Online Appendix of:
House prices and rents: a reappraisal

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A Proofs for model of section 2

Proof of Proposition 1. The first order conditions of problem (1) write:

$$1 - \frac{\mathbb{E}_t \beta_{it+1} R_t u'_{c_{it+1}}}{u'_{c_{it}}} - \mu_{it} = 0, \tag{A.1}$$

$$\begin{aligned} &-p_t + \frac{\mathbb{E}_t \beta_{it+1} p_{t+1} [(1-\delta_x) + l_{t+1}(1-h_{it+1})] u'_{c_{it+1}}}{u'_{c_{it}}} + \frac{\mathbb{E}_t \beta_{it+1} h_{it+1} u'_{hs_{it+1}}}{u'_{c_{it}}} \\ &+ \mu_{it} m (1-\delta_x) \mathbb{E}_t p_{t+1} + \pi^x_{it} = 0, \end{aligned} \tag{A.2}$$

$$-p_t l_t x_{it-1} + x_{it-1} \frac{u'_{hs_{it}}}{u'_{c_{it}}} + \pi^h_{it} - \pi^{1-h}_{it} = 0, \tag{A.3}$$

$$-p_t l_t + \phi \frac{u'_{hs_{it}}}{u'_{c_{it}}} + \pi^z_{it} = 0, \tag{A.4}$$

where marginal utilites are denoted $u'_{c_{it}}$ and $u'_{hs_{it}}$, and where μ_{it}, π^x_{it}, π^z_{it}, π^h_{it} and π^{1-h}_{it} are the multipliers (divided by $u'_{c_{it}}$) associated to non negativity constraints. The complementary slackness conditions are:

$$\mu_{it} [m(1-\delta_x) x_{it} \mathbb{E}_t p_{t+1} - d_{it}] = \pi^x_{it} x_{it} = \pi^z_{it} z_{it} = \pi^h_{it} h_{it} = \pi^{1-h}_{it} (1-h_{it}) = 0. \tag{A.5}$$

Let us first ignore μ_{it} and list the possible cases according to the remaining multipliers.

Case 1. $\pi^x_{it} > 0$ (and thus $x_{it} = 0$); in that case, the assumption on marginal utilities implies $z_{it+1} > 0$ (and thus $\pi^z_{it+1} = 0$). Note that the study of π^h_{it+1} and π^{1-h}_{it+1} is useless.

Case 2. $x_{it} > 0$ (and thus $\pi^x_{it} = 0$); we note that having simultaneously $\pi^h_{it+1} > 0$ and $\pi^{1-h}_{it+1} > 0$ is not possible. We therefore have five sub cases: *i)* $\pi^h_{it+1} > 0$ (and $h_{it+1} = 0$); the assumption on marginal utilities implies $z_{it+1} > 0$ (and thus $\pi^z_{it+1} = 0$). *ii)* $\pi^{1-h}_{it+1} > 0$ (and $h_{it+1} = 1$) and $z_{it+1} > 0$ (and thus $\pi^z_{it+1} = 0$). *iii)* $\pi^{1-h}_{it+1} > 0$ (and $h_{it+1} = 1$) and $z_{it+1} = 0$ (and thus $\pi^z_{it+1} \geq 0$). *iv)* $\pi^h_{it+1} = \pi^{1-h}_{it+1} = 0$ (and $h_{it+1} \in (0,1)$) and $z_{it+1} > 0$ (and thus $\pi^z_{it+1} = 0$). *v)* $\pi^h_{it+1} = \pi^{1-h}_{it+1} = 0$ (and $h_{it+1} \in (0,1)$) and $z_{it+1} = 0$ (and thus $\pi^z_{it+1} \geq 0$).

Rearranging (A.4) and (A.3) by eliminating $p_t l_t$, we obtain:

$$x_{it}\left((1-\phi)\frac{u'_{hs_{it+1}}}{u'_{c_{it+1}}}-\pi^z_{it+1}\right)+\pi^h_{it+1}-\pi^{1-h}_{it+1}=0. \tag{A.6}$$

If $\pi^h_{it+1}>0$, then $\pi^{1-h}_{it+1}=0$ and thus, using (A.6), we conclude that $\pi^z_{it+1}>0$; thus, *Case 2i* must be eliminated. If $\pi^h_{it+1}=\pi^{1-h}_{it+1}=0$, we conclude using (A.6) that $\pi^z_{it+1}>0$; thus, *Case 2iv* must be eliminated. □

