Appendices

Not for publication – for referees' use

A The model

A.1 The population

The participant population of the pension fund consists of individuals of ages 25 to 99 years. Individuals enter the fund at the age of 25 and remain with the fund for the rest of their life. Further, we assume that they retire at age a_R. The number of male and female participants of age a at time t is denoted as M_t^a and F_t^a, respectively, where $a \in [25, 99]$. Using projections of survival probabilities we can calculate the size of these cohorts in the future. Concretely, $M_{t+n}^{a+n} = q_{a,t}^{m,n} M_t^a$ and $F_{t+n}^{a+n} = q_{a,t}^{f,n} F_t^a$, where $q_{a,t}^{m,n}$ and $q_{a,t}^{f,n}$ are the probabilities that respectively a male and female person aged a in period t will survive another n years. The survival probabilities are deterministic. Hence, there is no longevity risk.

A.2 Wages

The wage level of the cohort of age a at time t is W_t^a and it is updated each time period. We assume a uniform wage level within each cohort, while over one's life the wage level follows a certain career profile. We set the wage levels of males and females equal. The nominal wage level evolves as follows:

$$W_t^a = W_{t-1}^{a-1} w_{t-1}^{a-1},$$

where w_{t-1}^{a-1} is the gross wage growth rate from period $t-1$ to t for a cohort aged $a-1$ in period $t-1$. This factor is the product of the economy-wide gross wage growth rate w_{t-1} and a component $\tilde{w}_{t-1}^{a-1}$ attributable to the progression of the career of someone of age $a-1$ in period $t-1$:

$$w_{t-1}^{a-1} = w_{t-1} \tilde{w}_{t-1}^{a-1}. \tag{13}$$

We refer to $\tilde{w}_{t-1}^{a-1}$ as the (gross) promotion rate from period $t-1$ to t for someone aged $a-1$ in period $t-1$. Note that $\tilde{w}_{t-1}^{a-1}$ is always greater than one if the career profile has an upward sloping shape. We assume that the career profile remains constant over time, which implies that $\tilde{w}_{t-1}^{a-1} = \tilde{w}^{a-1}$ in all periods. The economy-wide wage growth rate w_{t-1} is stochastic and is modelled as explained in Section 4.

A.3 The pension fund

A.3.1 Assets

Market value of the assets The market value of the fund's assets at the beginning of the next year, A_{t+1}, is equal to the asset value A_t at the beginning of this year, multiplied by its gross rate of return R_t, plus the net money inflow times the gross return over the half year over which it is on average invested:

$$A_{t+1} = A_t R_t + (C_t - B_t) R_t^{1/2}, \tag{14}$$

where C_t is the total amount of contributions received (calculated below) and B_t the total amount of benefits paid out. Since the benefits and contributions are (usually) paid on a monthly basis, while our model runs on a yearly basis, we assume that the payment of the benefits and contributions takes place in the middle of the calendar year and, hence, the net money inflow is invested on average for half a year until the beginning of the next year.

Calculation of the actuarial assets Pension fund assets in the U.S. are not measured at their market value when they are used as an input for pension policy. Rather, pension funds in the U.S. apply a smoothing procedure to come up with an actuarial value of their assets A_t^{act} in period t. Define the Investment Income Amount Of Immediate Recognition ($IIAOIR$) as a target investment income based on the *expected* gross return $\bar{R}_t$:

$$IIAOIR_t = A_t(\bar{R}_t - 1) + (C_t - B_t)(\bar{R}_t^{1/2} - 1). \tag{15}$$

The so-called Investment Income Market Total ($IIMT$) denotes the actual realization of investment income, i.e. the difference between the market value of the assets at the end of the year and the market value of the assets at the beginning of the year, less the net cash inflow associated with the contributions and the benefits. From equation (14) we get:

$$IIMT_t = A_{t+1} - A_t - (C_t - B_t) = A_t(R_t - 1) + (C_t - B_t)(R_t^{1/2} - 1). \tag{16}$$

Finally, the Investment Income Amount For Phased In Recognition ($IIAFPIR$) is realized investment income in excess of expected investment income:

$$IIAFPIR_t = IIMT_t - IIAOIR_t.$$

Hence, $IIAFPIR_t$ is positive when the actual investment return exceeds its expected value, and vice versa. The usual smoothing procedure to calculate actuarial assets involves taking the average of the excess investment incomes over the past. We define the Total Recognized

Investment Gain ($TRIG$) as the average of $IIAFPIR$ over a smoothing horizon of v years:

$$TRIG_t = \frac{1}{v}\sum_{i=0}^{v-1} IIAFPIR_{t-i}.$$

Then, the actuarial value of the assets at the beginning of year $t+1$ will be determined by adding to the actuarial value of the assets at the beginning of year t the net money inflows (contribution payments minus the benefit pay-outs), investment income of immediate recognition and the smoothed value of the excess investment income over the past v years:

$$A_{t+1}^{act} = A_t^{act} + (C_t - B_t) + IIAOIR_t + TRIG_t. \tag{17}$$

In short, the actuarial value of the assets at the end of the current year is equal to the actuarial value at the end of previous year, plus the net cash flows into the fund, plus the projected return on the assets, and a recognition of the smoothed difference between actual and expected investment income. In the special case that the initial actuarial assets equal the initial market value of the assets, $A_0^{act} = A_0$, and the smoothing period is shrunk to a single period, one has that $A_t^{act} = A_t$, for all $t \geq 0$. Hence, in this specific case the process of the actuarial assets coincides with that of the market value of the assets.

Calculation of the actuarial liabilities This subsection sketches the calculation of the actuarial liabilities, closely following Munnell et al. (2008b) and Novy-Marx and Rauh (2011). This requires the calculation of the (projected) benefits of the pension fund participants. The group of pension fund participants comprises the employees and the retired.

