Online appendix for the paper

Adding Partial Functions to Constraint Logic Programming with Sets

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Appendix A

Introduction

This document contains the complete collection of the rewrite and inference rules for dealing with $\mathcal{PF}$-constraints which are used in the mentioned paper.

The rules have the following general form

$$\frac{\textit{pre-conditions}}{\{C_1,\ldots,C_n\} \to \{C'_1,\ldots,C'_m\}}$$

where C_i and C'_i are primitive $(\mathcal{SET},\mathcal{PF})$-constraints and *pre-conditions* are (possibly empty) boolean conditions on the terms occurring in $C_1,\ldots,C_n$. In order to apply the rule, all *pre-conditions* need to be satisfied. $\{C_1,\ldots,C_n\} \to \{C'_1,\ldots,C'_m\}$ $(n,\, m \geq 0)$ represents the changes in the constraint store caused by the rule application.

The following conventions and auxiliary predicates are used in the rules:

- $\mathcal{V}$ represents the set of variables
- $\mathsf{empty}(s)$ is an auxiliary predicate which is defined as follows: $s = \emptyset \vee (s = \mathsf{int}(x,y) \wedge x > y)$ (note that, $\neg\mathsf{empty}(s)$ holds also if s is an unbound variable).
- $\mathsf{is\_ground}(t)$ is an auxiliary predicate which is true when the term t does not contain any unbound variable;
- $\mathsf{is\_pfun}(r)$ is an auxiliary predicate which is defined as follows:

$$\mathsf{is\_pfun}(r) \text{ is true if and only if } \forall x, y_1, y_2([x,y_1] \in r \wedge [x,y_2] \in r \Rightarrow y_1 = y_2)$$

$\mathsf{is\_pfun}(r)$ is used to test whether a relation r, with a ground domain, is a partial function or not.

Rewrite rules for partial function constraints

$$\frac{r \in \mathcal{V}}{\{\mathsf{dom}(r, r)\} \to \{r = \emptyset\}} \tag{A1}$$

$$\frac{\mathsf{empty}(a)}{\{\mathsf{dom}(r, a)\} \to \{r = \emptyset\}} \tag{A2}$$

$$\frac{\mathsf{empty}(r)}{\{\mathsf{dom}(r, a)\} \to \{a = \emptyset\}} \tag{A3}$$

$$\frac{r = \{[x, y]|rr\} \qquad \neg\mathsf{empty}(a)}{\{\mathsf{dom}(r, a)\} \to \{a = \{x|rs\}, [x, y] \ \mathsf{nin} \ rr, \mathsf{dom}(rr, rs)\}} \tag{A4}$$

$$\frac{r \in \mathcal{V} \qquad a = \{x|rs\}}{\{\mathsf{dom}(r, a)\} \to \{r = \{[x, y]|rr\}, x \ \mathsf{nin} \ rs, \mathsf{dom}(rr, rs)\}} \tag{A5}$$

$$\frac{r \in \mathcal{V}}{\{\mathsf{ran}(r, r)\} \to \{r = \emptyset\}} \tag{A6}$$

$$\frac{\mathsf{empty}(a)}{\{\mathsf{ran}(r, a)\} \to \{r = \emptyset\}} \tag{A7}$$

$$\frac{\mathsf{empty}(r)}{\{\mathsf{ran}(r, a)\} \to \{a = \emptyset\}} \tag{A8}$$

$$\frac{r = \{[x, y]|rr\} \qquad \neg\mathsf{empty}(a)}{\{\mathsf{ran}(r, a)\} \to \{a = \{y|rs\}, [x, y] \ \mathsf{nin} \ rr, \mathsf{ran}(rr, rs)\}} \tag{A9}$$

$$\frac{\mathsf{empty}(r)}{\{\mathsf{comp}(r,s,q)\} \to \{q=\emptyset\}} \tag{A10}$$

$$\frac{\mathsf{empty}(s) \qquad \neg\mathsf{empty}(r)}{\{\mathsf{comp}(r,s,q)\} \to \{q=\emptyset\}} \tag{A11}$$

$$\frac{\mathsf{empty}(q) \qquad \neg\mathsf{empty}(r) \qquad \neg\mathsf{empty}(s)}{\{\mathsf{comp}(r,s,q)\} \to \{\mathsf{ran}(r,rr), \mathsf{dom}(s,ds), \mathsf{disj}(rr,ds)\}} \tag{A12}$$

$$\frac{q=\{[x,z]|rq\} \qquad \neg\mathsf{empty}(r) \qquad \neg\mathsf{empty}(s)}{\begin{array}{c}\{\mathsf{comp}(r,s,q)\} \to \{r=\{[x,y]|rr\}, \\ s=\{[y,z]|rs\}, [x,z] \ \mathsf{nin}\ rq, [y,z]\ \mathsf{nin}\ rs, \mathsf{comp}(rr,s,rq)\}\end{array}} \tag{A13}$$

$$\frac{q \in \mathcal{V} \qquad r=\{[x,y]|rr\} \qquad \neg\mathsf{empty}(s) \qquad s \notin \mathcal{V}}{\begin{array}{c}\{\mathsf{comp}(r,s,q)\} \to \{s=\{[y,z]|rs\}, \\ q=\{[x,z]|rq\}, [x,y]\ \mathsf{nin}\ rr, [y,z]\ \mathsf{nin}\ rs, \mathsf{comp}(rr,s,rq)\} \\ \text{or} \\ \{\mathsf{comp}(r,s,q)\} \to \{\mathsf{dom}(s,ds), y\ \mathsf{nin}\ ds, [x,y]\ \mathsf{nin}\ rr, \\ \mathsf{comp}(rr,s,q)\}\end{array}} \tag{A14}$$