Proof of Proposition 2. At the deterministic steady state, the first order conditions of problem (1) write:

$$1-\beta_i R-\mu_i=0 \tag{A.7}$$

$$-p+\beta_i p\left[(1-\delta_x)+l(1-h_i)\right]+\beta_i h_i\frac{u'_{hs_i}}{u'_{c_i}}+\mu_i m(1-\delta_x)p+\pi^x_i=0 \tag{A.8}$$

$$-plx_i+x_i\frac{u'_{hs_i}}{u'_{c_i}}+\pi^h_i-\pi^{1-h}_i=0 \tag{A.9}$$

$$-pl+\phi\frac{u'_{hs_i}}{u'_{c_i}}+\pi^z_i=0 \tag{A.10}$$

The proof proceed by proving a sequence of claims.

Claim 1. The dominant consumer satisfies $\mu_1=0$ while all other agents satisfy $\mu_i>0$.

The proof is standard. It relies on the fact that if some agents lend to others, their μ_i should be zero. Then, the only possible configuration given by (A.7) is $\mu_1=0$, $\beta_1=1/R$ and $\mu_i=1-\beta_i/\beta_1>0$ for all $i=2,3,...N$. Thus, from (A.5), we conclude that $d_i=m(1-\delta_x)x_i p$ for all $i=2,3,...N$, while $d_1<0$.

Claim 2. There is only one landlord, who is the dominant consumer.

A landlord is defined such that $\pi^h_i-\pi^{1-h}_i=0$, $\pi^x_i=0$ and $x_i>0$. Replacing these relations into (A.7), (A.8) and (A.9) and rearanging give:

$$-1+\beta_i\left[(1-\delta_x)+l\right]+\left(1-\frac{\beta_i}{\beta_1}\right)m(1-\delta_x)=0. \tag{A.11}$$

Provided that the steady-state exists, we see there is at most one β_i that solves (A.11). It is denoted β^*_i and satisfies:

$$\beta^*_i=\frac{1-m(1-\delta_x)}{\left[l+\left(1-\frac{m}{\beta_1}\right)(1-\delta_x)\right]}. \tag{A.12}$$

We observe that:

$$\beta_i^* \leq \beta_1 \Leftrightarrow -1 + \beta_1 \left[(1 - \delta_x) + l \right] \geq 0. \tag{A.13}$$

We now proceed by contradiction by showing that if the dominant consumer is not the landlord, she cannot be neither tenant nor owner-occupier. A tenant is defined by $x_i = \pi_i^z = 0$. Replacing these relations into (A.7), (A.8) and (A.10) and rearanging give:

$$-1 + \beta_i \left[(1 - \delta_x) + l \right] + \beta_i h_i l \left(\frac{1}{\phi} - 1 \right) + \left[1 - \frac{\beta_i}{\beta_1} \right] m \left(1 - \delta_x \right) + \frac{\pi_i^x}{p} = 0. \tag{A.14}$$

Therefore, if $\beta_i^* < \beta_1$, the LHS of (A.14) is strictly positive for $\beta_i = \beta_1$. Similarly, a owner-occupier is defined by $\pi_i^{1-h} > 0$ and $\pi_i^h = \pi_i^x = 0$. Replacing these relations into (A.7), (A.8) and (A.9) and rearranging give:

$$-1 + \beta_i \left[(1 - \delta_x) + l \right] + \beta_i h_i \frac{\pi_i^{1-h}}{p x_i} + \left[1 - \frac{\beta_i}{\beta_1} \right] m \left(1 - \delta_x \right) = 0. \tag{A.15}$$

if $\beta_i^* < \beta_1$, the LHS of (A.15) is strictly positive for $\beta_i = \beta_1$. Consequently, $\beta_i^* = \beta_1$ and

$$-1 + \beta_1 \left[(1 - \delta_x) + l \right] = 0. \tag{A.16}$$

Claim 3. There exists a unique value for β_i such that the $x_i > 0$ and $z_i > 0$; this value, denoted $\overline{\beta}_i$, belongs to $(0, \beta_1)$.