The fund's actuarial liabilities are the sum of the actuarial liabilities $L_t^{act,m}$ and $L_t^{act,f}$ to the male and female participants. The actuarial liabilities to each gender, in turn, are calculated by multiplying the individual actuarial liability $L_t^{act,a,\zeta}$ to an age-a and gender-ζ individual by the number of gender-ζ individuals in this cohort and then summing over the cohorts. Hence, the fund's actuarial liabilities are calculated as:

$$\begin{aligned} L_t^{act} &= L_t^{act,m} + L_t^{act,f} = \sum_{a=25}^{99}\left(M_t^a L_t^{act,a,m} + F_t^a L_t^{act,a,f}\right), \\ L_t^{act,a,\zeta} &= \sum_{i=\max(a_R-a,0)}^{99-a} \left(\tilde{R}_t^{(i)}\right)^{-i} q_{a,t}^{\zeta,i} K_{t,t+i}^{a+i}, \end{aligned} \tag{18}$$

where $K_{t,t+i}^{a+i}$ is the projection at time t of the pension pay-out i years ahead for a participant of age a and $\tilde{R}_t^{(i)}$ is the gross interest rate that the *pension fund* uses to discount to period t the cash flows materializing i periods into the future. It will be based on the median discount rate used by the pension funds in our dataset. The discount rate used by U.S. public pension plans is usually flat over the entire projection horizon and equal to the expected asset portfolio return. We notice that $\tilde{R}_t^{(0)} = 1$ and $q_{a,t}^{\zeta,0} = 1$. Summarizing, the liabilities to an individual of a particular cohort depend on the number of years he/she will (still) receive benefits, the level of the benefits, the discount rate and the survival probabilities.

Calculation of the pensioners' benefits The current pay-out to a retiree equals the current pension rights, B_t^a, while the projected pay-out i periods from now equals the current pension rights adjusted for projected future indexation:

$$K_{t,t}^{a} = B_t^a, \forall a \in [a_R, 99],$$
$$K_{t,t+i}^{a+i} = B_t^a \prod_{j=1}^{i} \left(1 + \pi_{t,t+j}^{p}\right), \forall i \in [1, 99-a], \forall a \in [a_R, 98],$$

where $\pi_{t,t+j}^{p}$ is the projection in period t of the COLA in period $t+j$. It is based on the average *actuarial* annual inflation projection used by the pension funds in our dataset. Current pension rights B_t^a equal the pension rights at the moment of retirement increased by the past *realized* indexation since then:

$$B_t^a = B_{t-(a-a_R)}^{a_R} \prod_{j=0}^{a-(a_R+1)} (1 + COLA_{t-j}), \forall a \in [a_R + 1, 99].$$

For example, the pension rights of a 70-year old person, who retired at the age $a_R = 65$, are equal to the benefit calculated when that person reached the age of 65, plus the indexation that has been awarded over the 5 years since then.

A common feature of the state pension plans in the U.S. is that benefits at the moment of retirement are based on the average wage preceding the moment of exit from the workforce. Concretely, the benefit of somebody retiring in year $t-(a-a_R)$ can be calculated as the product of the accrual rate ε, the number of years in the workforce (here, 40), and the average wage level over the past z years:

$$B_{t-(a-a_R)}^{a_R} = \frac{40\varepsilon}{z} \sum_{l=1}^{z} W_{t-(a-a_R)-l}^{a_R-l}, \forall a \in [a_R, 99].$$

The averaging period z varies from one to five years, with the majority of public plans applying a three–year average (Munnell et al., 2012).

Calculation of the workers' benefits The projected (at time t) pay-out to someone of age a is given by its current pension rights B_t^a adjusted for the actuarially-projected COLAs during retirement:

$$
\begin{aligned}
K_{t,t+i}^{a+i} &= B_t^a, i = a_R - a, \forall a \in [25, a_R - 1]\,, \\
K_{t,t+i}^{a+i} &= B_t^a \prod_{j=a_R+1-a}^{i} \left(1 + \pi_{t,t+j}^p\right), \forall i \in [a_R + 1 - a, 99 - a]\,, \forall a \in [25, a_R - 1]\,,
\end{aligned}
$$

where the first line gives the projected benefit during the first year of retirement, while the second line gives the projected benefit during the ensuing years in retirement, which thus takes into account the projection of the COLAs during retirement.

Methods for recognizing liabilities The value of a worker's pension rights B_t^a depends on the method used to recognizing liabilities. Under the ABO method only the pension rights accrued until time t are taken into account:

$$
\begin{aligned}
B_t^{25} &= 0, \\
B_t^a &= \frac{(a-25)\varepsilon}{\min(a-25, z)} \sum_{l=1}^{\min(a-25,z)} W_{t-l}^{a-l}, \forall a \in [26, a_R]\,.
\end{aligned}
$$

The youngest cohort of age 25 has just entered the fund and has no rights accrued yet. The rights of the other young cohorts who do not yet have z years in service are based on the average of the available wage history. For the cohorts that have at least z years of service, the pension rights are the product of the years in the workforce, the accrual rate and the average pay over the past z years. Hence, for an individual worker the ABO pension rights increase with each additional year of service.

Under the *PBO method*, we also take into account the effect of expected future salary increases on the rights accrued up to now. Hence, under this method B_t^a is the projected benefit level at retirement when the *actuarially-projected* salary advances are taken into account:

$$
\begin{aligned}
B_t^{25} &= 0, \\
B_t^a &= \frac{(a-25)\varepsilon}{z} \sum_{l=1}^{z} W_{t,t+(a_R-a)-l}^{p,a_R-l}, \forall a \in [26, a_R - 1]\,.
\end{aligned}
$$

where $W_{t,t+(a_R-a)-l}^{p,a_R-l}$ is the period-t actuarial projection of wage growth $W_{t+(a_R-a)-l}^{a_R-l}$ of someone aged $a_R - l$ in period $t + (a_R - a) - l$. The actuarial projection of the wage growth that took place in the past or the current wage growth (e.g., if $a = a_R - 1$ and $l = 1$), is equal to the realized wage growth. Again, the youngest cohort, the 25 years old, has no accrual yet and, hence, it has no pension rights in terms of the PBO. However, for a given age and a positive nominal wage growth projection, the pension rights of the other working cohorts are higher under the PBO method than under the ABO method.

Finally, state civil service jobs are relatively secure, so that the pension fund might in addition consider the rights that the employees will acquire in the future if they continue working in their job until retirement. The *PVB method* takes this into account. Therefore, it defines the pension rights B_t^a including future accrual due to new service:

$$B_t^a = \frac{40\varepsilon}{z} \sum_{l=1}^{z} W_{t,t+(a_R-a)-l}^{p,a_R-l}, \forall a \in [25, a_R - 1]\,. \tag{19}$$

Hence, this measure is based on the accrual over a full working life.

