	**B**	0.5	0.1	0.3278	0.0537	0.2101	0.0350
Weight on Leisure in utility	ψ	**N**	2	0.75	1.3936	0.6148	2.1395	0.4105
Calvo's price	ξ_p	**B**	0.50	0.10	0.5347	0.0584	0.4425	0.0359
Calvo's wage	ξ_w	**B**	0.50	0.10	0.3609	0.0589	0.4839	0.0428
Price indexation	γ_p	**B**	0.50	0.10	0.4302	0.1058	0.4055	0.0895
Wage indexation	γ_w	**B**	0.50	0.10	0.5258	0.1055	0.6297	0.0882
Capital utilisation	γ_2	**B**	0.50	0.15	0.8306	0.0586	0.8537	0.0410
Profit	F	**N**	0.25	0.250	0.4992	0.1010	0.3698	0.0564
Feedback inflation	θ_π	**N**	2	0.25	2.0304	0.1649	2.5073	0.1421
Lagged interest rate	ρ_r	**B**	0.70	0.10	0.6962	0.0553	0.7323	0.0275
Feedback output gap	θ_y	**N**	0.125	0.05	0.0204	0.0180	-0.0036	0.0090
Feedback output growth	θ_{dy}	**N**	0.125	0.05	0.1507	0.0451	0.2039	0.0427
DSGE prior weight	λ	**Unif**	0.950	0.5485	0.3264	0.0413		

We next compare the ZLB mandate results between the estimated DSGE and estimated DSGE-VAR. Overall, the OSR of the DSGE-VAR also converges to a price-level rule. However, the optimal inflation target of the DSGE is significantly smaller than that of the DSGE-VAR.

Table 2: Comparing the welfare between the estimated DSGE and estimated DSGE-VAR. The CEV of the DSGE-VAR is calculated from the associated welfare of the Ramsey policy equaling to 71555.55, and Steady State Consumption Equivalent (CE) equaling to 76.76.

	(C) OSR with ZLB Mandate - PURE DSGE									
Regimes	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	Π^*	Act wel	CEV (%)	p_zlb	w_r^*	MPS
(C1) OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0	1.1	0.01	0.13	1.0094	-2639.69	-0.061	0.01	16	0.0
(C2) OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	1.0	1.73	0.02	0.26	1.006	-2639.48	-0.0334	0.05	8	0.0
(C3) OSR with ZLB ($\bar{p}_{zlb} = 0.096$)	1.0	2.64	0.03	0.44	1.004	-2639.41	-0.024	0.096	4	0.0

	(C) OSR with ZLB Mandate - DSGE-VAR									
Regimes	ρ_r^*	α_π^*	α_y^*	α_{dy}^*	Π^*	Act wel	CEV (%)	p_zlb	w_r^*	MPS
(C1) OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0	0.754	0.00	0.014	1.005	71554.1	-0.0189	0.01	12	0.0
(C2) OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	1.0	1.337	0.003	0.066	1.003	71554.8	-0.0098	0.05	4	0.0
(C3) OSR with ZLB ($\bar{p}_{zlb} = 0.096$)	1.0	1.733	0.01	0.104	1.0019	71555.0	-0.0072	0.096	2	0.0

E The Linear Quadratic Approximation Approach

This Section provides details of the LQ approach as used by Debortoli *et al.* (2019) and discussed in the Introduction. From Levine *et al.* (2008a) consider the following general deterministic optimization problem

$$\max \sum_{t=0}^{\infty} \beta^t U(X_t, W_t) \ \ s.t. \ \ X_t = f(X_{t-1}, W_t) \tag{E.1}$$

where X_t is vector of state variables and W_t a vector of instruments.(**) There are given initial and the usual tranversality conditions. For our purposes, we consider this as including models with forward-looking expectations, so that the optimal solution to the latter setup is the pre-commitment solution. Suppose the solution converges to a steady state X, W as $t \to \infty$ for the states X_t and the policies W_t. Define $x_t = X_t - X$ and $w_t = W_t - W$ as representing the first-order approximation to absolute deviations of states and policies from their steady states.(***)