By replacing $\mu_i = 1 - \beta_i/\beta_1$, (A.10) and (A.16) in (A.8), we obtain the following condition:

$$\beta_i h_i \left[(1 - \phi) \frac{u'_{hs_i}}{u'_{c_i}} - \pi_i^z \right] - \left[1 - \frac{\beta_i}{\beta_1} \right] \left[1 - m \left(1 - \delta_x \right) \right] p + \pi_i^x = 0, \tag{A.17}$$

which holds for all $i = 2, 3, .., N$. If $x_i > 0$ and $z_i > 0$, one has $\pi_i^z = \pi_i^x = 0$, $h_i = 1$ and using (A.10) and (A.16), (A.17) can be rewritten as:

$$\beta_i \left(\frac{1}{\phi} - 1 \right) \left[\frac{1}{\beta_1} - (1 - \delta_x) \right] - \left[1 - \frac{\beta_i}{\beta_1} \right] \left[1 - m \left(1 - \delta_x \right) \right] = 0. \tag{A.18}$$

The latter expression hence holds for a unique β_i, denoted $\overline{\beta}_i$, which satisfies:

$$\overline{\beta}_i = \beta_1 \frac{\phi \left[1 - m \left(1 - \delta_x \right) \right]}{\left[(1 - \phi) \left[1 - \beta_1 \left(1 - \delta_x \right) \right] + \phi \left[1 - m \left(1 - \delta_x \right) \right] \right]}. \tag{A.19}$$

Using (A.19), it is obvious that $\overline{\beta}_i \in (0, \beta_1)$.

Claim 4. For all $i = 2, 3, ..., N$, $\pi_i^z > 0$ *if* $\beta_i > \overline{\beta}_i$.

The case $\pi_i^z > 0$ corresponds to $z_i = \pi_i^x = 0$ and $h_i = 1$. Using (A.17) and replacing (A.10) and (A.16) in it, we have:

$$\beta_i \left[\left(\frac{1}{\phi} - 1 \right) \left(\frac{1}{\beta_1} - (1 - \delta_x) \right) - \frac{\pi_i^z}{p\phi} \right] - \left[1 - \frac{\beta_i}{\beta_1} \right] [1 - m(1 - \delta_x)] = 0, \tag{A.20}$$

from whitch we deduce that $\pi_i^z \geq 0$ if and only if $\beta_i \geq \overline{\beta}_i$. □

Proof of Proposition 3. In the neighborhood of the deterministic steady state, the partition of the population described in Proposition 2 still holds, and in particular, the dominant consumer is the a unique landlord. The first order conditions (A.1) and (A.3) of this agent can be rewritten as:

$$1 - \frac{\mathbb{E}_t \beta_{1t+1} R_t u'_{c_{1t+1}}}{u'_{c_{1t}}} = 0 \qquad \text{and} \qquad -p_t l_t + \frac{u'_{hs_{1t}}}{u'_{c_{1t}}} = 0, \tag{A.21}$$

which can be replaced in (A.2) to obtain (2). □

B Derivation of the model of Section 3

This section describes the first-order conditions, constraints, production functions, shocks and market clearing conditions that define the equilibrium.

B.1 First order conditions of agent 1

The Lagrangean is:

$$\begin{aligned}
\mathcal{L} = & \mathbb{E}_0 \textstyle\sum_{t=0}^{\infty} \beta_1^t \tilde{\beta}_t \left[\ln c_{1t} + j_t \ln (h_t x_{1t-1}) - \chi_1 \frac{n_{1t}^\eta}{\eta} \right] \\
& + \beta_1^t \tilde{\beta}_t \lambda_{1t} \Big(p_t [(1 - \delta_x) + l_t (1 - h_t)] x_{1t-1} + r_t^c k_{t-1}^c + r_t^h k_{t-1}^h + p_t^q (1 + l_t^q) q_{t-1} \\
& + d_{1t} + w_t n_{1t} - c_{1t} - R_{t-1}^d d_{1t-1} - i_t^c - i_t^h - p_t x_{1t} - p_t^q q_t \Big) \\
& + \beta_1^t \tilde{\beta}_t \lambda_{1t} p_t^{kc} \left(\Upsilon_t^c i_t^c \left[1 - \frac{\phi_c}{2} \left(\frac{i_t^c}{i_{t-1}^c} - 1 \right)^2 \right] + (1 - \delta_c) k_{t-1}^c - k_t^c \right) \\
& + \beta_1^t \tilde{\beta}_t \lambda_{1t} p_t^{kh} \left(\Upsilon_t^h i_t^h \left[1 - \frac{\phi_h}{2} \left(\frac{i_{1t}^h}{i_{t-1}^h} - 1 \right)^2 \right] + (1 - \delta_h) k_{t-1}^h - k_t^h \right),
\end{aligned} \tag{B.1}$$