A.4 Benefits

The total amount of pension benefits to be paid out in year t is:

$$B_t = \sum_{a=a_R}^{99} (M_t^a + F_t^a) B_t^a.$$

A.5 Inputs for calculating the contributions

A.5.1 The entry-age normal costing method

The most common method for calculating the normal cost in public plans is the so-called *entry-age normal costing* (EAN) method. Under the EAN method, the employer's annual normal cost associated with an individual participant is calculated as a contribution throughout the projected years of service needed to finance the PVB obligation. Due to salary growth pension rights increase more than linearly over time. Hence, the method implies a component of front-loading, because the employer is pre-paying some of the future accrual (Munnell et al., 2008b).

The so-called *normal cost rate* (NCR) of an active participant is calculated at the entry age as the ratio of the actuarial (i.e., using the fund's discount rate) present values of the actuarially-projected benefits and career salary levels:

$$NCR_t^{25,\zeta} = \frac{L_t^{25,\zeta}}{PVW_t^{25,\zeta}}, \quad \zeta \in \{f; m\},$$

where $L_t^{25,\zeta}$ is the actuarial liability to someone of gender ζ who enters the labor force in period t as calculated in (18) on the basis of the PVB method, i.e. based on pension rights as calculated in (19), and $PVW_t^{25,\zeta}$ is the actuarial present value of all future wages throughout the participant's career as projected at entry. Hence, the normal cost rate is the percentage payment of a worker's projected career salary needed to cover the cost of the projected pension benefits for that worker at entry into the fund.

Further, the actuarial present value of actuarially-projected wages of a worker of age a and gender ζ is calculated as:

$$PVW_t^{a,\zeta} = \sum_{i=0}^{a_R-1-a} \left(\tilde{R}_t^{(i)}\right)^{-i} q_{a,t}^{\zeta,i} W_{t,t+i}^{p,a+i},$$

which takes account of the survival probabilities.

Finally, the (actuarial) *present value of the future normal cost* ($PVFNC$) of a worker of age a are the normal costs that will be recognized throughout her remaining years of service:

$$PVFNC_t^{a,\zeta} = PVW_t^{a,\zeta} \times NCR_{t-(a-25)}^{25,\zeta}.$$

Hence, this is the product of the actuarially discounted value of projected wages and the normal cost rate determined at the time when the worker entered the fund.

A.5.2 The actuarial accrued liability

The actuarial accrued liability (L_{accr}^{act}) of a participant is the difference between the actuarial liabilities to this participant, calculated as the actuarial liabilities, and the actuarial present value of the future normal cost:

$$L_{accr,t}^{act,a,\zeta} = L_t^{act,a,\zeta} - PVFNC_t^{a,\zeta}.$$

If we follow an individual worker over time, we will see that the actuarial present value of the future normal cost will decrease, as there will be fewer remaining years to pay the normal cost. Therefore, the actuarial accrued liability associated with an individual worker increases over time.

The actuarial accrued liability of pensioners is simply equal to the actuarial liabilities, since they should have paid the whole normal cost before reaching the retirement age. Therefore,

$$PVFNC_t^{a,\zeta} = 0 \Rightarrow L_{accr,t}^{act,a,\zeta} = L_t^{act,a,\zeta}, \forall a \in [a_R, 99].$$

Finally, the fund's actuarial accrued liability is the sum of the individual actuarial accrued liabilities over genders and cohorts:

$$L_{accr,t}^{act} = L_{accr,t}^{act,m} + L_{accr,t}^{act,f} = \sum_{a=25}^{99} \left(M_t^a L_{accr,t}^{act,a,m} + F_t^a L_{accr,t}^{act,a,f}\right). \tag{20}$$

A.6 Contributions

The aggregate volume of actuarial contributions by all the participants and the employer in year t is:

$$C_t^{act} = \sum_{a=25}^{a_R-1} \left(NCR_{t-(a-25)}^{25,m} M_t^a W_t^a + NCR_{t-(a-25)}^{25,f} F_t^a W_t^a \right) + \lambda AMORT_t,$$

where

$$AMORT_t = \begin{cases} \frac{1}{u} UAAL_t & \text{if } UAAL_t \geq 0, \\ 0 & \text{if } UAAL_t < 0, \end{cases}$$

and λ is the fraction of the required amortization payment *actually* paid. Notice that the first component of C_t^{act} is the sum over all working life ages and over the genders of the product of the gender-specific normal cost rate times the aggregate wage volume earned by each gender. The actuarial contribution rate c_t^{act} is expressed as a percentage of the total wage sum in year t:

$$c_t^{act} = \frac{C_t^{act}}{\sum_{a=25}^{a_R-1} (M_t^a + F_t^a) W_t^a}. \tag{21}$$

A.7 The sponsor support

We define the sponsor support contribution rate as:

$$c_t^{SS} = \frac{SS_t}{\sum_{a=25}^{a_R-1} (M_t^a + F_t^a) W_t^a}, \tag{22}$$

where

$$SS_t = \begin{cases} 0 & \text{if } A_t \geq B_t - C_t, \\ (B_t - C_t) - A_t & \text{if } A_t < B_t - C_t. \end{cases}$$

Notice that the sponsor support is included in the calculation of the actuarial assets.

B Simulation procedure

This appendix provides the details on the simulation of both the classic ALM and the value-based ALM model. In both cases we draw a set economic scenarios. Each scenario involves drawing a path of 300 quarters of our state vector. Below we first discuss the pricing framework, followed by the calculation of the fund's portfolio returns. Finally, we discuss the risk-neutral sampling procedure used for the risk-neutral valuation.

