

Online Appendix to
“A Guide to Estimating the Canonical Income
Process in Quasidifferences”

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APPENDIX

A Biases in the variances fixed effects and shocks

Tables [A-1](#)–[A-3](#) document the results from regression analyses for the biases in the variance of fixed effects, persistent, and transitory shocks, respectively. Since we consider more than one true value for those variables, we express biases in each variable in percentages to their true values.

The variance of fixed effects is poorly identified when the true persistence is close to unity¹—there is a large upward bias of at least 150% regardless of the weighting matrix used and sample size in terms of the number of individuals; see columns (7)–(12) of [Table A-1](#). Biases are smaller when the true persistence is low; see columns (1)–(6). When the number of individuals is large and true persistence is low, optimal weighting results in a small downward bias that does not vary much with changes in the model variances. The biases are positive but still not large for equal and diagonal weighting and go down when the variance of transitory shocks is lower or the true variance of fixed effects is higher—[Table A-1](#).

The biases are typically small for the variances of persistent and transitory shocks, especially when N is large—[Tables A-2](#)–[A-3](#). For example, the biggest bias for the variance of persistent shocks is about 8% when the true persistence is low, N is small, and the last fifteen periods are used in estimation—column (6) of [Table A-2](#). Thus, for the variance of persistent shocks of 0.01, its biased estimate using quasidifferences is 0.0108. The biases vary with the size of the model parameters, but these effects are small. The variance of transitory shocks is biased downward for all the experiments we considered, although those biases are very small and become negligible when N is large—see the estimated constants in [Table A-3](#). Biases typically become smaller when the variance of fixed effects is higher and the true variance of transitory shocks is smaller.

Table A-1: BIAS IN THE ESTIMATED VARIANCE OF FIXED EFFECTS. REGRESSION ANALYSIS

		$\rho = 0.995$											
		$\rho = 0.90$				Panel A: $N = 1000$				Panel B: $N = 10000$			
Weighting	t =	Equal		Optimal		Diagonal		Equal		Optimal		Diagonal	
		[1-15], [16-30]	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
σ_α^2		-5.130*** (0.500)	-6.300*** (0.872)	-0.458 (1.561)	-9.434*** (2.773)	-4.514*** (0.427)	-6.701*** (0.685)	-32.439*** (3.262)	-78.226*** (4.280)	-107.474*** (11.234)	-67.056*** (16.530)	-40.088*** (3.123)	-60.505*** (4.987)
σ_η^2		9.473*** (0.914)	8.570*** (1.722)	4.074*** (1.379)	6.716** (2.605)	11.039*** (0.647)	17.222*** (1.094)	59.315*** (4.855)	14.237** (6.629)	-9.208 (7.587)	-7.331 (10.352)	102.283*** (3.321)	46.486*** (5.691)
σ_ϵ^2		17.496*** (0.914)	26.395*** (1.722)	0.364 (1.379)	-0.972 (2.605)	14.330*** (0.647)	18.515*** (1.094)	12.107** (4.855)	10.773 (6.629)	-12.222 (7.587)	-10.482 (10.352)	-14.513*** (3.321)	17.329*** (5.691)
Const.		19.580*** (0.758)	24.553*** (1.415)	-14.733*** (1.282)	-23.638*** (2.386)	27.716*** (0.541)	40.514*** (0.909)	269.445*** (4.100)	344.895*** (5.588)	204.682*** (7.989)	162.624*** (11.315)	266.978*** (2.969)	264.997*** (5.061)
σ_α^2		-5.956*** (0.307)	-7.818*** (0.415)	0.822 (0.730)	-0.292 (1.388)	-5.964*** (0.265)	-7.552*** (0.359)	-56.570*** (4.024)	-98.346*** (4.187)	-133.699*** (10.625)	-125.736*** (17.472)	-33.838*** (3.801)	-80.047*** (4.331)
σ_η^2		0.419 (0.448)	0.624 (0.781)	0.330 (0.544)	-0.448 (1.198)	3.565*** (0.374)	4.757*** (0.691)	-42.974*** (6.371)	-56.388*** (8.976)	-95.197*** (7.392)	-50.806*** (11.464)	14.416** (5.629)	-31.424*** (8.090)
σ_ϵ^2		8.272*** (0.448)	11.542*** (0.781)	-1.176** (0.544)	-3.183*** (1.198)	9.807*** (0.375)	14.355*** (0.691)	6.915 (6.371)	5.040 (8.976)	-2.413 (7.392)	-7.924 (11.464)	31.126*** (5.629)	13.442* (8.090)
Const.		10.882*** (0.383)	15.252*** (0.648)	-3.455*** (0.543)	-7.245*** (1.111)	17.818*** (0.321)	24.678*** (0.567)	269.569*** (5.355)	319.788*** (7.368)	279.107*** (7.673)	236.318*** (12.206)	272.102*** (4.737)	303.122*** (6.688)
No. obs.		1000	500	1000	500	1000	500	1000	500	1000	500	1000	500

