Online appendix for the paper

Dual-normal Logic Programs – the Forgotten Class

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Appendix

Proof of Theorem 6

Proof. Let Z be a set of atoms, $\mathcal{S} \subseteq \mathcal{S}_Z$ a set of SE-interpretations that is complete and closed under here-union, and $\mathcal{Y} = \{Y : (X, Y) \in \mathcal{S}\}$. Consider $\hat{Y} \subseteq Z$ such that $(\hat{Y}, \hat{Y}) \notin \mathcal{S}$. Since $\mathcal{S}$ is complete, for every $Y \in \mathcal{Y}$, $(Y, Y) \in \mathcal{S}$. Thus, for every $Y \in \mathcal{Y}$, $Y \neq \hat{Y}$. We define

$$\mathcal{Y}' = \{Y \in \mathcal{Y} : Y \subseteq \hat{Y}\} \text{ and } \mathcal{Y}'' = \{Y \in \mathcal{Y} : Y \setminus \hat{Y} \neq \emptyset\}.$$

Clearly, $\mathcal{Y}'' \cap \mathcal{Y}' = \emptyset$ and $\mathcal{Y}' \cup \mathcal{Y}'' = \mathcal{Y}$. For each $Y \in \mathcal{Y}'$, we select an element $b_Y \in \hat{Y} \setminus Y$ (it is possible, as $Y \neq \hat{Y}$). Similarly, for each $Y \in \mathcal{Y}''$, we select an element $c_Y \in Y \setminus \hat{Y}$. We set $B_{\hat{Y}} = \{b_Y : Y \in \mathcal{Y}'\}$ and $C_{\hat{Y}} = \{c_Y : Y \in \mathcal{Y}''\}$, and we define

$$r_{\hat{Y}} = \ \leftarrow B_{\hat{Y}}, \neg C_{\hat{Y}}.$$

We note that for every $(X, Y) \in \mathcal{S}$, $(X, Y) \models_{SE} r_{\hat{Y}}$. Indeed, if $Y \in \mathcal{Y}'$, then $b_Y \in B_{\hat{Y}} \setminus Y$ and so, Condition (2) of Lemma 1 holds. Otherwise, $Y \in \mathcal{Y}''$ and $c_Y \in C_{\hat{Y}} \cap Y$. Thus, Condition (1) of that lemma holds. On the other hand, $(\hat{Y}, \hat{Y}) \not\models_{SE} r_{\hat{Y}}$. Indeed, $C_{\hat{Y}} \cap \hat{Y} = \emptyset$ and $B_{\hat{Y}} \subseteq \hat{Y}$, so neither Condition (1) nor Condition (2) holds. Moreover, neither Condition (3) nor Condition (4) holds, as $r_{\hat{Y}}$ is a constraint.

Next, let us consider $(\hat{X}, \hat{Y}) \notin \mathcal{S}$, where $\hat{Y} \in \mathcal{Y}$, and let us define $\mathcal{X} = \{X : (X, \hat{Y}) \in \mathcal{S}\}$. We set

$$\mathcal{X}' = \{X \in \mathcal{X} : X \subseteq \hat{X}\} \text{ and } \mathcal{X}'' = \{X \in \mathcal{X} : X \setminus \hat{X} \neq \emptyset\}.$$

If $\mathcal{X}' \neq \emptyset$, let $X_0 = \bigcup \mathcal{X}'$. Since $\mathcal{S}$ is closed under here-union, X_0 is a *proper* subset of X. We select an arbitrary element $b \in \hat{X} \setminus X_0$ and define $B = \{b\}$. Otherwise, we define $B = \emptyset$.

If $\mathcal{X}'' \neq \emptyset$, for each $X \in \mathcal{X}''$, we select $a_X \in X \setminus \hat{X}$, and we define $A = \{a_X \colon X \in \mathcal{X}''\}$. Otherwise, we select any element $a \in \hat{Y} \setminus \hat{X}$ and define $A = \{a\}$. We note that by construction, $A \subseteq \hat{Y}$.

Next, we define

$$\mathcal{Z} = \{Y \in \mathcal{Y} \setminus \{\hat{Y}\} \colon Y \setminus \hat{Y} \neq \emptyset\}.$$

For each $Y \in \mathcal{Z}$, we select $c_Y \in Y \setminus \hat{Y}$ and set $C = \{c_Y \colon Y \in \mathcal{Y}'\}$.