$$\frac{\mathsf{empty}(r)}{\{\mathsf{pfun}(r)\} \to \{\}} \tag{A15}$$

$$\frac{r=\{t_1,\dots,t_n\} \qquad n \ge 1 \qquad \mathsf{is\_ground}(\operatorname{dom} r) \qquad \mathsf{is\_pfun}(r)}{\{\mathsf{pfun}(r)\} \to \{\}} \tag{A16}$$

$$\frac{r=\{[x,y]|rr\} \qquad \neg(\mathsf{is\_ground}(\operatorname{dom} r))}{\begin{array}{c}\{\mathsf{pfun}(r)\} \to \{\mathsf{dom}(rr,d), x\ \mathsf{nin}\ d, [x,y]\ \mathsf{nin}\ rr, \mathsf{pfun}(rr)\} \\ \text{or} \\ \{\mathsf{pfun}(r)\} \to \{rr=\{[x,y]|s\}, \\ \mathsf{dom}(s,d), x\ \mathsf{nin}\ d, [x,y]\ \mathsf{nin}\ s, \mathsf{pfun}(s)\}\end{array}} \tag{A17}$$

$$\frac{n \text{ is not an integer number}}{\{\mathsf{pfun}(r,n)\} \to \mathsf{false}} \tag{A18}$$

$$\frac{n=0}{\{\mathsf{pfun}(r,n)\} \to \{r=\emptyset\}} \tag{A19}$$

$$\frac{n \text{ is an integer number} > 0}{\{\mathsf{pfun}(r,n)\} \to \{\mathsf{pfun}(r), \mathsf{size}(r,m), m\ \mathsf{in}\ \mathsf{int}(0,n)\}} \tag{A20}$$

Inference rules for partial function constraints

$$\frac{}{\{\mathsf{dom}(r,a),\mathsf{dom}(r,b)\} \to \{\mathsf{dom}(r,a), a = b\}} \tag{A21}$$

$$\frac{}{\{\mathsf{ran}(r,a),\mathsf{ran}(r,b)\} \to \{\mathsf{ran}(r,a), a = b\}} \tag{A22}$$

$$\frac{}{\{\mathsf{dom}(r,a), a \ \mathsf{neq}\ \emptyset\} \to \{\mathsf{dom}(r,a), a \ \mathsf{neq}\ \emptyset, r \ \mathsf{neq}\ \emptyset\}} \tag{A23}$$

$$\frac{}{\{\mathsf{dom}(r,a), r \ \mathsf{neq}\ \emptyset\} \to \{\mathsf{dom}(r,a), r \ \mathsf{neq}\ \emptyset, a \ \mathsf{neq}\ \emptyset\}} \tag{A24}$$

$$\frac{}{\{\mathsf{ran}(r,a), a \ \mathsf{neq}\ \emptyset\} \to \{\mathsf{ran}(r,a), a \ \mathsf{neq}\ \emptyset, r \ \mathsf{neq}\ \emptyset\}} \tag{A25}$$

$$\frac{a \in \mathcal{V}}{\{\mathsf{ran}(r,a), r \ \mathsf{neq}\ \emptyset\} \to \{\mathsf{ran}(r,a), r \ \mathsf{neq}\ \emptyset, a \ \mathsf{neq}\ \emptyset\}} \tag{A26}$$

$$\frac{a \notin \mathcal{V}}{\{\mathsf{ran}(r,a)\} \to \{\mathsf{ran}(r,a), r \ \mathsf{neq}\ \emptyset\}} \tag{A27}$$

$$\frac{}{\{\mathsf{dom}(r,a)\} \to \{\mathsf{dom}(r,a), \mathsf{size}(r,n), \mathsf{size}(a,n)\}} \tag{A28}$$

$$\frac{}{\{\mathsf{ran}(r,a)\} \to \{\mathsf{ran}(r,a), \mathsf{size}(r,n), \mathsf{size}(a,m), m =< n\}} \tag{A29}$$

$$\frac{}{\begin{array}{ll}\{\mathsf{comp}(r,s,q)\} \to & \{\mathsf{comp}(r,s,q), \\ & \ \mathsf{dom}(q,a), \mathsf{dom}(r,b), \mathsf{subset}(a,b)\}\end{array}} \tag{A30}$$

$$\frac{}{\begin{array}{ll}\{\mathsf{comp}(r,s,q)\} \to & \{\mathsf{comp}(r,s,q), \\ & \ \mathsf{ran}(q,a), \mathsf{ran}(s,b), \mathsf{subset}(a,b)\}\end{array}} \tag{A31}$$

$$\frac{}{\{\mathsf{comp}(r,s,q)\} \to \{\mathsf{comp}(r,s,q), \mathsf{size}(q,n), \mathsf{size}(r,m), n \le m\}} \tag{A32}$$

$$\frac{}{\{\mathsf{un}(r,s,q),\mathsf{pfun}(q)\} \rightarrow \{\mathsf{un}(r,s,q),\mathsf{pfun}(q), \mathsf{dom}(r,dr),\mathsf{dom}(s,ds),\mathsf{dom}(q,dq), \mathsf{un}(dr,ds,dq)\}} \quad \text{(A33)}$$

$$\frac{}{\{\mathsf{un}(r,s,q),\mathsf{pfun}(q)\} \rightarrow \{\mathsf{un}(r,s,q),\mathsf{pfun}(q), \mathsf{ran}(r,rr),\mathsf{ran}(s,rs),\mathsf{ran}(q,rq), \mathsf{un}(rr,rs,rq)\}} \quad \text{(A34)}$$