8 APPENDIX

8.1 Proof of Theorem 1

The proof of Theorem 1 is done using a set of lemmas. In this sections we present those lemmas and then use them to prove Theorem 1.

Lemma 8.1. Let $\mathcal{G}_{\mathcal{S}} = (\mathbb{V}_{\mathcal{S}}, \mathbb{E}_{\mathcal{S}}, f_{\mathcal{S}})$ be a graphical representation of the sequence of sentences in a WSC problem. Then, Step 1 of the WiSCR Algorithm extracts a subgraph $\mathcal{G}'_{\mathcal{S}}$ of $\mathcal{G}_{\mathcal{S}}$ such that $\mathcal{G}'_{\mathcal{S}} = (\mathbb{V}'_{\mathcal{S}}, \mathbb{E}'_{\mathcal{S}}, f'_{\mathcal{S}})$ where $\mathbb{V}'_{\mathcal{S}} = \mathbb{V}_{\mathcal{S}} - \mathbb{V}^c_{\mathcal{S}}$, $\mathbb{V}^c_{\mathcal{S}}$ is a set of all the class nodes in $\mathcal{G}_{\mathcal{S}}$, $f'_{\mathcal{S}} = f_{\mathcal{S}}$, $\mathbb{E}'_{\mathcal{S}} = \mathbb{E}_{\mathcal{S}} - \mathbb{E}^c_{\mathcal{S}}$, and $e \in \mathbb{E}^c_{\mathcal{S}}$ if $f(e) = instance\_of$.

Proof. According to the Step 1 of the WiSCR algorithm, given a graph $\mathcal{G}_{\mathcal{S}} = (\mathbb{V}_{\mathcal{S}}, \mathbb{E}_{\mathcal{S}}, f_{\mathcal{S}})$, a subgraph of it is extracted. Let $\mathcal{G}'_{\mathcal{S}} = (\mathbb{V}'_{\mathcal{S}}, \mathbb{E}'_{\mathcal{S}}, f'_{\mathcal{S}})$ be the extracted subgraph. $\mathbb{V}'_{\mathcal{S}}$ contains all the nodes from $\mathcal{G}_{\mathcal{S}}$ which are not class nodes, i.e., $\mathbb{V}'_{\mathcal{S}} = \mathbb{V}_{\mathcal{S}} - \mathbb{V}^c_{\mathcal{S}}$, $\mathbb{V}^c_{\mathcal{S}}$ is a set of all the class nodes in $\mathcal{G}_{\mathcal{S}}$. Also, $\mathbb{E}'_{\mathcal{S}}$ contains all the edges between the nodes in $\mathbb{V}'_{\mathcal{S}}$. So, by Definition 4 $\mathbb{E}'_{\mathcal{S}} = \mathbb{E}_{\mathcal{S}} - \mathbb{E}^c_{\mathcal{S}}$ where $e \in \mathbb{E}^c_{\mathcal{S}}$ if $f(e) = instance\_of$. Furthermore, no new edges or nodes are added to $\mathcal{G}'_{\mathcal{S}}$ so $f'_{\mathcal{S}} = f_{\mathcal{S}}$.

Hence, the step 1 of the WiSCR Algorithm extract a subgraph $\mathcal{G}'_{\mathcal{S}}$ from $\mathcal{G}_{\mathcal{S}}$ such that if $\mathcal{G}_{\mathcal{S}} = (\mathbb{V}_{\mathcal{S}}, \mathbb{E}_{\mathcal{S}}, f_{\mathcal{S}})$ then $\mathcal{G}'_{\mathcal{S}} = (\mathbb{V}'_{\mathcal{S}}, \mathbb{E}'_{\mathcal{S}}, f'_{\mathcal{S}})$ where $\mathbb{V}'_{\mathcal{S}} = \mathbb{V}_{\mathcal{S}} - \mathbb{V}^c_{\mathcal{S}}$, $\mathbb{V}^c_{\mathcal{S}}$ is a set of all the class nodes in $\mathcal{G}_{\mathcal{S}}$, $f'_{\mathcal{S}} = f_{\mathcal{S}}$, $\mathbb{E}'_{\mathcal{S}} = \mathbb{E}_{\mathcal{S}} - \mathbb{E}^c_{\mathcal{S}}$, and $e \in \mathbb{E}^c_{\mathcal{S}}$ iff $f(e) = instance\_of$. □