The Lagrangian for the general problem is defined as,

$$\sum_{t=0}^{\infty} \beta^t [U(X_t, W_t) - \lambda_t'(X_t - f(X_{t-1}, W_t))] \tag{E.2}$$

where λ_t is the Lagrange multiplier so that a necessary condition for the solution to (E.1) is that the Lagrangian is stationary at all $\{X_s\}$, $\{W_s\}$ i.e.

$$U_W + \lambda_t' f_W = 0 \qquad U_X - \lambda_t' + \beta \lambda_t' f_X = 0 \tag{E.3}$$

(**) An alternative representation of the problem is $U(X_t, W_t)$ and $\mathbb{E}_t[X_{t+1}] = f(X_t, W_t)$ where X_t includes forward-looking non-predetermined variables and $\mathbb{E}_t[X_{t+1}] = X_{t+1}$ for the deterministic problem where perfect foresight applies. Whichever one uses, it is easy to switch from one to the other by a simple re-definition. As we demonstrate in Levine *et al.* (2008b), although the inclusion of forward-looking variables significantly alters the nature of the optimization problem, these changes only affect the boundary conditions and the second-order conditions, but not the steady state of the optimum which is all we require for LQ approximation.

(***) Alternatively $x_t = (X_t - X)/X$ and $w_t = (W_t - W)/W$, depending on the nature of the economic variable. Then the Theorem follows in a similar way with an appropriate adjustment to the Jacobian Matrix.

Assume a steady state λ for the Lagrange multipliers exists as well. Now define the Hamiltonian $H_t = U(X_t, W_t) - \lambda_t' f(X_{t-1}, W_t)$. The following is the discrete time version of Magill (1977):

Theorem: If a steady state solution (X, W, M) to the optimization problem (E.1) exists, then any perturbation (x_t, w_t) about this steady state can be expressed as the solution to

$$\max \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \begin{bmatrix} x_t' & w_t' \end{bmatrix} \begin{bmatrix} H_{XX} & H_{XW} \\ H_{WX} & H_{WW} \end{bmatrix} \begin{bmatrix} x_t \\ w_t \end{bmatrix} \quad s.t. \;\; x_t = f_X x_t + f_W w_t \tag{E.4}$$

where H_{XX}, etc denote second-order derivatives evaluated at (X, W). This can be directly extended to the case incorporating disturbances.

Thus our general procedure is as follows:

1. Set out the deterministic non-linear problem for the Ramsey Problem, to maximize the representative agents' utility subject to non-linear dynamic constraints.

2. Write down the Lagrangian for the problem.

3. Calculate the first order conditions. We do not require the initial conditions for an optimum since we ultimately only need the steady-state of the Ramsey problem.

4. Calculate the steady state of the first-order conditions. The terminal condition implied by this procedure is such that the system converges to this steady state.

5. Calculate a second-order Taylor series approximation, about the steady state, of the Hamiltonian associated with the Lagrangian in 2. **Note this involves only the steady state M of the multipliers**.

6. Calculate a first-order Taylor series approximation, about the steady state, of the first-order conditions and the original constraints.

7. Use 4. to eliminate the steady-state Lagrangian multipliers in 5. By appropriate elimination both the Hamiltonian and the constraints can be expressed in minimal form. This then gives us the accurate LQ approximation of the original non-linear optimization problem in the form of a minimal linear state-space representation of the constraints and a quadratic form of the utility expressed in terms of the states.