Finally, we define a rule $r_{(\hat{X},\hat{Y})}$ as

$$r_{(\hat{X},\hat{Y})} = A \leftarrow B, \neg C.$$

It is easy to see that $(\hat{X},\hat{Y}) \not\models_{SE} r_{(\hat{X},\hat{Y})}$. Indeed, by construction, $\hat{Y} \cap C = \emptyset$, $B \subseteq \hat{X} \subseteq \hat{Y}$, and $A \cap \hat{X} = \emptyset$. The second condition implies that $B \setminus \hat{Y} = \emptyset$ and $B \setminus \hat{X} = \emptyset$. Thus, none of the Conditions (1)–(4) of Lemma 1 holds.

We will show that for every $(X,Y) \in \mathcal{S}$, $(X,Y) \models_{SE} r_{(\hat{X},\hat{Y})}$. First, assume that $Y \setminus \hat{Y} \neq \emptyset$. It follows that $c_Y \in C \cap Y$ and so, $C \cap Y \neq \emptyset$. Thus, $(X,Y) \models_{SE} r_{(\hat{X},\hat{Y})}$ by Condition (1).

Assume that $Y \subseteq \hat{Y}$. Since $(X,Y) \in \mathcal{S}$ and $(\hat{Y},\hat{Y}) \in \mathcal{S}$, $(X,\hat{Y}) \in \mathcal{S}$. Thus, $X \in \mathcal{X}$. If $X \setminus \hat{X} \neq \emptyset$, then $\mathcal{X} \in \mathcal{X}''$ and so, $X \cap A \neq \emptyset$. Consequently, $(X,Y) \models_{SE} r_{(\hat{X},\hat{Y})}$ by Condition (3). Otherwise, $X \in \mathcal{X}'$ and $B = \{b\}$, for some $b \in \hat{X} \setminus X_0$. In particular, $B \setminus X \neq \emptyset$. Since $(X,Y) \in \mathcal{S}$, $(Y,Y) \in \mathcal{S}$ and so, $(Y,\hat{Y}) \in \mathcal{S}$. Consequently, $Y \in \mathcal{X}$. If $Y \in \mathcal{X}''$, then $Y \cap A \neq \emptyset$ and $(X,Y) \models_{SE} r_{(\hat{X},\hat{Y})}$ by Condition (4). If $Y \in \mathcal{X}'$, then $b \in \hat{X} \setminus Y$ and so, $B \setminus Y \neq \emptyset$. Thus, $(X,Y) \models_{SE} r_{(\hat{X},\hat{Y})}$ by Condition (2).

Let P consist of all rules $r_{\hat{Y}}$, where $\hat{Y} \subseteq Z$ and $Y \notin Y$ and of all rules $r_{(\hat{X},\hat{Y})}$ such that $\hat{X},\hat{Y} \subseteq Z$, $\hat{X} \subseteq \hat{Y}$ and $(\hat{X},\hat{Y}) \notin \mathcal{S}$. Clearly, $\mathcal{S} \subseteq SE(P)$. Let $(\hat{X},\hat{Y}) \notin \mathcal{S}$. If $\hat{Y} \notin \mathcal{Y}$, then $(\hat{Y},\hat{Y}) \not\models_{SE} r_{\hat{Y}}$. Thus, $(\hat{X},\hat{Y}) \notin SE(P)$. If $\hat{Y} \in \mathcal{Y}$, then $(\hat{X},\hat{Y}) \not\models_{SE} r_{(\hat{X},\hat{Y})}$. Thus, $(\hat{X},\hat{Y}) \notin SE(P)$. It follows that $SE(P) = \mathcal{S}$. ∎

Proof of Theorem 8

Proof. For every Z such that $(Z,Z) \in \mathcal{U}$, we define

$$\mathcal{U}_Z = \{X \colon (X,Y) \in \mathcal{U}, \text{ for some } Y \subseteq Z\}.$$

and we denote by $cl(\mathcal{U}_Z)$ the closure of $\mathcal{U}_Z$ under union. Finally, we define the *SE-closure* $\overline{\mathcal{U}}$ of $\mathcal{U}$ by setting

$$\overline{\mathcal{U}} = \{(X,Z) \colon X \in cl(\mathcal{U}_Z)\}.$$

We note that if $(X,Z) \in \overline{\mathcal{U}}$, then $X \in cl(\mathcal{U}_Z)$. Thus, $\mathcal{U}_Z$ is defined, that is, $(Z,Z) \in \mathcal{U}$. Consequently, $Z \in cl(\mathcal{U}_Z)$ and $(Z,Z) \in \overline{\mathcal{U}}$.