where λ_{1t}, $\lambda_{1t} Q_t^c$ and $\lambda_{1t} Q_t^h$ are the multiplyiers associated with constraints (8) and (10). The first-order conditions with respect to c_{1t}, h_t, n_{1t}, x_{1t}, d_{1t}, q_t, i_t^s and k_t^s with $s = \{c, h\}$, are given by:

$$\frac{1}{c_{1t}} - \lambda_{1t} = 0, \tag{B.2}$$

$$\frac{j_t}{h_{1t}} - \lambda_{1t} p_t l_t x_{1t-1} = 0, \tag{B.3}$$

$$\chi_1 n_{1t}^{\eta-1} - \lambda_{1t} w_t = 0, \tag{B.4}$$

$$p_t - \beta_1 \mathbb{E}_t \frac{\tilde{\beta}_{t+1}}{\tilde{\beta}_t} \frac{\lambda_{1t+1}}{\lambda_{1t}} \times \left[\frac{j_{t+1}}{x_{1t} \lambda_{1t+1}} + p_{t+1} \left[1 - \delta_x + l_{t+1} \left(1 - h_{t+1}\right)\right] \right] = 0, \tag{B.5}$$

$$\tilde{\beta}_t \lambda_{1t} - \beta_1 R_t^d \mathbb{E}_t \tilde{\beta}_{t+1} \lambda_{1t+1} = 0, \tag{B.6}$$

$$\lambda_{1t} \tilde{\beta}_t p_t^q - \beta_1 \mathbb{E}_t \lambda_{1t+1} \tilde{\beta}_{t+1} p_{t+1}^q \left(1 + l_{t+1}^q\right) = 0, \tag{B.7}$$

$$\begin{aligned} &-1 + p_t^{ks} \Upsilon_t^s \left[1 - \frac{\phi_s}{2} \left(\frac{i_t^s}{i_{t-1}^s} - 1\right)^2 - \phi_s \left(\frac{i_t^s}{i_{t-1}^s} - 1\right) \frac{i_t^s}{i_{t-1}^s}\right] \\ &\quad + \beta_1 \mathbb{E}_t \frac{\tilde{\beta}_{t+1}}{\tilde{\beta}_t} \frac{\lambda_{1t+1}}{\lambda_{1t}} p_{t+1}^{ks} \Upsilon_{t+1}^s \phi_s \left(\frac{i_{t+1}^s}{i_t^s} - 1\right) \left(\frac{i_{t+1}^s}{i_t^s}\right)^2 = 0, \end{aligned} \tag{B.8}$$

$$p_t^{ks} - \beta_1 \mathbb{E}_t \frac{\tilde{\beta}_{t+1}}{\tilde{\beta}_t} \frac{\lambda_{1t+1}}{\lambda_{1t}} \left[p_{t+1}^{ks} \left(1 - \delta_s\right) + r_{t+1}^s\right] = 0. \tag{B.9}$$

Using (B.2), conditions (B.3), (B.4), (B.6), (B.7), (B.8) and (B.9) can be rewritten as:

$$p_t l_t h_t x_{1t-1} = j_t c_{1t}, \tag{B.10}$$

$$\chi_1 n_{1t}^{\eta-1} = \frac{w_t}{c_{1t}}, \tag{B.11}$$

$$1 = R_t^d \beta_1 \mathbb{E}_t \frac{\tilde{\beta}_{t+1}}{\tilde{\beta}_t} \frac{c_{1t}}{c_{1t+1}}, \tag{B.12}$$

$$1 = \beta_1 \mathbb{E}_t \frac{\tilde{\beta}_{t+1}}{\tilde{\beta}_t} \frac{c_{1t}}{c_{1t+1}} \frac{p_{t+1}^q}{p_t^q} \left(1 + l_{t+1}^q\right), \tag{B.13}$$

$$\begin{aligned} 1 = &\, p_t^{ks} \Upsilon_t^s \left[1 - \frac{\phi_s}{2} \left(\frac{i_t^s}{i_{t-1}^s} - 1\right)^2 - \phi_s \left(\frac{i_t^s}{i_{t-1}^s} - 1\right) \frac{i_t^s}{i_{t-1}^s}\right] \\ &+ \beta_1 \mathbb{E}_t \frac{\tilde{\beta}_{t+1}}{\tilde{\beta}_t} \frac{c_{1t}}{c_{1t+1}} p_{t+1}^{ks} \Upsilon_{t+1}^s \phi_s \left(\frac{i_{t+1}^s}{i_t^s} - 1\right) \left(\frac{i_{t+1}^s}{i_t^s}\right)^2, \end{aligned} \tag{B.14}$$