Lemma 8.2. Let $\mathcal{G}_{\mathcal{K}} = (\mathbb{V}_{\mathcal{K}}, \mathbb{E}_{\mathcal{K}}, f_{\mathcal{K}})$ be a graphical representation of a knowledge (By Definition 6). Then, Step 2 of the WiSCR Algorithm extracts a subgraph $\mathcal{G}'_{\mathcal{K}}$ from $\mathcal{G}_{\mathcal{K}}$ such that $\mathcal{G}'_{\mathcal{K}} = (\mathbb{V}'_{\mathcal{K}}, \mathbb{E}'_{\mathcal{K}}, f'_{\mathcal{K}})$ where $\mathbb{V}'_{\mathcal{K}} = \mathbb{V}_{\mathcal{K}} - \mathbb{V}^c_{\mathcal{K}}$, $\mathbb{V}^c_{\mathcal{K}}$ is a set of all the class nodes in $\mathcal{G}_{\mathcal{K}}$, $f'_{\mathcal{K}} = f_{\mathcal{K}}$, $\mathbb{E}'_{\mathcal{K}} = \mathbb{E}_{\mathcal{K}} - \mathbb{E}^c_{\mathcal{K}}$, and $e \in \mathbb{E}^c_{\mathcal{K}}$ if $f(e) \in \{instance\_of, is\_same\_as\}$.

Proof. According to the Step 2 of the WiSCR algorithm, given a graphical representation of a knowledge $\mathcal{G}_{\mathcal{K}} = (\mathbb{V}_{\mathcal{K}}, \mathbb{E}_{\mathcal{K}}, f_{\mathcal{K}})$, a subgraph of it is extracted. Let $\mathcal{G}'_{\mathcal{K}} = (\mathbb{V}'_{\mathcal{K}}, \mathbb{E}'_{\mathcal{K}}, f'_{\mathcal{K}})$ be the extracted subgraph. $\mathbb{V}'_{\mathcal{K}}$ contains all the nodes from $\mathcal{G}_{\mathcal{K}}$ which are not class nodes, i.e., $\mathbb{V}'_{\mathcal{K}} = \mathbb{V}_{\mathcal{K}} - \mathbb{V}^c_{\mathcal{K}}$, $\mathbb{V}^c_{\mathcal{K}}$ is a set of all the class nodes in $\mathcal{G}_{\mathcal{K}}$. Also, $\mathbb{E}'_{\mathcal{K}}$ contains all the edges between the nodes in $\mathbb{V}'_{\mathcal{K}}$ except the ones labeled as *'is_same_as'*. So, by Definition 6 $\mathbb{E}'_{\mathcal{K}} = \mathbb{E}_{\mathcal{K}} - \mathbb{E}^c_{\mathcal{K}}$, and $e \in \mathbb{E}^c_{\mathcal{K}}$ if $f(e) \in \{instance\_of, is\_same\_as\}$. Furthermore, no new edges or nodes are added to $\mathcal{G}'_{\mathcal{K}}$ so $f'_{\mathcal{K}} = f_{\mathcal{K}}$.

Hence, the step 2 of the WiSCR Algorithm extract a subgraph $\mathcal{G}'_{\mathcal{K}}$ from $\mathcal{G}_{\mathcal{K}}$ such that if $\mathcal{G}_{\mathcal{K}} = (\mathbb{V}_{\mathcal{K}}, \mathbb{E}_{\mathcal{K}}, f_{\mathcal{K}})$ then $\mathcal{G}'_{\mathcal{K}} = (\mathbb{V}'_{\mathcal{K}}, \mathbb{E}'_{\mathcal{K}}, f'_{\mathcal{K}})$ where $\mathbb{V}'_{\mathcal{K}} = \mathbb{V}_{\mathcal{K}} - \mathbb{V}^c_{\mathcal{K}}$, $\mathbb{V}^c_{\mathcal{K}}$ is a set of all the class nodes in $\mathcal{G}_{\mathcal{K}}$, $f'_{\mathcal{K}} = f_{\mathcal{K}}$, $\mathbb{E}'_{\mathcal{K}} = \mathbb{E}_{\mathcal{K}} - \mathbb{E}^c_{\mathcal{K}}$, and $e \in \mathbb{E}^c_{\mathcal{K}}$ if $f(e) \in \{instance\_of, is\_same\_as\}$. □