Notes: The table contains the results from a regression of the bias in the variance of fixed effects measured in percent, $100 \cdot (\hat{\sigma}_\alpha^2 - \sigma_\alpha^2) / \sigma_\alpha^2$, on the standardized variances of fixed effects, σ_α^2 , persistent shocks, σ_η^2 , and transitory shocks, σ_ϵ^2 . Columns $t = [1-15], [16-30]$ utilize estimation data based on the first fifteen or thirty (last fifteen) observations from Tables 1-2. Standard errors are in parentheses. *** (**) [*] significant at the 1% (5%) [10%] level.

Table A-2: BIAS IN THE ESTIMATED VARIANCE OF PERSISTENT SHOCKS. REGRESSION ANALYSIS

		$\rho = 0.995$													
		$\rho = 0.90$				Equal				Optimal					
Weighting	t =	Equal		Diagonal		Equal		Optimal		Equal		Optimal		Diagonal	
		[1-15], [16-30]	[1-15], [16-30]	[1-15], [16-30]	[1-15], [16-30]	[1-15], [16-30]	[1-15], [16-30]	[1-15], [16-30]	[1-15], [16-30]	[1-15], [16-30]	[1-15], [16-30]	[1-15], [16-30]	[1-15], [16-30]	[1-15], [16-30]	[1-15], [16-30]
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)			
Panel A: N = 1000															
σ_α^2		-1.124*** (0.077)	-1.215*** (0.134)	0.133** (0.056)	-0.877*** (0.222)	-1.532*** (0.093)	-3.010*** (0.479)	-0.048 (0.067)	0.250** (0.111)	0.195*** (0.034)	0.068 (0.077)	-0.033 (0.122)	-0.505*** (0.229)		
σ_η^2		-0.061 (0.044)	-0.178** (0.074)	-0.697*** (0.045)	1.628*** (0.215)	-0.405*** (0.047)	2.145*** (0.324)	0.202*** (0.036)	0.273*** (0.064)	-0.685*** (0.035)	-0.047 (0.076)	0.111** (0.051)	1.195*** (0.114)		
σ_ϵ^2		0.624*** (0.044)	0.359*** (0.074)	0.116** (0.045)	-0.446** (0.215)	1.016*** (0.047)	2.719*** (0.324)	0.129*** (0.036)	0.237*** (0.064)	-0.009 (0.035)	0.204*** (0.076)	0.588*** (0.051)	1.248*** (0.114)		
Const.		1.365*** (0.050)	1.224*** (0.086)	-0.898*** (0.044)	2.636*** (0.201)	1.996*** (0.057)	8.226*** (0.342)	0.162*** (0.043)	0.060 (0.073)	-1.048*** (0.032)	-0.039 (0.070)	0.523*** (0.070)	2.374*** (0.140)		
Panel B: N = 10000															
σ_α^2		-0.195*** (0.025)	-0.265*** (0.043)	-0.001 (0.014)	-0.425*** (0.125)	-0.419*** (0.028)	-2.011*** (0.282)	0.115*** (0.022)	0.184*** (0.035)	0.028** (0.011)	-0.018 (0.029)	0.066*** (0.023)	0.191*** (0.057)		
σ_η^2		-0.074*** (0.013)	-0.029 (0.021)	-0.064*** (0.011)	1.298*** (0.166)	-0.125*** (0.014)	1.265*** (0.210)	0.167*** (0.012)	0.194*** (0.022)	-0.001 (0.010)	0.029 (0.021)	0.191*** (0.012)	0.381*** (0.044)		
σ_ϵ^2		0.126*** (0.013)	0.054*** (0.021)	0.004 (0.011)	0.127 (0.166)	0.243*** (0.014)	1.096*** (0.210)	0.006 (0.012)	-0.011 (0.022)	0.013 (0.010)	0.036* (0.021)	0.024* (0.012)	0.226*** (0.044)		
Const.		0.259*** (0.016)	0.240*** (0.026)	-0.093*** (0.011)	2.289*** (0.144)	0.522*** (0.017)	4.664*** (0.214)	-0.077*** (0.014)	-0.143*** (0.024)	-0.099*** (0.010)	0.041* (0.021)	-0.085*** (0.014)	0.080* (0.044)		
No. obs.		1000	500	1000	500	1000	500	1000	500	1000	500	1000	500	1000	

Notes: The table contains the results from a regression of the bias in the variance of persistent shocks measured in percent, $100 \cdot (\hat{\sigma}_\eta^2 - \sigma_\eta^2) / \sigma_\eta^2$, on the standardized variances of fixed effects, σ_α^2 , persistent shocks, σ_η^2 , and transitory shocks, σ_ϵ^2 . Columns t=[1-15], [1-30] utilize estimation data based on the first fifteen or thirty (last fifteen) observations from Tables 1-2. Standard errors are in parentheses. *** (**) [*] significant at the 1% (5%) [10%] level.