$$p_t^{ks} = \beta_1 \mathbb{E}_t \frac{\tilde{\beta}_{t+1}}{\tilde{\beta}_t} \frac{c_{1t}}{c_{1t+1}} \left[p_{t+1}^{ks} \left(1 - \delta_s\right) + r_{t+1}^s\right], \tag{B.15}$$

while using (B.2) and (B.3), condition (B.5) can be rewritten as:

$$1 = \beta_1 \mathbb{E}_t \frac{\tilde{\beta}_{t+1}}{\tilde{\beta}_t} \frac{c_{1t}}{c_{1t+1}} \frac{p_{t+1}}{p_t} \left[\left(1 - \delta_x + l_{t+1}\right)\right]. \tag{B.16}$$

There are therefore 13 equations given by (8), (10), (B.2) and (B.10)-(B.16).

B.2 First order conditions of agent 2

The Lagrangean is:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_2^t \tilde{\beta}_t \left[\ln c_{2t} + j_t \ln x_{2t-1} - \chi_2 \frac{n_{2t}^{\eta}}{\eta} \right]$$
$$+\beta_2^t \tilde{\beta}_t \lambda_{2t} \Big(p_t \left(1 - \delta_x\right) x_{2t-1} + d_{2t} + w_t n_{2t}$$
$$-c_{2t} - R_{t-1}^d d_{2t-1} - p_t x_{2t} \Big)$$
$$+\beta_2^t \tilde{\beta}_t \lambda_{2t} \mu_{2t} \mathbb{E}_t \left(m_t p_{t+1} \left(1 - \delta_x\right) x_{2t} - d_{2t} \right),$$

where λ_{2t} and $\lambda_{2t}\mu_{2t}$ are the multiplyers associated with constraints (12) and (13). The first-order conditions with respect to c_{2t}, n_{2t}, x_{2t}, d_{2t}, are given by:

$$\frac{1}{c_{2t}} = \lambda_{2t}, \tag{B.17}$$
$$\chi_2 n_{2t}^{\eta-1} = \frac{w_t}{c_{2t}}, \tag{B.18}$$
$$p_t - \mu_{2t} m_t \left(1 - \delta_x\right) \mathbb{E}_t p_{t+1} = \beta_2 \mathbb{E}_t \frac{\tilde{\beta}_{t+1}}{\tilde{\beta}_t} \left[\frac{j_{t+1} c_{2t}}{x_{2t}} + \frac{c_{2t}}{c_{2t+1}} p_{t+1} \left[1 - \delta_x\right] \right], \tag{B.19}$$
$$\mu_{2t} = 1 - R_t^d \beta_2 \mathbb{E}_t \left[\frac{\tilde{\beta}_{t+1}}{\tilde{\beta}_t} \frac{c_{2t}}{c_{2t+1}} \right]. \tag{B.20}$$

There are therefore 6 equations given by (12), (13) and (B.17)-(B.20).

B.3 First order conditions of agent 3

The Lagrangean is:

$$\mathcal{L} = \ln c_{3t} + j \ln \left(\phi z_t\right) - \chi_3 \frac{n_{3t}^{\eta}}{\eta} + \lambda_{3t} \left[w_t n_{3t} - c_{3t} - p_t l_t z_t \right], \tag{B.21}$$

where λ_{3t} is the multiplyer associated to (15). The first-order conditions with respect to z_t and n_{3t} are:

$$-\frac{p_t l_t}{c_{3t}} + j \frac{1}{z_t} = 0, \tag{B.22}$$
$$-\chi_3 n_{3t}^{\eta-1} + \frac{w_t}{c_{3t}} = 0. \tag{B.23}$$

There are therefore 3 equations given by (15), (B.22) and (B.23).