Lemma 8.3. Let $\mathcal{G}_{\mathcal{S}} = (\mathbb{V}_{\mathcal{S}},\mathbb{E}_{\mathcal{S}},f_{\mathcal{S}})$ be a graphical representation of a sequence of sentences in a WSC problem, $\mathcal{G}'_{\mathcal{S}} = (\mathbb{V}'_{\mathcal{S}},\mathbb{E}'_{\mathcal{S}},f'_{\mathcal{S}})$ be a subgraph of $\mathcal{G}_{\mathcal{S}}$ such that $\mathbb{V}'_{\mathcal{S}} = \mathbb{V}_{\mathcal{S}} - \mathbb{V}^c_{\mathcal{S}}$ where $\mathbb{V}^c_{\mathcal{S}}$ is the set of all the class nodes in $\mathcal{G}_{\mathcal{S}}$, $f'_{\mathcal{S}} = f_{\mathcal{S}}$ and $\mathbb{E}'_{\mathcal{S}} = \mathbb{E}_{\mathcal{S}} - \mathbb{E}^c_{\mathcal{S}}$ where $e \in \mathbb{E}^c_{\mathcal{S}}$ iff $f_{\mathcal{S}}(e) =$ "*instance_of*". Let $\mathcal{G}_{\mathcal{K}} = (\mathbb{V}_{\mathcal{K}},\mathbb{E}_{\mathcal{K}},f_{\mathcal{K}})$ be a graphical representation of a knowledge where $f_{\mathcal{K}}$ is defined using $f_{\mathcal{S}}$, $\mathcal{G}'_{\mathcal{K}} = (\mathbb{V}'_{\mathcal{K}},\mathbb{E}'_{\mathcal{K}},f'_{\mathcal{K}})$ be a subgraph of $\mathcal{G}_{\mathcal{K}}$ such that $\mathbb{V}'_{\mathcal{K}} = \mathbb{V}_{\mathcal{K}} - \mathbb{V}^c_{\mathcal{K}}$ where $\mathbb{V}^c_{\mathcal{K}}$ is the set of all the class nodes in $\mathcal{G}_{\mathcal{K}}$, $f'_{\mathcal{K}} = f_{\mathcal{K}}$ and $\mathbb{E}'_{\mathcal{K}} = \mathbb{E}_{\mathcal{K}} - \mathbb{E}^c_{\mathcal{K}}$ where $e \in \mathbb{E}^c_{\mathcal{K}}$ iff $f_{\mathcal{K}}(e) \in \{is\_same\_as, instance\_of\}$. Then, Step 3 of the WiSCR algorithm extracts the all possible sets of node pairs of the form (a,b) such that either there does not exist such a non-empty set or if $\mathbb{M}_i$ is one such non-empty set then,

- for each $(a, b) \in \mathbb{M}_i$, $a \in \mathbb{V}'_{\mathcal{S}}$ and $b \in \mathbb{V}'_{\mathcal{K}}$,

- for each $(a, b) \in \mathbb{M}_i$, a and b are instances of same class, i.e., $(a, i) \in \mathbb{E}_{\mathcal{S}}$, $(b, i) \in \mathbb{E}_{\mathcal{K}}$, $f_{\mathcal{S}}((a, i)) = instance\_of$ and $f_{\mathcal{K}}((b, i)) = instance\_of$
- if for every pair $(a,b) \in \mathbb{M}_i$, a is replaced by b in $\mathbb{V}'_{\mathcal{S}}$ then $\mathcal{G}'_{\mathcal{K}}$ becomes a subgraph of the node-replaced $\mathcal{G}'_{\mathcal{S}}$