B.1 Pricing framework

Consider some derivative with a pay-off of Z_τ at time τ, which is a function of the path $X_1, X_2, ...$ of the state vector. The price of the derivative at time q is then given by[13]

$$P_t = \sum_{\tau=q+1}^{\infty} \mathrm{E}_q \left[Z_\tau \exp \sum_{s=q+1}^{\tau} m_s \right],$$

where $-m_{q+1}$ is the stochastic discount rate for the real-world scenarios. In line with the literature (Campbell et al., 1996), we assume that the stochastic discount rate $-m_{q+1}$ in period $q+1$ for the real-world scenarios is given by the following function of the state vector generated by our VAR model and the shocks to this state vector:

$$-m_{q+1} = e_y' X_q + \frac{1}{2}(\beta_0 + \beta_1 X_q)' \Sigma (\beta_0 + \beta_1 X_q) + (\beta_0 + \beta_1 X_q)' \varepsilon_{q+1}, \tag{23}$$

where β_0 and β_1 are respectively a vector and a matrix of parameters and e_y indicates the position of the short rate in the state vector:

$$e_y = (1, 0, 0, 0)'.$$

In a complete market it is possible to sell the derivative at time $q + 1$ for its price P_{q+1}. Hence the following must hold:

$$P_q = \mathrm{E}_q \left[P_{q+1} \exp \left(m_{q+1} \right) \right], \tag{24}$$

where P_{q+1} is the total price based on the total return index where any pay-off is reinvested in the same index. If we use lower-case letters to denote log-values so that

$$p_q = \log P_q,$$

knowing that X_{q+1} has a Gaussian distribution, and using the properties of the log-normal distribution, we can derive from equation (24):

$$\begin{aligned} \exp p_q &= \mathrm{E}_q \left[\exp p_{q+1} \exp m_{q+1} \right] = \mathrm{E}_q \left[\exp(p_{q+1} + m_{q+1}) \right] \\ &= \exp \left(\mathrm{E}_q \left[p_{q+1} + m_{q+1} \right] + 1/2 \, \mathrm{Var}_q \left[p_{q+1} + m_{q+1} \right] \right), \end{aligned}$$

Hence,

$$p_q = \mathrm{E}_q \left[p_{q+1} + m_{q+1} \right] + \tfrac{1}{2} \mathrm{Var}_q \left[p_{q+1} + m_{q+1} \right]. \tag{25}$$

[13]Here, we index time by q to indicate that we now count time in terms of quarters (running from 1 to 300).

Note that

$$\mathrm{E}_q\left[m_{q+1}\right] = -e_y' X_q - \tfrac{1}{2}(\beta_0 + \beta_1 X_q)'\Sigma(\beta_0 + \beta_1 X_q), \tag{26}$$

$$\mathrm{Var}_q\left[m_{q+1}\right] = (\beta_0 + \beta_1 X_q)'\Sigma(\beta_0 + \beta_1 X_q). \tag{27}$$

We will now apply our pricing framework to the various assets that are relevant for us.

B.1.1 Nominal bonds

We determine the term structure using an affine model based on the state variables (e.g., see Dai and Singleton (2000), Ang and Piazzesi (2003), Ang et al. (2008) and Le et al. (2010)). Denote by $p_q^{(n)}$ the quarter-q log price of a zero coupon nominal bond that matures at time $q+n$ and pays one unit of currency at maturity date. We assume that it is an affine function of the state variables:

$$p_q^{(n)} = -D_n - H_n' X_q. \tag{28}$$

Its nominal yield $Y_q^{(n)}$ satisfies the following relationship:

$$\left(1 + Y_q^{(n)}\right)^{-n} = P_q^n. \tag{29}$$

Denote $y_q^{(n)} \equiv \ln(1 + Y_q^{(n)})$. Hence, equations (28) and (29) imply

$$y_q^{(n)} = -\frac{1}{n} p_q^{(n)} = \frac{1}{n} D_n + \frac{1}{n} H_n' X_q. \tag{30}$$

Applying the general pricing equation (25) to zero-coupon bonds and using equation (28) we get:

$$\begin{aligned} p_q^{(n)} &= \mathrm{E}_q\left[p_{q+1}^{(n-1)} + m_{q+1}\right] + 1/2\,\mathrm{Var}_q\left[p_{q+1}^{(n-1)} + m_{q+1}\right] \\ &= -D_{n-1} - H_{n-1}'\,\mathrm{E}_q\left[X_{q+1}\right] \\ &+ \mathrm{E}_q\left[m_{q+1}\right] \\ &+ \frac{1}{2} Var_q\left[-H_{n-1}' X_{q+1}\right] \\ &+ \frac{1}{2}\,\mathrm{Var}_q\left[m_{q+1}\right] \\ &+ \mathrm{Cov}_q\left[-H_{n-1}' X_{q+1}, m_{q+1}\right]. \end{aligned} \tag{31}$$

Using (26), (27) and $\mathrm{E}_q\left[X_{q+1}\right] = (I - \Gamma)\,\mu + \Gamma X_q$, we can rewrite (31) as

$$
\begin{aligned}
p_q^{(n)} &= -D_{n-1} - H'_{n-1}\left((I-\Gamma)\mu + \Gamma X_q\right) \\
&- e'_y X_q - \frac{1}{2}(\beta_0 + \beta_1 X_q)'\Sigma(\beta_0 + \beta_1 X_q) \\
&+ \frac{1}{2}\left(H'_{n-1}\Sigma H_{n-1}\right) \\
&+ \frac{1}{2}(\beta_0 + \beta_1 X_q)'\Sigma(\beta_0 + \beta_1 X_q) \\
&+ H'_{n-1}\Sigma(\beta_0 + \beta_1 X_q),
\end{aligned}
$$

where the last part follows from

$$
\begin{aligned}
\mathrm{Cov}_q\left[-H'_{n-1}X_{q+1}, m_{q+1}\right] &= \mathrm{Cov}_q[-H'_{n-1}\left((I-\Gamma)\mu + \Gamma X_q + \varepsilon_{q+1}\right), \\
&- e'_y X_q - \frac{1}{2}(\beta_0 + \beta_1 X_q)'\Sigma(\beta_0 + \beta_1 X_q) - (\beta_0 + \beta_1 X_q)'\varepsilon_{q+1}] \\
&= \mathrm{E}_q\left[(-H'_{n-1}\varepsilon_{q+1})(-(\beta_0 + \beta_1 X_q)'\varepsilon_{q+1})\right] \\
&= \mathrm{E}_q\left[(H'_{n-1}\varepsilon_{q+1})(\varepsilon'_{q+1}(\beta_0 + \beta_1 X_q))\right] \\
&= H'_{n-1}\,\mathrm{E}_q\left[\varepsilon_{q+1}\varepsilon'_{q+1}\right](\beta_0 + \beta_1 X_q) \\
&= H'_{n-1}\Sigma(\beta_0 + \beta_1 X_q).
\end{aligned}
$$