B.4 First order conditions of firms

Profit in the consumption sector is written as:

$$A_{ct}K_{ct}^{\gamma_c}L_{ct}^{\alpha_c} - r_t^c K_{ct} - w_t L_{ct}. \tag{B.24}$$

Its maximization implies that the following conditions are satisfied:

$$\gamma_c \frac{Y_{ct}}{K_{ct}} = r_t^c, \tag{B.25}$$

$$\alpha_c \frac{Y_{ct}}{L_{ct}} = w_t. \tag{B.26}$$

Profit in the housing sector is:

$$p_t A_{ht} K_{ht}^{\gamma_h} L_{ht}^{\alpha_h} Q_t^{1-\alpha_h-\gamma_h} - r_t^h K_{ht} - w_t L_{ht} - p_t^q l_t^q Q_t.$$

The maximization implies:

$$\gamma_h \frac{Y_{ht}}{K_{ht}} = \frac{r_t^h}{p_t} \tag{B.27}$$

$$\alpha_h \frac{Y_{ht}}{L_{ht}} = \frac{w_t}{p_t} \tag{B.28}$$

$$(1-\alpha_h-\gamma_h)\frac{Y_{ht}}{Q_t} = \frac{p_t^q l_t^q}{p_t} \tag{B.29}$$

C Data

C.1 Data sources for estimation and computation of moments

Data for the estimation and the computation of moments stem from the following sources (in italics, we indicate for which variable these data are used):

- Board of Governors of the Federal Reserve System (US), 3-Month Treasury Bill: Secondary Market Rate [TB3MS], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/TB3MS, January 14, 2021. *Used to construct the series for the interest rate.*

- Board of Governors of the Federal Reserve System (US), Households and Nonprofit Organizations; One-to-Four-Family Residential Mortgages; Liability, Level [HHMSDODNS], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/HHMSDODNS, January 14, 2021. *Used to construct the series for household's debt.*

- Lincoln Institute of Land Policy, rent-price ratio (FHFA serie) https://www.lincolninst.edu. See also

  https://www.aei.org/historical-land-price-indicators/.

  Used for calibration targets requiring the level of the rent-price ratio.

- U.S. Bureau of Labor Statistics, Consumer Price Index for All Urban Consumers: Rent of Primary Residence in U.S. City Average [CUUR0000SEHA], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/CUUR0000SEHA, January 14, 2021. *Used to construct the series for the rent-price ratio for estimation.*

- U.S. Bureau of Economic Analysis, "Table 1.1. Current-Cost Net Stock of Fixed Assets and Consumer Durable Goods" (accessed January 14, 2021). *Used to compute ratio of residential real estate over output in calibration.*

- U.S. Bureau of Economic Analysis, "Table 1.1.5. Gross Domestic Product" (accessed January 14, 2021). *Used to construct the series for consumption, residential and non-residential investments and output.*

- U.S. Bureau of Labor Statistics, Nonfarm Business Sector: Implicit Price Deflator [IPDNBS], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/IPDNBS, January 14, 2021. *Used as price deflator.*

- U.S. Bureau of Labor Statistics, Population Level [CNP16OV], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/CNP16OV,January14,2021. *Used to express aggregates in per capita terms.*

- U.S. Census Bureau, Homeownership Rate for the United States [RHORUSQ156N], retrieved from FRED, Federal Reserve Bank of St. Louis;

https://fred.stlouisfed.org/series/RHORUSQ156N, January 14, 2021.*
Used to construct series on homeownership rate.

- U.S. Census Bureau house price index on single family homes: https://www.census.gov/construction/nrs/historical_data/index.html
Used to construct series for house prices.

C.2 Construction of data used for estimation and moments

To estimate the model and compute moments, we compute aggregates at quarterly frequency (when annualized), express them in real terms and in per capita terms when appropriate. Here is a summary of how the aggregates are computed in the paper:

- **Consumption**: BEA seasonally adjusted data on personal consumption expenditures deflated by the non-farm business sector implicit price deflator and divided by the US non-institutional population
- **Non-residential investment**: BEA seasonally adjusted data on non-residential fixed investment deflated by the non-farm business sector implicit price deflator and divided by the US non-institutional population
- **Residential investment**: BEA seasonally adjusted data on residential fixed investment deflated by the non-farm business sector implicit price deflator and divided by the US non-institutional population
- **GDP**: BEA seasonally adjusted data on GDP deflated by the non-farm business sector implicit price deflator and divided by the US non-institutional population
- **House prices**: U.S. Census Bureau house price index divided by implicit price deflator
- **Rent-price ratio**:
    - for estimation: rent index from BLS divided by U.S. Census Bureau house price index

*These data have been further deseasonalized using The the X-13ARIMA-SEATS Seasonal Adjustment Program (https://www.census.gov/srd/www/x13as/)

– for calibration: rent-price ratio from Lincoln institute[†]

- **Real interest rate**: constructed from 3-month T-bill and one-quarter ahead change in the non-farm business sector implicit price deflator

- **Debt**: series for One-to-Four-Family Residential Mortgages divided deflated by the non-farm business sector implicit price deflator and divided by the US non-institutional population

C.3 Data Sources for Figure 5

The sources used to construct figure 5 are:

- **For house prices**: U.S. Federal Housing Finance Agency, Purchase Only House Price Index for the United States [HPIPONM226S], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/HPIPONM226S, January 14, 2021.