Proof. (i) Given a graphical representation of the sentences in a WSC problem (say $\mathcal{G}_{\mathcal{S}} = (\mathbb{V}_{\mathcal{S}},\mathbb{E}_{\mathcal{S}},f_{\mathcal{S}})$) and Lemma 8.1, the Step 1 of the WiSCR algorithm produces a subgraph of $\mathcal{G}_{\mathcal{S}}$ (say $\mathcal{G}'_{\mathcal{S}} = (\mathbb{V}'_{\mathcal{S}},\mathbb{E}'_{\mathcal{S}},f'_{\mathcal{S}})$) such that $\mathbb{V}'_{\mathcal{S}} = \mathbb{V}_{\mathcal{S}} - \mathbb{V}^c_{\mathcal{S}}$ where $\mathbb{V}^c_{\mathcal{S}}$ is a set of all the class nodes in $\mathcal{G}_{\mathcal{S}}$, $f'_{\mathcal{S}} = f_{\mathcal{S}}$ and $\mathbb{E}'_{\mathcal{S}} = \mathbb{E}_{\mathcal{S}} - \mathbb{E}^c_{\mathcal{S}}$ where $e \in \mathbb{E}^c_{\mathcal{S}}$ if $f_{\mathcal{S}}(e) = instance\_of$.

(ii) Given a graphical representation of a knowledge (say $\mathcal{G}_{\mathcal{K}} = (\mathbb{V}_{\mathcal{K}},\mathbb{E}_{\mathcal{K}},f_{\mathcal{K}})$) and Lemma 8.2, the step 2 of the WiSCR algorithm produces a subgraph of $\mathcal{G}_{\mathcal{K}}$ (say $\mathcal{G}'_{\mathcal{K}} = (\mathbb{V}'_{\mathcal{K}},\mathbb{E}'_{\mathcal{K}},f'_{\mathcal{K}})$) such that $\mathbb{V}'_{\mathcal{K}} = \mathbb{V}_{\mathcal{K}} - \mathbb{V}^c_{\mathcal{K}}$ where $\mathbb{V}^c_{\mathcal{K}}$ is a set of all the class nodes in $\mathcal{G}_{\mathcal{K}}$, $f'_{\mathcal{K}} = f_{\mathcal{K}}$ and $\mathbb{E}'_{\mathcal{K}} = \mathbb{E}_{\mathcal{K}} - \mathbb{E}^c_{\mathcal{K}}$ where $e \in \mathbb{E}^c_{\mathcal{K}}$ if $f_{\mathcal{K}}(e) \in \{instance\_of, is\_same\_as\}$.

(iii) Given $\mathcal{G}'_{\mathcal{S}}$ and $\mathcal{G}'_{\mathcal{K}}$ are the graphs generated by the steps 1 and 2 of the WiSCR algorithm respectively, then according to the Step 3 of the WiSCR algorithm, it extracts all possible graph-subgraph isomorphisms between $\mathcal{G}'_{\mathcal{S}}$ and $\mathcal{G}'_{\mathcal{K}}$. In other words, it extracts all possible sets of pairs of the form (a, b) such that either there does not exist such a non-empty set or if $\mathbb{M}_i$ is one such non-empty set then,

- for each $(a, b) \in \mathbb{M}_i$, $a \in \mathbb{V}'_{\mathcal{S}}$ and $b \in \mathbb{V}'_{\mathcal{K}}$,
- for each $(a, b) \in \mathbb{M}_i$, a and b are instances of same class, i.e., $(a, i) \in \mathbb{E}_{\mathcal{S}}$, $(b, i) \in \mathbb{E}_{\mathcal{K}}$, $f_{\mathcal{S}}((a, i)) = instance\_of$ and $f_{\mathcal{K}}((a, i)) = instance\_of$, and
- if for every pair $(a, b) \in \mathbb{M}_i$, a is replaced by b in $\mathbb{V}'_{\mathcal{S}}$ then $\mathcal{G}'_{\mathcal{K}}$ becomes a subgraph of the node-replaced $\mathcal{G}'_{\mathcal{S}}$