We can further rewrite $p_q^{(n)}$ as

$$
\begin{aligned}
p_q^{(n)} &= -D_{n-1} - H'_{n-1}(I-\Gamma)\mu - H'_{n-1}\Gamma X_q \\
&- e'_y X_q \\
&+ \frac{1}{2}H'_{n-1}\Sigma H_{n-1} \\
&+ H'_{n-1}\Sigma\beta_0 + H'_{n-1}\Sigma\beta_1 X_q \\
&= -D_{n-1} - H'_{n-1}(I-\Gamma)\mu + \frac{1}{2}H'_{n-1}\Sigma H_{n-1} + H'_{n-1}\Sigma\beta_0 \\
&- (e'_y + H'_{n-1}\Gamma - H'_{n-1}\Sigma\beta_1)X_q.
\end{aligned}
$$

The last equation is already of the affine structure as in equation (28) with parameters

$$
\begin{aligned}
D_n = D_{n-1} + H'_{n-1}(I-\Gamma)\mu - \frac{1}{2}H'_{n-1}\Sigma H_{n-1} - H'_{n-1}\Sigma\beta_0, \\
H_n = e_y + (\Gamma - \Sigma\beta_1)'H_{n-1}.
\end{aligned}
\tag{32}
$$

For $n = 0$ we have that:

$$
p_q^{(0)} = \ln P_q^{(0)} = \ln 1 = 0,
$$

which is given by equation (28) with parameters

$$
\begin{aligned}
D_0 &= 0, \\
H_0 &= 0.
\end{aligned}
\tag{33}
$$

Using (33) in (32) we get

$$D_1 = 0,$$
$$H_1 = e_y. \tag{34}$$

This implies

$$p_q^{(1)} = -e_y' X_q = -y_q^{(1)}. \tag{35}$$

The deflator is calibrated to the short rate so that this constraint is satisfied.

B.1.2 Real bonds

Denote by $P_{s,q}^{r(n)}$ the price of a real bond at time q issued at time $s \leq q$ and maturing at time $q+n$. Such a bond pays Π_{q+n}/Π_s at maturity, where Π_q is the price index at time q. This implies that:

$$P_{q-1,q}^{r(n)} = \frac{\Pi_q}{\Pi_{q-1}} P_{q,q}^{r(n)}, \tag{36}$$

which can be expressed in terms of logarithms as:

$$p_{q-1,q}^{r(n)} = \pi_q + p_{q,q}^{r(n)}. \tag{37}$$

At maturity the nominal pay-off of the real bond is equal to the inflation during the bond's life, so in real terms the pay-off is equal to one. The real n-period yield is thus:

$$Y_q^{r(n)} = \left(\frac{1}{P_{q,q}^{r(n)}} \right)^{\frac{1}{n}}, \tag{38}$$

which in logarithmic terms is:

$$y_q^{r(n)} = -\frac{1}{n} p_{q,q}^{r(n)}. \tag{39}$$

Analogously to (28), we assume that $p_{q,q}^{r(n)}$ is of affine structure:

$$p_{q,q}^{r(n)} = -D_n^r - H_n^{r\prime} X_q. \tag{40}$$

For $n = 0$ we have that:

$$p_{q,q}^{r(0)} = \ln P_{q,q}^{r(0)} = \ln 1 = 0, \tag{41}$$

which by equation (40) always holds if

$$D_0^r = 0,$$
$$H_0^r = 0. \tag{42}$$

We will assume that (40) is valid for n and deduce its validity for $n+1$.

According to the general pricing formula (24), and using (37):

$$P_{q,q}^{r(n+1)} = \mathrm{E}_q\left[P_{q,q+1}^{r(n)} \exp m_{q+1}\right] = \mathrm{E}_q\left[\exp(\pi_{q+1} + p_{q+1,q+1}^{r(n)} + m_{q+1})\right]. \tag{43}$$

Applying (40) we get:

$$\begin{aligned} p_{q,q}^{r(n+1)} &= \mathrm{E}_q\left[\pi_{q+1} + p_{q+1,q+1}^{r(n)} + m_{q+1}\right] + 1/2\,\mathrm{Var}_q\left[\pi_{q+1} + p_{q+1,q+1}^{r(n)} + m_{q+1}\right] \\ &= \mathrm{E}_q\left[\pi_{q+1}\right] \\ &- D_n^r - H_n^{r\prime}\,\mathrm{E}_q\left[X_{q+1}\right] \\ &+ \mathrm{E}_q\left[m_{q+1}\right] \\ &+ \frac{1}{2} Var_q\left[\pi_{q+1} - H_n^{r\prime} X_{q+1}\right] \\ &+ \frac{1}{2}\,\mathrm{Var}_q\left[m_{q+1}\right] \\ &+ \mathrm{Cov}_q\left[\pi_{q+1} - H_n^{r\prime} X_{q+1}, m_{q+1}\right]. \end{aligned} \tag{44}$$

Let e_π indicate the position of the inflation in the state vector:

$$e_\pi = (0, 0, 1, 0)', \tag{45}$$

so that

$$\pi_{q+1} = e_\pi' X_{q+1}. \tag{46}$$

Using (46), (26), (27) and $\mathrm{E}_q\left[X_{q+1}\right] = (\mathrm{I} - \Gamma)\mu + \Gamma X_q$, we can rewrite (44) as

$$\begin{aligned} p_{q,q}^{r(n+1)} &= \mathrm{E}_q\left[\pi_{q+1} + p_{q+1,q+1}^{r(n)} + m_{q+1}\right] + 1/2\,\mathrm{Var}_q\left[\pi_{q+1} + p_{q+1,q+1}^{r(n)} + m_{q+1}\right] \\ &= e_\pi'\left((I - \Gamma)\mu + \Gamma X_q\right) \\ &- D_n^r - H_n^{r\prime}\left((I - \Gamma)\mu + \Gamma X_q\right) \\ &- e_y' X_q - \tfrac{1}{2}(\beta_0 + \beta_1 X_q)'\Sigma(\beta_0 + \beta_1 X_q) \\ &+ \frac{1}{2}(e_\pi' - H_n^{r\prime})\Sigma(e_\pi - H_n^r) \\ &+ \frac{1}{2}(\beta_0 + \beta_1 X_q)'\Sigma(\beta_0 + \beta_1 X_q) \\ &+ (H_n^{r\prime} - e_\pi')\Sigma(\beta_0 + \beta_1 X_q). \end{aligned} \tag{47}$$