The Lagrangian for the NK model is

$$
\begin{aligned}
\sum_{t=0}^{\infty}\beta^t\Big[U(C_t,H_t) &+ \lambda_{1,t}\left(Y_t-\frac{(A_tH_t)^\alpha}{\Delta_t}\right)+\lambda_{2,t}\left(\frac{U_{H,t}}{U_{C,t}}+W_t\right)\\
&+ \lambda_{3,t}\left(W_t-\alpha MC_tA_t^\alpha H_t^{\alpha-1}\right)+\lambda_{4,t}\left(\Lambda_{t,t+1}-\beta\frac{U_{C,t+1}}{U_{C,t}}\right)\\
&+ \lambda_{5,t}\left(JJ_t-\xi\mathbb{E}_t[\Lambda_{t,t+1}\Pi_{t+1}^{\zeta-1}JJ_{t+1}]-Y_t\right)\\
&+ \lambda_{6,t}\left(J_t-\xi\mathbb{E}_t[\Lambda_{t,t+1}\Pi_{t+1}^{\zeta}J_{t+1}]-\left(\frac{1}{1-\frac{1}{\zeta}}\right)Y_tMC_tMS_t\right)\\
&+ \lambda_{7,t}\left(1-\xi\Pi_t^{\zeta-1}+(1-\xi)\left(\frac{J_t}{JJ_t}\right)^{1-\zeta}\right)\\
&+ \lambda_{8,t}\left(MC_t-\frac{W_t}{\alpha A_t^\alpha H_t^{\alpha-1}}\right)\\
&+ \lambda_{9,t}\left(\Delta_t-\xi\Pi_t^\zeta\Delta_{t-1}-(1-\xi)\left(\frac{J_t}{JJ_t}\right)^{-\zeta}\right)+\lambda_{10,t}(Y_t-C_t-C_t)\Big]
\end{aligned}
$$

From Levine *et al.* (2008a) with no habit ($h_C=0$) and $\Phi\equiv 1-\frac{1}{\zeta}$ the inefficiency wedge in the model here with wage flexibility (defined as α in Levine *et al.* (2008a)) we finally arrive at the *correct quadratic approximation to the nonlinear Ramsey problem* as the maximization of $E_0\left[\sum_{t=0}^{\infty}\beta^tU_t\right]$ with respect to $\{\pi_t\}$, subject to

$$
\pi_t=\beta\pi_{t+1}+\frac{(1-\xi)(1-\beta\xi)}{\xi}((\phi+\sigma)y_t-(1+\phi)a_t) \tag{E.5}
$$

where

$$
\begin{aligned}
U_t &= -\frac{1}{2\Phi}\frac{Y^{1+\phi}}{A^{1+\phi}}\Big[\sigma y_t^2+\phi(\Phi+\lambda_6(1+\phi))y_t^2-2(1+\phi)(\Phi+\lambda_6(1+\phi))y_ta_t+2\lambda_6\sigma y_t^2\\
&- \lambda_6\sigma(\sigma+1)y_t^2+\frac{\xi\zeta}{(1-\xi)(1-\beta\xi)}(\Phi+(1+\phi)\lambda_6)\pi_t^2\Big]
\end{aligned} \tag{E.6}
$$

where $\lambda_6=\frac{1-\Phi}{\sigma+\phi}$. Then in the efficient case we have $\Phi=1$ and $\lambda_6=0$ in which case the term in big brackets in (E.6) reduces to

$$
-\frac{1}{2}\left((\sigma+\phi)\left(y_t-\frac{1+\phi}{\sigma+\phi}a_t\right)^2+\frac{\xi\zeta}{(1-\beta\xi)(1-\xi)}\pi_t^2\right) \tag{E.7}
$$

plus terms in $\hat{a}_t$ which are independent of policy. Since $\frac{1+\phi}{\sigma+\phi}a$ is the flexible-price level of output this is a quadratic loss function in terms of inflation and the output gap. In fact it is the welfare-based loss function of the canonical NK model as derived by Woodford (2003).