□

Theorem 1. Let $\mathcal{S}$ be a sequence of sentences in a WSC problem $\mathcal{P}$, $\mathcal{G}_{\mathcal{S}} = (\mathbb{V}_{\mathcal{S}}, \mathbb{E}_{\mathcal{S}}, f_{\mathcal{S}})$ be a graphical representation of $\mathcal{S}$, p be a node in $\mathcal{G}_{\mathcal{S}}$ such that it represents the pronoun to be resolved in $\mathcal{P}$, a_1 and a_2 be two nodes in $\mathcal{G}_{\mathcal{S}}$ such that they represent the two answer choices for $\mathcal{P}$, and $\mathcal{G}_{\mathcal{K}} = (\mathbb{V}_{\mathcal{K}}, \mathbb{E}_{\mathcal{K}}, f_{\mathcal{K}})$ be a graphical representation of a knowledge such that $f_{\mathcal{K}}$ is defined using $f_{\mathcal{S}}$. Then, the Winograd Schema Challenge Reasoning (WiSCR) algorithm outputs,

- a_1 as the answer of $\mathcal{P}$, if only a_1 provides the 'most natural resolution' (By Definition 7) for p in $\mathcal{G}_{\mathcal{S}}$,
- a_2 as the answer of $\mathcal{P}$, if only a_2 provides the 'most natural resolution' for p in $\mathcal{G}_{\mathcal{S}}$,
- No answer otherwise

Proof. If $\mathcal{G}_{\mathcal{S}} = (\mathbb{V}_{\mathcal{S}}, \mathbb{E}_{\mathcal{S}}, f_{\mathcal{S}})$ is a graphical representation of the sequence of sentences in a WSC problem then by Lemma 8.1, we have that
Step 1 of the WiSCR Algorithm extract a subgraph $\mathcal{G}'_{\mathcal{S}}$ from $\mathcal{G}_{\mathcal{S}}$ such that $\mathcal{G}'_{\mathcal{S}} = (\mathbb{V}'_{\mathcal{S}}, \mathbb{E}'_{\mathcal{S}}, f'_{\mathcal{S}})$ where $\mathbb{V}'_{\mathcal{S}} = \mathbb{V}_{\mathcal{S}} - \mathbb{V}^c_{\mathcal{S}}$, $\mathbb{V}^c_{\mathcal{S}}$ is a set of all the class nodes in $\mathcal{G}_{\mathcal{S}}$, $f'_{\mathcal{S}} = f_{\mathcal{S}}$, $\mathbb{E}'_{\mathcal{S}} = \mathbb{E}_{\mathcal{S}} - \mathbb{E}^c_{\mathcal{S}}$, and $e \in \mathbb{E}^c_{\mathcal{S}}$ if $f(e) = instance\_of$.

If $\mathcal{G}_{\mathcal{K}} = (\mathbb{V}_{\mathcal{K}}, \mathbb{E}_{\mathcal{K}}, f_{\mathcal{K}})$ is a graphical representation of a knowledge then by Lemma 8.2,

we have that
Step 2 of the WiSCR Algorithm extracts a subgraph $\mathcal{G}'_{\mathcal{K}}$ from $\mathcal{G}_{\mathcal{K}}$ such that $\mathcal{G}'_{\mathcal{K}} = (\mathbb{V}'_{\mathcal{K}}, \mathbb{E}'_{\mathcal{K}}, f'_{\mathcal{K}})$ where $\mathbb{V}'_{\mathcal{K}} = \mathbb{V}_{\mathcal{K}} - \mathbb{V}^c_{\mathcal{K}}$, $\mathbb{V}^c_{\mathcal{K}}$ is a set of all the class nodes in $\mathcal{G}_{\mathcal{K}}$, $f'_{\mathcal{K}} = f_{\mathcal{K}}$, $\mathbb{E}'_{\mathcal{K}} = \mathbb{E}_{\mathcal{K}} - \mathbb{E}^c_{\mathcal{K}}$, and $e \in \mathbb{E}^c_{\mathcal{K}}$ if $f(e) \in \{instance\_of, is\_same\_as\}$.