- **For rent-price ratio**: U.S. Bureau of Labor Statistics, Consumer Price Index for All Urban Consumers: Rent of Primary Residence in U.S. City Average [CUUR0000SEHA], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/CUUR0000SEHA, January 14, 2021. (NB: we divide this series by the above series for house prices to obtain the rent-price ratio)

- **For interest rate**: Board of Governors of the Federal Reserve System (US), 3-Month Treasury Bill: Secondary Market Rate [TB3MS], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/TB3MS, January 14, 2021.

- **For household debt**: Board of Governors of the Federal Reserve System (US), Households and Nonprofit Organizations; One-to-Four-Family Residential Mortgages; Liability, Level [HHMSDODNS], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/HHMSDODNS, January, 2021.

[†]In steady state we have the following relation: $l = (\frac{1}{\beta_1} - 1) + \delta_x$. The parameter δ_x can therefore be calibrated if we have information on the *levels* of the interest rate and the rent-price ratio. We cannot recover the level of the rent-price ratio from the two indices used to construct the rent-price ratio series used for estimation. We therefore use the series from the Lincoln Institute to calibrate δ_x.

- **For residential investment**: U.S. Bureau of Economic Analysis, Private Residential Fixed Investment [PRFI], retrieved from FRED, Federal Reserve Bank of St. Louis; `https://fred.stlouisfed.org/series/PRFI`, January 14, 2021.

D Robustness

D.1 Alternative period not including the Great Recession

Table D.1 checks the ability of our model to match the data from the period before the Great Recession (1965-2006). The first column provides the moments for our series (1965-2006). Columns 2 and 3 show the simulation results for our model estimated on the series of the same time interval, by using a first order and second order approximation, respectively.

This shorter period is characterized (perhaps unsurprisingly) by less volatility for house prices, the rent-price ratio, residential investment and household debt relative to the results we obtain for the period 1965-2019 in the main text. One noticeable difference compared with the longer time period is that the correlation between house prices and interest rates is here -0.10 for the data and -0.14 for the model. Our model can thus match the low and positive correlation between interest rates and house prices for the period before the financial crisis.

D.2 Alternative series for house prices

Table D.2 provides the moments calculated for 1965-2016 on the series we use in our baseline estimation except for house prices. We use here the FHFA series from the Lincoln Institute.

The model is overall quite successful in matching the data moments with this alternative series. Both the series of house prices and rent-price ratios are more volatile with respect to the baseline ones and the model has hard time to reproduce this feature. Moreover, the series of house prices are a bit more correlated with consumption and less correlated with residential output.

Table D.1: Baseline model, moments for period 1965-2006.

	Data	Baseline model	
		1st order	2nd order
Std output	1.75	1.10	1.14
Standard deviations relative to output:			
House prices	1.19	1.43	1.50
Rent-price ratio	1.27	2.04	2.14
Consumption	0.83	0.76	0.69
Non-residential investment	2.52	3.10	2.98
Residential investment	5.89	8.50	14.62
Private debt	1.57	1.96	2.14
Interest rate	0.21	0.26	0.25
Cross-correlation between output and			
Consumption	0.92	0.80	0.75
Residential investment	0.72	0.39	0.39
Non-residential invesment	0.70	0.76	0.73
House prices	0.52	0.13	0.06
Cross-correlation between house prices and			
Consumption	0.44	0.27	0.24
Residential investment	0.45	0.29	0.12
Rent-price ratio	-0.84	-0.56	-0.62
Interest rates	-0.10	-0.21	-0.14

"Data": Data include series for the period 1965-2006. "1st order": predictions from model simulated with a first order approximation. "2nd order": predictions from model simulated with a second order approximation with pruning. Results are provided after all shocks. Results come from a simulation of the model of a 30000-periods length. The first 500 periods have been truncated. One-sided HP filter, lambda = 1600. Aggregates are expressed in *per capita* units.