Next, assume that $(X,Y) \in \overline{\mathcal{U}}$, $(Z,Z) \in \overline{\mathcal{U}}$, and $Y \subset Z$. It follows that $X \in cl(\mathcal{U}_Y)$. Thus, there are sets $X_1,\ldots,X_k$ such that $X = \bigcup_{i=1}^{n} X_i$ and $X_i \in \mathcal{U}_Y$, for every $i =$

$1, \ldots, k$. Let us consider any such set X_i. By definition, there is a set Y' such that $(X_i, Y') \in \mathcal{U}$ and $Y' \subseteq Y$. Since $Y \subseteq Z$, $Y' \subseteq Z$. It follows that $X_i \in \mathcal{U}_Z$. Thus, $X_1, \ldots, X_k \in \mathcal{U}_Z$. Consequently, $X \in cl(\mathcal{U}_Z)$ and $(X, Z) \in \overline{\mathcal{U}}$.

Thus, $\overline{\mathcal{U}}$ is complete and, by the construction, closed under here-unions. It follows that there is a dual-normal program P such that $SE(P) = \overline{\mathcal{U}}$. We will show that $UE(P) = \mathcal{U}$.

First, let $(X, Y) \in \mathcal{U}$. It follows that $X \in \mathcal{U}_Y$. Thus, $X \in cl(\mathcal{U}_Y)$ and $(X, Y) \in \overline{\mathcal{U}}$. Consequently, $(X, Y) \in SE(P)$. Let us assume that for some $(X', Y) \in SE(P)$, $X \subset X' \subset Y$. Since $(X', Y) \in SE(P)$, $(X', Y) \in \overline{\mathcal{U}}$ and so, $X' \in cl(\mathcal{U}_Y)$. Thus, $X' = X_1 \cup \ldots \cup X_k$, where $X_1, \ldots, X_k \in \mathcal{U}_Y$ or, equivalently, $(X_1, Y), \ldots, (X_k, Y) \in \mathcal{U}$. Since $X' \subset Y$, it follows by splittability that there is $(Y', Y) \in \mathcal{U}$ such that $Y' \subset Y$ and $X_1 \cup \ldots \cup X_k \subseteq Y'$. Since $(X_1, Y) \in \mathcal{U}$ and $X_1 \subseteq Y' \subset Y$, it follows that $X_1 = Y'$. Consequently, $X' = X_1 \cup \ldots \cup X_k = Y'$. Thus, $(X', Y) \in \mathcal{U}$, a contradiction. It follows that $(X, Y) \in UE(P)$.

Conversely, let $(X, Y) \in UE(P)$. It follows that $(X, Y) \in SE(P)$ and, since $SE(P) = \overline{\mathcal{U}}$, $(X, Y) \in \overline{\mathcal{U}}$. By the definition, $X \in cl(\mathcal{U}_Y)$. Since $\mathcal{U}_Y$ is defined, $(Y, Y) \in \mathcal{U}$. Thus, if $X = Y$, the assertion follows. Otherwise, $X \subset Y$. In this case, we reason as follows. Since $X \in cl(\mathcal{U}_Y)$, as before we have $X = X_1 \cup \ldots \cup X_k$, for some sets X_i, $1 \leq i \leq k$, such that $(X_i, Y) \in \mathcal{U}$. By splittability, there is Y' such that $X_1 \cup \ldots \cup X_k \subseteq Y'$, $Y' \subset Y$ and $(Y', Y) \in \mathcal{U}$. Again as before, we obtain that $X_1 = Y'$ and so, $X = X_1 \cup \ldots \cup X_k = Y'$. Thus, $(X, Y) \in \mathcal{U}$. ∎