If $\mathcal{G}_{\mathcal{S}}$, $\mathcal{G}_{\mathcal{S}}{}'$, $\mathcal{G}_{\mathcal{K}}$ and $\mathcal{G}_{\mathcal{K}}{}'$ are inputs to the Step 3 of the WiSCR algorithm then by Lemma 8.3, we have that
Step 3 of the WiSCR algorithm produces all the possible sets of node pairs of the form (a, b) such that either there does not exist such a non-empty set or if $\mathbb{M}_i$ is one such non-empty set then,

- for each $(a, b) \in \mathbb{M}_i$, $a \in \mathbb{V}'_{\mathcal{S}}$ and $b \in \mathbb{V}'_{\mathcal{K}}$, and
- for each $(a, b) \in \mathbb{M}_i$, a and b are instances of same class, i.e., $(a, i) \in \mathbb{E}_{\mathcal{S}}$, $(b, i) \in \mathbb{E}_{\mathcal{K}}$, $f_{\mathcal{S}}((a, i)) = instance\_of$ and $f_{\mathcal{K}}((a, i)) = instance\_of$
- if for every pair $(a,b) \in \mathbb{M}_i$, a is replaced by b in $\mathbb{V}'_{\mathcal{S}}$ then $\mathcal{G}'_{\mathcal{K}}$ becomes an induced subgraph of $\mathcal{G}'_{\mathcal{S}}$

If $p \in \mathbb{V}_{\mathcal{S}}$ represents the pronoun to be resolved, $a_1, a_2 \in \mathbb{V}_{\mathcal{S}}$ represent the two answer choices. Then by Step 4 of the WiSCR algorithm and for each possible non-empty set of pairs (say $\mathbb{M}_i$) produced by Step 3, we have that

1. a_1 as an answer if,
   - $(p, n_1) \in \mathbb{M}_i$,
   - $(a_1, n_2) \in \mathbb{M}_i$,
   - $(n_1, n_2) \in \mathbb{E}_{\mathcal{K}}$ and $f_{\mathcal{K}}((n_1, n_2)) = is\_same\_as$, or $(n_2, n_1) \in \mathbb{E}_{\mathcal{K}}$ and $f_{\mathcal{K}}((n_2, n_1)) = is\_same\_as$, and
   - there does not exist an n and an x ($x \neq a_1$) such that $(x, n) \in \mathbb{M}_i$ and either $f_{\mathcal{K}}((n, n_1)) = is\_same\_as$ or $f_{\mathcal{K}}((n_1, n)) = is\_same\_as$.
2. a_2 as an answer if,
   - $(p, n_1) \in \mathbb{M}_i$,
   - $(a_2, n_2) \in \mathbb{M}_i$,
   - $(n_1, n_2) \in \mathbb{E}_{\mathcal{K}}$ where $f_{\mathcal{K}}((n_1, n_2)) = is\_same\_as$, or $(n_2, n_1) \in \mathbb{E}_{\mathcal{K}}$ where $f_{\mathcal{K}}((n_2, n_1)) = is\_same\_as$, and
   - there does not exist an n and an x ($x \neq a_2$) such that $(x, n) \in \mathbb{M}_i$ and either $f_{\mathcal{K}}((n, n_1)) = is\_same\_as$ or $f_{\mathcal{K}}((n_1, n)) = is\_same\_as$.
3. not answer is produced if neither a_1 nor a_2 are found as an answer

Then, the Step 4 of the WiSCR algorithm outputs a_1 as the final answer if only a_1 is found as an answer with respect to the possible node pairs extracted in the Step 3. The Step 4 of the WiSCR algorithm outputs a_2 as the final answer if only a_2 is found as an answer with respect to the possible node pairs extracted in the Step 3. The Step 4 of the algorithm does not answer anything otherwise.