F Quadratic Loss Function Mandates

Tables 3-6 set out the numerical results for our 4 quadratic mandates. The optimal mandate is described by the choice of weights (w_r^*, w_{dy}^*) for mandate I, w_r^* for mandate II, (w_r^*, w_{dw}^*) for mandate III and (w_r^*, w_{dh}^*) for mandate IV. In the tables we report the welfare-optimal mandate for the cases $\bar{p}_{zlb} = 0.01$, 0.05 and 0.096 by choosing appropriate grids for the pair of weights then we comparing the pair of weights that produce lowest welfare cost for each given $\bar{p}_{zlb}$. Table (4) is then the mandate equilibrium given the grids on the weights.

Table 3: Results for Mandate I

	OSR with Quadratic Mandate $(w_{dy}^* = 0.2)$							
Regimes	ρ_r^*	α_π^*	α_{dy}^*	Π^*	Ω	CEV (%)	$\bar{p}_{zlb}$	w_r^*
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0	0.78	0.08	1.0087	-2639.71	-0.0634	0.01	1
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	1.0	0.78	0.08	1.0046	-2639.53	-0.0399	0.05	1
OSR with ZLB ($\bar{p}_{zlb} = 0.096$)	1.0	0.78	0.08	1.0026	-2639.50	-0.0360	0.096	1
	OSR with Quadratic Mandate $(w_{dy}^* = 0.5)$							
Regimes	ρ_r^*	α_π^*	α_{dy}^*	Π^*	Ω	CEV (%)	$\bar{p}_{zlb}$	w_r^*
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0	0.95	0.23	1.0092	-2639.72	-0.0647	0.01	1
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	1.0	0.95	0.23	1.0050	-2639.53	-0.0399	0.05	1
OSR with ZLB ($\bar{p}_{zlb} = 0.096$)	1.0	0.95	0.23	1.0029	-2639.47	-0.0321	0.096	1
	OSR with Quadratic Mandate $(w_{dy} = 1.0)$							
Regimes	ρ_r^*	α_π^*	α_{dy}^*	Π^*	Ω	CEV (%)	$\bar{p}_{zlb}$	w_r^*
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0	1.23	0.5	1.01	-2639.80	-0.0752	0.01	1
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	1.0	1.23	0.5	1.0056	-2639.58	-0.0465	0.05	1
OSR with ZLB ($\bar{p}_{zlb} = 0.096$)	1.0	1.22	0.5	1.0033	-2639.51	-0.0373	0.096	1

Table 4: Results for Mandate II

	OSR with Quadratic Mandate							
Regimes	ρ_r^*	α_π^*	α_{dy}^*	Π^*	Ω	CEV (%)	$\bar{p}_{zlb}$	w_r^*
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0	1.02	0.0	1.0093	-2639.73	-0.0660	0.01	0.6
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	1.0	1.36	0.0	1.0055	-2639.53	-0.0399	0.05	0.4
OSR with ZLB ($\bar{p}_{zlb} = 0.096$)	1.0	1.36	0.0	1.0033	-2639.46	-0.0308	0.096	0.4

