#2 APPENDIX

UNIFORM to LOGNORMAL and BACK with Shannon ENTROPY

Discovered June 10, 2015 at 2:58 a.m. by Claudio Maccone.
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(%i1) kill(all);

(%o0) *done*

Equivalence between UNIFORM and LOGNORMAL distribution.

Equivalence of the two mean values.

(%i1) assume(d>b,sigma>0);

(%o1) $[d>b,\sigma>0]$

(%i2) mean_value_equivalence:(b+d)/2=%e^(mu+sigma^2/2);

(%o2) $\frac{d+b}{2}=\%e^{\frac{\sigma^2}{2}+\mu}$

Equivalence of the two standard deviations.

(%i3) st_dev_equivalence:(d-b)/(2*sqrt(3))=%e^(mu+sigma^2/2)*sqrt(%e^(sigma^2)-1);

(%o3) $\frac{d-b}{2\sqrt{3}}=\%e^{\frac{\sigma^2}{2}+\mu}\sqrt{\%e^{\sigma^2}-1}$

Resolving equation in sigma only.

(%i4) resolving_equation_in_sigma_only:st_dev_equivalence/mean_value_equivalence;

(%o4) $\frac{d-b}{\sqrt{3}\,(d+b)}=\sqrt{\%e^{\sigma^2}-1}$

(%i5) towards_sigma:first(factor(solve(resolving_equation_in_sigma_only^2+1,%e^sigma^2)));

(%o5) $\%e^{\sigma^2}=\frac{4\,(d^2+b\,d+b^2)}{3\,(d+b)^2}$

(%i6) def_sigma_square:log(towards_sigma);

(%o6) $\sigma^2=\log\left(\frac{4\,(d^2+b\,d+b^2)}{3\,(d+b)^2}\right)$

(%i7) def_sigma:sqrt(def_sigma_square);

(%o7) $\sigma=\sqrt{\log\left(\frac{4\,(d^2+b\,d+b^2)}{3\,(d+b)^2}\right)}$

(%i8) resolving_equation_in_mu_only:mean_value_equivalence,def_sigma_square;

(%o8) $\frac{d+b}{2}=\%e^{\mu+\frac{\log\left(\frac{4\,(d^2+b\,d+b^2)}{3\,(d+b)^2}\right)}{2}}$

(%i9) towards_mu:factor(first(solve(radcan(resolving_equation_in_mu_only),%e^mu)));

(%o9) $\%e^{\mu}=\frac{\sqrt{3}\,(d+b)^2}{4\,\sqrt{d^2+b\,d+b^2}}$

(%i10) def_mu:log(towards_mu);

(%o10) $\mu=\log\left(\frac{\sqrt{3}\,(d+b)^2}{4\,\sqrt{d^2+b\,d+b^2}}\right)$

Checking the Final LOGNORMAL MEAN VALUE.

(%i11) lognormal_mean_value:lhs(mean_value_equivalence),def_sigma_square,def_mu;

(%o11) $\frac{d+b}{2}$

Checking the Final LOGNORMAL STANDARD DEVIATION.

(%i12) lognormal_st_dev:lhs(st_dev_equivalence),def_sigma_square,def_mu;

(%o12) $\frac{d-b}{2\,\sqrt{3}}$

INVERTING the system, i.e. finding a and b vs. my and sigma.

(%i13) def_d_vs_mu_and_sigma:factor((2*mean_value_equivalence+(2*sqrt(3))*st_dev_equivalence))/2

(%o13) $d = \%e^{\frac{\sigma^2}{2}+\mu}\left(\sqrt{3}\,\sqrt{\%e^{\sigma^2}-1}+1\right)$

(%i14) def_b_vs_mu_and_sigma:factor((2*mean_value_equivalence-(2*sqrt(3))*st_dev_equivalence))/2

(%o14) $b = -\%e^{\frac{\sigma^2}{2}+\mu}\left(\sqrt{3}\,\sqrt{\%e^{\sigma^2}-1}-1\right)$

ENTROPY CHANGE in passing from the UNIFORM to the LOGNORMAL distribution,

(%i15) def_uniform_entropy:uniform_entropy=integrate(-(1/(d-b)*log(1/(d-b)))/log(2),x,b,d);

(%o15) $uniform\_entropy = \frac{\log(d-b)}{\log(2)}$

(%i16) def_lognormal_entropy:lognormal_entropy=(log(sqrt(2*%pi)*sigma)+mu+1/2)/log(2);

(%o16) $lognormal\_entropy = \frac{\log(\sqrt{2}\,\sqrt{\pi}\,\sigma)+\mu+\frac{1}{2}}{\log(2)}$

(%i17) lognormal_entropy_vs_b_and_d:def_lognormal_entropy,def_sigma,def_mu;

(%o17) $lognormal\_entropy = \frac{\log\left(\sqrt{2}\,\sqrt{\pi}\,\sqrt{\log\left(\frac{4\,(d^2+b\,d+b^2)}{3\,(d+b)^2}\right)}\right)+\log\left(\frac{\sqrt{3}\,(d+b)^2}{4\,\sqrt{d^2+b\,d+b^2}}\right)+\frac{1}{2}}{\log(2)}$

(%i18) logcontract(radcan(lognormal_entropy_vs_b_and_d));

(%o18) $lognormal\_entropy = -\frac{\log\left(\frac{8\,(d^2+b\,d+b^2)}{3\,\pi\,(d+b)^4\,\log\left(\frac{4\,(d^2+b\,d+b^2)}{3\,(d+b)^2}\right)}\right)-1}{2\,\log(2)}$

(%i19) def_uniform_entropy,def_b_vs_mu_and_sigma,def_d_vs_mu_and_sigma;

(%o19) $uniform\_entropy = \frac{\log\left(\%e^{\frac{\sigma^2}{2}+\mu}\left(\sqrt{3}\,\sqrt{\%e^{\sigma^2}-1}+1\right)+\%e^{\frac{\sigma^2}{2}+\mu}\left(\sqrt{3}\,\sqrt{\%e^{\sigma^2}-1}-1\right)\right)}{\log(2)}$

(%i20) def_uniform_entropy_vs_mu_and_sigma:radcan(%);

(%o20) $uniform\_entropy = \dfrac{\log\left(\%e^{\sigma^2}-1\right)+\sigma^2+2\,\mu+\log(3)+2\,\log(2)}{2\,\log(2)}$

(%i21) def_uniform_entropy_vs_mu_and_sigma-def_lognormal_entropy;

(%o21) $uniform\_entropy - lognormal\_entropy = \dfrac{\log\left(\%e^{\sigma^2}-1\right)+\sigma^2+2\,\mu+\log(3)+2\,\log(2)}{2\,\log(2)} - \dfrac{\log(\sqrt{2}\,\sqrt{\pi}\,\sigma)+\mu+\frac{1}{2}}{\log(2)}$

(%i22) logcontract(radcan(%));

(%o22) $uniform\_entropy - lognormal\_entropy = \dfrac{\log\left(\dfrac{6\,(\%e^{\sigma^2}-1)}{\pi\,\sigma^2}\right)+\sigma^2-1}{2\,\log(2)}$

(%i23) limit(%,sigma,0);

(%o23) $uniform\_entropy - lognormal\_entropy = \dfrac{\log\left(\dfrac{6}{\pi}\right)-1}{2\,\log(2)}$

(%i24) ev(%,numer);

(%o24) $uniform\_entropy - lognormal\_entropy = -0.25461433482006$