By definition of 'most natural resolution' and above details of the Step 4 of the WiSCR algorithm, we have that

- a_1 is the answer of $\mathcal{P}$, if only a_1 provides the 'most natural resolution' (By Definition 7) for p in $\mathcal{G}_{\mathcal{S}}$,
- a_2 is the answer of $\mathcal{P}$, if only a_2 provides the 'most natural resolution' for p in $\mathcal{G}_{\mathcal{S}}$,
- No answer otherwise

The theorem is proved. □

8.2 Proof of Theorem 2

Theorem 2. Let $\mathcal{S}$ be a sequence of sentences in a WSC problem $\mathcal{P}$, $\mathbb{T}(S)$ be the set of tokens in $\mathcal{S}$, $p \in \mathbb{T}(S)$ be the token which represents the pronoun to be resolved, $a_1, a_2 \in \mathbb{T}(S)$ be two tokens which represent the two answer choices, $\mathcal{G}_{\mathcal{S}} = (\mathbb{V}_{\mathcal{S}}, \mathbb{E}_{\mathcal{S}}, f_{\mathcal{S}})$ be a graphical representation of $\mathcal{S}$, and $\mathcal{G}_{\mathcal{K}} = (\mathbb{V}_{\mathcal{K}}, \mathbb{E}_{\mathcal{K}}, f_{\mathcal{K}})$ be a representation of a knowledge such that $f_{\mathcal{K}}$ is defined using $f_{\mathcal{S}}$. Also, $\Pi(\mathcal{G}_S, \mathcal{G}_K, p, a_1, a_2)$ be the AnsProlog program for WiSCR algorithm and AnswerFinder be the python procedure defined in Section 4.2.5. Then, the WiSCR algorithm produces an answer x to the input WSC problem iff $\Pi(\mathcal{G}_S, \mathcal{G}_K, p, a_1, a_2)$ and AnswerFinder together output the answer x.

Proof. (i) Given the ASP encoding of a graphical representation of the sequence of sentences in a WSC problem, the rules `s11-s13` extract a subgraph such that it contains only the non class nodes from the original graphs and the edges which connect them. The nodes of the subgraph are represented using the predicate `node_G_s` and the edges are represented using the binary predicate `edge_G_S`. In other words, the rules `s11-s13` implement the Step 1 of the WiSCR algorithm.
(ii) Similar to (i) the rules `s21-s23` implement the Step 2 of the WiSCR algorithm.
(iii) Given the outputs of the rules `s11-s23`, and the ASP representations of the sequence of sentences in a WSC problem and a knowledge, the rules `s31-s37` first generate all possible matching pairs corresponding to the nodes of the graph of WSC sentences and the graph of knowledge, then a set of constraints are used to remove the possibilities which do not represent an isomorphism between the subgraphs of WSC sentences and knowledge. In other words, the rules `s31-s37` implement the Step 3 of the WiSCR algorithm.
(iv) Given an output of the rules `s31-s37`, and the ASP representations of the sequence of sentences in a WSC problem and a knowledge, the rules `s41-s49`
- output $ans(a_1)$ if $matches(p, n_1)$, $matches(a_1, n_2)$ are true and $has\_k(n_1, "is\_same\_as", n_2)$ or $has\_k(n_2, ``is\_same\_as'', n_1)$ is true, and there does not exist an n and an x ($x \neq a_1$) such that $matches(x, n)$ is true and either $has\_k(n_1, "is\_same\_as", n)$ or $has\_k(n, "is\_same\_as", n_1)$ is true.
- output $ans(a_2)$ if $matches(p, n_1)$, $matches(a_2, n_2)$ are true and $has\_k(n_1, "is\_same\_as", n_2)$ or $has\_k(n_2, ``is\_same\_as'', n_1)$ is true, and there does not exist an n and an x ($x \neq a_2$) such that $matches(x, n)$ is true and either $has\_k(n_1, "is\_same\_as", n)$ or $has\_k(n, "is\_same\_as", n_1)$ is true.
- do not satisfy the current interpretation

If more than one answers are produced and all of them correspond to one answer then AnswerFinder module outputs that as the final answer. Otherwise if zero answers are produced, or not all among the multiple answers correspond to a common answer then the AnswerFinder module does not output anything.

In other words, the rules `s41-s49` and the AnswerFinder module together implement the step 4 of the WiSCR algorithm.

By (i), (ii), (iii) and (iv), the WiSCR algorithm produces an answer x to the input WSC problem iff $\Pi(\mathcal{G}_S, \mathcal{G}_K, p, a_1, a_2)$ and AnswerFinder together output the answer x.

The theorem is proved.

□