Table 5: Results for Mandate III

	OSR with Quadratic Mandate ($w_{dw} = 0.1$)							
Regimes	ρ_r^*	α_π^*	α_{dw}^*	Π^*	Ω	CEV (%)	$\bar{p}_{zlb}$	w_r^*
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0	1.15	0.03	1.0095	-2639.71	-0.0634	0.01	0.5
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	1.0	1.15	0.03	1.0052	-2639.51	-0.0373	0.05	0.5
OSR with ZLB ($\bar{p}_{zlb} = 0.096$)	1.0	1.15	0.03	1.0031	-2639.45	-0.0295	0.096	0.5
	OSR with Quadratic Mandate ($w_{dw} = 0.5$)							
Regimes	ρ_r^*	α_π^*	α_{dw}^*	Π^*	Ω	CEV (%)	$\bar{p}_{zlb}$	w_r^*
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0	1.24	0.24	1.0096	-2639.61	-0.0504	0.01	0.5
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	1.0	4.56	1.00	1.007	-2639.41	-0.0243	0.05	0.0
OSR with ZLB ($\bar{p}_{zlb} = 0.096$)	1.0	4.57	1.00	1.0045	-2639.33	-0.0138	0.096	0.0
	OSR with Quadratic Mandate ($w_{dw}^* = 1$)							
Regimes	ρ_r^*	α_π^*	α_{dw}^*	Π^*	Ω	CEV (%)	$\bar{p}_{zlb}$	w_r^*
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0	1.37	0.50	1.0098	**-2639.58**	-0.0465	0.01	0.5
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	1.0	2.73	1.00	1.0063	**-2639.36**	-0.0178	0.05	0.0
OSR with ZLB ($\bar{p}_{zlb} = 0.096$)	1.0	2.74	1.00	1.0039	**-2639.28**	-0.0073	0.096	0.0
	OSR with Quadratic Mandate ($w_{dw}^* = 1.5$)							
Regimes	ρ_r^*	α_π^*	α_{dw}^*	Π^*	Ω	CEV (%)	$\bar{p}_{zlb}$	w_r^*
OSR with ZLB ($\bar{p}_{zlb} = 0.01$)	1.0	0.90	0.43	1.0092	-2639.60	-0.0491	0.01	1
OSR with ZLB ($\bar{p}_{zlb} = 0.05$)	1.0	1.47	0.74	1.0056	-2639.38	-0.0204	0.05	0.5
OSR with ZLB ($\bar{p}_{zlb} = 0.096$)	1.0	1.46	0.73	1.0034	-2639.31	-0.0112	0.096	0.5

Table 6: Results for Mandate IV

	OSR with Quadratic Mandate ($w_{dh}=0.1$)							
Regimes	ρ_r^*	α_π^*	α_{dh}^*	Π^*	Ω	CEV (%)	$\bar{p}_{zlb}$	w_r^*
OSR with ZLB ($\bar{p}_{zlb}=0.01$)	1.0	1.27	0.05	1.0098	-2639.73	-0.0660	0.01	0.5
OSR with ZLB ($\bar{p}_{zlb}=0.05$)	1.0	1.26	0.05	1.0054	-2639.52	-0.0386	0.05	0.5
OSR with ZLB ($\bar{p}_{zlb}=0.096$)	1.0	1.26	0.05	1.0032	-2639.46	-0.0308	0.096	0.5
	OSR with Quadratic Mandate ($w_{dh}^*=0.2$)							
Regimes	ρ_r^*	α_π^*	α_{dh}^*	Π^*	Ω	CEV (%)	$\bar{p}_{zlb}$	w_r^*
OSR with ZLB ($\bar{p}_{zlb}=0.01$)	1.0	0.75	0.15	1.0090	-2639.74	-0.0674	0.01	1.5
OSR with ZLB ($\bar{p}_{zlb}=0.05$)	1.0	1.05	0.24	1.0056	-2639.53	-0.0399	0.05	1
OSR with ZLB ($\bar{p}_{zlb}=0.096$)	1.0	1.88	0.49	1.0041	-2639.45	-0.0295	0.096	0.5
	OSR with Quadratic Mandate ($w_{dh}=0.5$)							
Regimes	ρ_r^*	α_π^*	α_{dh}^*	Π^*	Ω	CEV (%)	$\bar{p}_{zlb}$	w_r^*
OSR with ZLB ($\bar{p}_{zlb}=0.01$)	1.0	0.78	0.26	1.0095	-2639.79	-0.0739	0.01	2
OSR with ZLB ($\bar{p}_{zlb}=0.05$)	1.0	1.0	0.36	1.0057	-2639.57	-0.0452	0.05	1.5
OSR with ZLB ($\bar{p}_{zlb}=0.096$)	1.0	1.42	0.55	1.0040	-2639.49	-0.0347	0.096	1

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