Table A-3: BIAS IN THE ESTIMATED VARIANCE OF TRANSITORY SHOCKS. REGRESSION ANALYSIS

		$\rho = 0.995$											
		$\rho = 0.90$				Equal				Optimal			
Weighting	t =	Equal	Optimal	Diagonal	Equal	Optimal	Diagonal	Equal	Optimal	Diagonal	Equal	Optimal	Diagonal
		[1-15], [16-30]	[1-15], [16-30]	[1-15], [16-30]	[1-15], [16-30]	[1-15], [16-30]	[1-15], [16-30]	[1-15], [16-30]	[1-15], [16-30]	[1-15], [16-30]	[1-15], [16-30]	[1-15], [16-30]	[1-15], [16-30]
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: N = 1000													
σ_α^2		0.810*** (0.063)	0.917*** (0.108)	0.771*** (0.058)	0.653*** (0.082)	0.806*** (0.064)	0.895*** (0.109)	0.109* (0.058)	-0.096 (0.092)	0.710*** (0.058)	0.311*** (0.070)	0.125** (0.059)	0.009 (0.094)
σ_η^2		-0.016 (0.035)	0.024 (0.055)	-0.017 (0.034)	-0.057 (0.050)	-0.045 (0.035)	-0.057 (0.056)	0.032 (0.027)	-0.023 (0.045)	0.054* (0.030)	-0.042 (0.039)	0.068** (0.028)	-0.022 (0.047)
σ_ϵ^2		-0.505*** (0.035)	-0.437*** (0.055)	-0.702*** (0.034)	-0.385*** (0.050)	-0.494*** (0.035)	-0.479*** (0.056)	-0.141*** (0.027)	-0.216*** (0.045)	-0.660*** (0.030)	-0.520*** (0.039)	-0.187*** (0.028)	-0.327*** (0.047)
Const.		-1.074*** (0.040)	-1.122*** (0.067)	-1.689*** (0.038)	-1.202*** (0.055)	-1.151*** (0.041)	-1.348*** (0.068)	-0.324*** (0.035)	-0.274*** (0.055)	-1.661*** (0.037)	-1.137*** (0.045)	-0.427*** (0.036)	-0.546*** (0.057)
Panel B: N = 10000													
σ_α^2		0.185*** (0.020)	0.257*** (0.035)	0.074*** (0.016)	0.083*** (0.024)	0.325*** (0.020)	0.315*** (0.033)	0.025 (0.018)	0.028 (0.029)	0.105*** (0.012)	0.085*** (0.020)	0.113*** (0.018)	0.108*** (0.032)
σ_η^2		0.051*** (0.011)	0.007 (0.017)	0.002 (0.009)	0.001 (0.015)	0.065*** (0.010)	0.046** (0.018)	-0.011 (0.009)	-0.014 (0.015)	-0.016** (0.007)	-0.026** (0.012)	-0.012 (0.008)	0.004 (0.014)
σ_ϵ^2		-0.134*** (0.011)	-0.131*** (0.017)	-0.073*** (0.009)	-0.023 (0.015)	-0.243*** (0.010)	-0.231*** (0.018)	-0.033*** (0.009)	-0.033*** (0.015)	-0.077*** (0.007)	-0.061*** (0.012)	-0.054*** (0.008)	-0.067*** (0.014)
Const.		-0.254*** (0.013)	-0.296*** (0.021)	-0.171*** (0.010)	-0.098*** (0.016)	-0.499*** (0.013)	-0.492*** (0.021)	-0.108*** (0.011)	-0.106*** (0.018)	-0.210*** (0.008)	-0.182*** (0.013)	-0.173*** (0.011)	-0.198*** (0.019)
No. obs.		1000	500	1000	500	1000	500	1000	500	1000	500	1000	500

Notes: The table contains the results from a regression of the bias in the variance of transitory shocks measured in percent, $100 \cdot (\hat{\sigma}_\epsilon^2 - \sigma_\epsilon^2) / \sigma_\epsilon^2$, on the standardized variances of fixed effects, σ_α^2 , persistent shocks, σ_η^2 , and transitory shocks, σ_ϵ^2 . Columns t=[1-15], [1-30] utilize estimation data based on the first fifteen or thirty (last fifteen) observations from Tables 1-2. Standard errors are in parentheses. *** (**) [*] significant at the 1% (5%) [10%] level.