where the last part follows from

$$
\begin{aligned}
\mathrm{Cov}_q\left[\pi_{q+1} - H_n^{r\prime} X_{q+1}, m_{q+1}\right] &= \mathrm{Cov}_q\left[(e_\pi' - H_n^{r\prime}) X_{q+1}, m_{q+1}\right] \\
&= \mathrm{Cov}_q[(e_\pi' - H_n^{r\prime})\left((I-\Gamma)\mu + \Gamma X_q + \varepsilon_{q+1}\right), \\
&- e_y' X_q - \frac{1}{2}(\beta_0 + \beta_1 X_q)'\Sigma(\beta_0 + \beta_1 X_q) - (\beta_0 + \beta_1 X_q)'\varepsilon_{q+1}] \\
&= \mathrm{E}_q\left[((e_\pi' - H_n^{r\prime})\varepsilon_{q+1})(-(\beta_0 + \beta_1 X_q)'\varepsilon_{q+1})\right] \\
&= \mathrm{E}_q\left[((H_n^{r\prime} - e_\pi')\varepsilon_{q+1})(\varepsilon_{q+1}'(\beta_0 + \beta_1 X_q))\right] \\
&= (H_n^{r\prime} - e_\pi')\,\mathrm{E}_q\left[\varepsilon_{q+1}\varepsilon_{q+1}'\right](\beta_0 + \beta_1 X_q) \\
&= (H_n^{r\prime} - e_\pi')\Sigma(\beta_0 + \beta_1 X_q).
\end{aligned}
\tag{48}
$$

We can further rewrite $p_{q,q}^{r(n+1)}$ as

$$
\begin{aligned}
p_{q,q}^{r(n+1)} &= -D_n^r + (e_\pi' - H_n^{r\prime})(I-\Gamma)\mu + (e_\pi' - H_n^{r\prime})\Gamma X_q \\
&- e_y' X_q \\
&+ \frac{1}{2}(e_\pi - H_n^r)'\Sigma(e_\pi - H_n^r) \\
&+ (H_n^{r\prime} - e_\pi')\Sigma\beta_0 + (H_n^{r\prime} - e_\pi')\Sigma\beta_1 X_q \\
&= -D_n^r + (e_\pi' - H_n^{r\prime})(I-\Gamma)\mu + \frac{1}{2}(e_\pi - H_n^r)'\Sigma(e_\pi - H_n^r) + (H_n^{r\prime} - e_\pi')\Sigma\beta_0 \\
&+ ((e_\pi' - H_n^{r\prime})\Gamma - e_y' + (H_n^{r\prime} - e_\pi')\Sigma\beta_1) X_q.
\end{aligned}
\tag{49}
$$

The last equation is already of the affine structure as in equation (28) with parameters

$$
\begin{aligned}
D_{n+1}^r = D_n^r + (H_n^r - e_\pi)'(I-\Gamma)\mu - \frac{1}{2}(H_n^r - e_\pi)'\Sigma(H_n^r - e_\pi) - (H_n^r - e_\pi)'\Sigma\beta_0, \\
H_{n+1}^r = e_y + (\Gamma - \Sigma\beta_1)'(H_n^r - e_\pi).
\end{aligned}
\tag{50}
$$

B.1.3 Stocks

The excess return on stocks is defined as follows and can be rearranged using (35):

$$
r_{q+1}^s - y_q^{(1)} = \ln\left(\frac{P_{q+1}^s}{P_q^s}\right) - y_q^{(1)} = p_{q+1}^s - p_q^s + p_q^{(1)} \tag{51}
$$

$$
\Longrightarrow p_{q+1}^s = r_{q+1}^s - y_q^{(1)} + p_q - p_q^{(1)}, \tag{52}
$$

where P_q^s is the stock price and r_q^s is its return. Note that both r_{q+1}^s and $y_q^{(1)}$ are returns going from period q to period $q+1$, with $y_q^{(1)}$ known in period q. Using (33) in (31) we get

$$
p_q^{(1)} = \mathrm{E}_q\left[m_{q+1}\right] + \frac{1}{2}\mathrm{Var}_q\left[m_{q+1}\right]. \tag{53}
$$

From (25), (51) and (53) it follows that

$$\begin{aligned}
\mathrm{E}_q\left[r^s_{q+1}-y^{(1)}_q\right] &= \mathrm{E}_q[p^s_{q+1}] - \left(\mathrm{E}_q\left[p^s_{q+1}+m_{q+1}\right] + 1/2\,\mathrm{Var}_q\left[p^s_{q+1}+m_{q+1}\right]\right) \\
&\quad + \mathrm{E}_q\left[m_{q+1}\right] + \frac{1}{2}\,\mathrm{Var}_q\left[m_{q+1}\right] \\
&= \mathrm{E}_q[p^s_{q+1}] - \left(\mathrm{E}_q\left[p^s_{q+1}\right] + \frac{1}{2}\,\mathrm{Var}_q\left[p^s_{q+1}\right] + \mathrm{Cov}_q\left[p^s_{q+1}, m_{q+1}\right]\right) \\
&= -\frac{1}{2}\,\mathrm{Var}_q\left[p^s_{q+1}\right] - \mathrm{Cov}_q\left[p^s_{q+1}, m_{q+1}\right].
\end{aligned}$$

Using (52) and the fact that p^s_q and $p^{(1)}_q$ are known at time q, so that

$$\mathrm{Var}_q[p^s_{q+1}] = \mathrm{Var}_q[r^s_{q+1} - y^{(1)}_q + p^s_q - p^{(1)}_q] = \mathrm{Var}_q[r^s_{q+1} - y^{(1)}_q], \tag{54}$$

we get

$$\mathrm{E}_q\left[r^s_{q+1}-y^{(1)}_q\right] = -\frac{1}{2}\,\mathrm{Var}_q\left[r^s_{q+1}-y^{(1)}_q\right] - \mathrm{Cov}_q\left[r^s_{q+1}-y^{(1)}_q, m_{q+1}\right]. \tag{55}$$