D.3 Model with shocks to the collateral constraint

Table D.3 shows the moments for an estimated version of the model in which we allow for a stochastic loan-to-value parameter in the collateral constraint. This model tends to generate too much volatility with respect to the data, especially for house prices, residential investment, private debt and interest rates.

Table D.2: Baseline model with FHFA house prices, moments for period 1965-2016.

	Data	Baseline model	
		1st order	2nd order
Std output	1.69	1.12	1.13
Standard deviations relative to output:			
House prices	2.14	1.05	1.12
Rent-price ratio	2.13	1.49	1.12
Consumption	0.84	0.73	0.73
Non-residential investment	2.77	3.08	3.09
Residential investment	6.91	6.16	8.81
Private debt	1.88	1.54	1.65
Interest rate	0.21	0.23	0.23
Cross-correlation between output and			
Consumption	0.93	0.81	0.80
Residential investment	0.70	0.43	0.38
Non-residential invesment	0.70	0.78	0.77
House prices	0.56	0.25	0.21
Cross-correlation between house prices and			
Consumption	0.50	0.40	0.37
Residential investment	0.54	0.41	0.26
Rent-price ratio	-0.95	-0.45	-0.49
Interest rates	–0.04	-0.28	-0.25

"Data": Data include series for the period 1965-2016. Predictions from model simulated with a second order approximation with pruning. Results are provided after all shocks. Results come from a simulation of the model of a 30000-periods length. The first 500 periods have been truncated. One-sided HP filter, lambda = 1600. Aggregates are expressed in per capita units. For house prices, we use the FHFA series from the Lincoln Institute.

D.4 Model estimated and simulated with a 2-sided HP filter

Table D.4 provides the simulated moments of the model when it is estimated and simulated with a 2-sided HP filter. The choice of alternative filters does not affect significantly the results of our analysis.

Table D.3: Model with shocks to collateral constraint, moments for period 1965-2019.

	Data	Baseline model	
		1st order	2nd order
Std output	1.63	1.09	1.13
Standard deviations relative to output:			
House prices	1.59	1.63	1.75
Rent-price ratio	1.66	2.30	2.47
Consumption	0.84	0.73	0.67
Non-residential investment	2.79	3.28	3.12
Residential investment	6.94	9.42	17.30
Private debt	1.92	3.53	3.50
Interest rate	0.21	0.30	0.28
Cross-correlation between output and			
Consumption	0.92	0.78	0.72
Residential investment	0.69	0.32	0.35
Non-residential invesment	0.70	0.77	0.73
House prices	0.52	0.08	0.04
Cross-correlation between house prices and			
Consumption	0.45	0.23	0.22
Residential investment	0.63	0.31	0.12
Rent-price ratio	-0.91	-0.58	-0.64
Interest rates	0.03	-0.13	-0.08

Notes: Data include series for the period 1965-2019. "1st order": predictions from model simulated with a first order approximation. "2nd order": predictions from model simulated with a second order approximation with pruning. Results are provided after all shocks. Results come from a simulation of the model of a 30000-periods length. The first 500 periods have been truncated. HP filter, lambda = 1600. Aggregates are expressed in *per capita* units.

Table D.4: Baseline model estimated with two-sided HP filter, moments for period 1965-2019.

	Data	Baseline model	
		1st order	2nd order
Std output	1.60	1.14	1.17
Standard deviations relative to output:			
House prices	1.40	1.53	1.66
Rent-price ratio	1.43	2.18	2.28
Consumption	0.81	0.77	0.74
Non-residential investment	2.80	3.00	2.98
Residential investment	6.49	7.74	11.97
Private debt	1.41	2.19	2.39
Interest rate	0.22	0.31	0.30
Cross-correlation between output and			
Consumption	0.91	0.81	0.78
Residential investment	0.71	0.39	0.35
Non-residential invesment	0.74	0.74	0.73
House prices	0.56	0.16	0.12
Cross-correlation between house prices and			
Consumption	0.49	0.26	0.24
Residential investment	0.58	0.35	0.31
Rent-price ratio	-0.90	-0.52	-0.58
Interest rates	0.02	-0.19	-0.13

"Data": Data include series for the period 1965-2019. "1st order": predictions from model simulated with a first order approximation. "2nd order": predictions from model simulated with a second order approximation with pruning. Results are provided after all shocks. Results come from a simulation of the model of a 30000-periods length. The first 500 periods have been truncated. HP filter, lambda = 1600. Aggregates are expressed in *per capita* units.