The excess return on stocks can also be written as

$$r^s_{q+1} - y^{(1)}_q = e'_{xs}X_{q+1} = e'_{xs}\left((\mathrm{I}-\Gamma)\mu + \Gamma X_q + \varepsilon_{q+1}\right), \tag{56}$$

where $e_{xs} = (0,1,0,0)'$ is a unit vector representing the location of the excess return on stocks in the state vector. Hence,

$$e'_{xs}\left((\mathrm{I}-\Gamma)\mu + \Gamma X_q\right) = \mathrm{E}_q\left[r^s_{q+1}-y^{(1)}_q\right]. \tag{57}$$

It follows from equation (23), (55), (56) and (57) that

$$\begin{aligned}
e'_{xs}\left((\mathrm{I}-\Gamma)\mu + \Gamma X_q\right) &= -\frac{1}{2}\,\mathrm{Var}_q\left[e'_{xs}\varepsilon_{q+1}\right] - \mathrm{Cov}_q\left[e'_{xs}\varepsilon_{q+1}, -(\beta_0+\beta_1 X_q)'\varepsilon_{q+1}\right] \\
&= -(\frac{1}{2}e'_{xs}\Sigma e_{xs}) - \mathrm{E}_q\left[(e'_{xs}\varepsilon_{q+1}-0)(-(\beta_0+\beta_1 X_q)'\varepsilon_{q+1}-0)\right] \\
&= -(\frac{1}{2}e'_{xs}\Sigma e_{xs}) - \mathrm{E}_q\left[-e'_{xs}\varepsilon_{q+1}\varepsilon'_{q+1}(\beta_0+\beta_1 X_q)\right] \\
&= -(\frac{1}{2}e'_{xs}\Sigma e_{xs}) + e'_{xs}\Sigma(\beta_0+\beta_1 X_q).
\end{aligned}$$

Hence,

$$\begin{aligned}
& e'_{xs}\left((\mathrm{I}-\Gamma)\mu + \Gamma X_q + \frac{1}{2}\Sigma e_{xs} - \Sigma\beta_0 - \Sigma\beta_1 X_q\right) = 0 \\
\Leftrightarrow\ & e'_{xs}\left(\left((\mathrm{I}-\Gamma)\mu - \Sigma\beta_0 + \frac{1}{2}\Sigma e_{xs}\right) + (\Gamma - \Sigma\beta_1)X_q\right) = 0.
\end{aligned}$$

This is satisfied for all values of X_q if:

$$\begin{aligned} e'_{xs}\left((\mathrm{I}-\Gamma)\mu - \Sigma\beta_0\right) + \frac{1}{2}e'_{xs}\Sigma e_{xs} = 0, \\ e'_{xs}\left(\Gamma - \Sigma\beta_1\right) = 0. \end{aligned} \tag{58}$$

The first equation yields one condition, whereas the second equation yields as many conditions as there are state variables. It follows that the conditions in (58) must be satisfied for stocks. These conditions determine the parameters β_0 and β_1 of the discount factor.

B.1.4 The inflation risk premium

We follow Grishchenko and Huang (2012) and assume that the inflation risk premium $RP_q^{\pi(n)}$ is defined by the following equation:

$$y_q^{(n)} - y_{q,q}^{r(n)} = \frac{1}{n}\sum_{i=1}^{n}\mathrm{E}_q[\pi_{q+i}] + \frac{1}{2}e'_\pi \Sigma e_\pi + RP_q^{\pi(n)}, \tag{59}$$

where on the left-hand side we have the difference between the nominal and the real yield. The first term on the right-hand side is n-period expected inflation and the second term is the so-called Jensen's correction - a convexity term that has little impact on the results.

Using the formula of the sum of a geometric series we have

$$\begin{aligned} \mathrm{E}_q[X_{q+i}] &= \Gamma^0(\mathrm{I}-\Gamma)\mu + \Gamma^1(\mathrm{I}-\Gamma)\mu + ... + \Gamma^{i-1}(\mathrm{I}-\Gamma)\mu + \Gamma^i X_q \\ &= (\mathrm{I}-\Gamma)\mu(\mathrm{I}-\Gamma^i)(\mathrm{I}-\Gamma)^{-1} + \Gamma^i X_q \\ &= \mu + \Gamma^i(X_q - \mu). \end{aligned} \tag{60}$$

It follows then that the expected inflation is

$$\mathrm{E}_q[\pi_{q+i}] = \mathrm{E}_q[e'_\pi X_{q+i}] = e'_\pi\mu + e'_\pi\Gamma^i(X_q - \mu), \tag{61}$$

and

$$\begin{aligned} \frac{1}{n}\sum_{i=1}^{n}\mathrm{E}_q[\pi_{q+i}] &= \frac{1}{n}\sum_{i=1}^{n}e'_\pi\mu + \frac{1}{n}\sum_{i=1}^{n}e'_\pi\Gamma^i(X_q-\mu) \\ &= e'_\pi\mu + \frac{1}{n}e'_\pi(\sum_{i=1}^{n}\Gamma^i(X_q-\mu)) \\ &= e'_\pi\mu + \frac{1}{n}e'_\pi((X_q-\mu)(\mathrm{I}-\Gamma^{n+1})(\mathrm{I}-\Gamma)^{-1} - (X_q-\mu)) \\ &= e'_\pi\mu + \frac{1}{n}e'_\pi\Gamma(\mathrm{I}-\Gamma^n)(\mathrm{I}-\Gamma)^{-1}(X_q-\mu). \end{aligned} \tag{62}$$

Inserting

$$y_q^{(n)} = \frac{1}{n}D_n + \frac{1}{n}H'_n X_q, \tag{63}$$

$$y_{q,q}^{r(n)} = \frac{1}{n} D_n^r + \frac{1}{n} H_n^{r\prime} X_q, \tag{64}$$

and (62) into (59), and multiplying by n, we obtain:

$$D_n + H_n' X_q - D_n^r - H_n^{r\prime} X_q = n e_\pi' \mu + e_\pi' \Gamma(\mathrm{I} - \Gamma^n)(\mathrm{I} - \Gamma)^{-1}(X_q - \mu) + \frac{n}{2} e_\pi' \Sigma e_\pi + n RP_q^{\pi(n)}, \tag{65}$$

or

$$\begin{aligned} 0 = {} & D_n^r - D_n + n e_\pi' \mu - e_\pi' \Gamma(\mathrm{I} - \Gamma^n)(\mathrm{I} - \Gamma)^{-1} \mu + \frac{n}{2} e_\pi' \Sigma e_\pi + n RP_q^{\pi(n)} \\ & + (H_n^{r\prime} - H_n' + e_\pi' \Gamma(\mathrm{I} - \Gamma^n)(\mathrm{I} - \Gamma)^{-1}) X_q. \end{aligned} \tag{66}$$

Note that the risk premium is constant over time, hence $RP_q^{\pi(n)} = RP^{\pi(n)}$ for all q. Assuming that this expression holds for all X_q, we can write for $n = 1$:

$$\begin{aligned} RP^\pi &= D_1 - D_1^r - e_\pi' \mu + e_\pi' \Gamma \mu - \frac{1}{2} e_\pi' \Sigma e_\pi, \\ H_1^{r\prime} &= H_1' - e_\pi' \Gamma. \end{aligned} \tag{67}$$

where we have defined $RP^\pi = RP^{\pi(1)}$. Using (34) and (50),

$$\begin{aligned} RP^\pi = e_\pi'(I - \Gamma)\mu + \frac{1}{2} e_\pi' \Sigma e_\pi - e_\pi' \Sigma \beta_0 - e_\pi' \mu + e_\pi' \Gamma \mu - \frac{1}{2} e_\pi' \Sigma e_\pi, \\ e_y - (\Gamma - \Sigma \beta_1)' e_\pi = e_y - \Gamma' e_\pi. \end{aligned} \tag{68}$$

Hence, we have the following constraints associated with the one-period inflation risk premium:

$$\begin{aligned} e_\pi'(\Sigma \beta_0) &= -RP^\pi, \\ e_\pi'(\Sigma \beta_1) &= 0. \end{aligned} \tag{69}$$

Following an analogous procedure, i.e. starting from an expression similar to (59) for the difference between the yield on a nominal bond and a bond indexed to real wage growth, we would obtain the following constraints associated with the one-period real-wage risk premium:

$$\begin{aligned} e_w'(\Sigma \beta_0) &= -RP^w, \\ e_w'(\Sigma \beta_1) &= 0. \end{aligned} \tag{70}$$

where

$$e_w = (0, 0, 0, 1)'. \tag{71}$$

B.2 Parameter optimization

In this subsection we obtain the model-induced values $\tilde{\beta}_0$ and $\tilde{\beta}_1$ for β_0 and β_1, respectively. First, we obtain empirical estimates $\hat{D}_n$ and $\hat{H}_n$ of D_n and H_n, respectively, through multivariate OLS estimation of equation (30) for some specific maturities using the historical time series of zero-coupon yields for those maturities and using the historical time series of the state variables. The estimation is again at the quarterly level from the third quarter of 1990 up to and including the second quarter of 2015. The same procedure is repeated for the real yields, in order to obtain empirical estimates $\hat{D}^r_n$ and $\hat{H}^r_n$. For this purpose we use the historical data of the real yields starting from 2003. The optimal $\tilde{\beta}_0$ and $\tilde{\beta}_1$ are obtained through an optimization procedure exploiting several model-implied restrictions. The optimal values for $\tilde{\beta}_0$ and $\tilde{\beta}_1$ imply specific values $\tilde{D}_n$, $\tilde{H}_n$, $\tilde{D}^r_n$ and $\tilde{H}^r_n$ for the model parameters D_n, H_n, D^r_n and H^r_n, respectively, by using the recursion in equations (32) and (50). Specifically, given $H_0 = 0$ and $\tilde{\beta}_1$, we can calculate $\tilde{H}_1$, $\tilde{H}_2$, and so on. Given $D_0 = 0$, $\tilde{\beta}_0$ and the path H_0, $\tilde{H}_1$, $\tilde{H}_2$,..., we calculate $\tilde{D}_1$, $\tilde{D}_2$, ... Similarly, given $H^r_0 = 0$ and $\tilde{\beta}_1$, we can calculate $\tilde{H}^r_1$, $\tilde{H}^r_2$, and so on. Given $D^r_0 = 0$, $\tilde{\beta}_0$ and the path H^r_0, $\tilde{H}^r_1$, $\tilde{H}^r_2$,..., we calculate $\tilde{D}^r_1$, $\tilde{D}^r_2$, ... The part of the objective function constructed in order to solve for $\tilde{\beta}_1$ aims at matching the historical yield exposures to the state variables to the exposures in the model, i.e. bringing $\tilde{H}_n$ and $\tilde{H}^r_n$ as close as possible to $\hat{H}_n$ and $\hat{H}^r_n$. The part of the objective function constructed in order to solve for $\tilde{\beta}_0$ aims at matching the latest interest rate level in our model to the latest interest rate level in our sample period, i.e. bringing $\tilde{D}_n$ and $\tilde{D}^r_n$ as close as possible to the levels implied by the most recent interest rates and $\tilde{H}_n$ and $\tilde{H}^r_n$. Once we have the series $\tilde{D}_n$ and $\tilde{H}_n$, we have constructed the nominal term structure that we use to calculate the returns on the fixed-income part of pension fund portfolio. Once we have the series $\tilde{D}^r_n$ and $\tilde{H}^r_n$, we have constructed the real term structure.

B.2.1 Optimization of β_1 and β_0

In the main analysis we use the $\hat{\mu}$ corresponding to the values indicated by the SPF, so that the simulations generated from the model are based on an outlook for the economy that is as close as possible to the current one. In our robustness analysis we use the direct estimate $\hat{\mu}$ of μ. Further, note that the model constraints in the second lines of (58) and (69) are expressed in terms of $\Sigma\beta_1$, so it is easier to obtain first an estimate $(\widetilde{\Sigma\beta_1})$ of the combination $\Sigma\beta_1$ than to obtain a direct estimate $\tilde{\beta}_1$ of β_1 itself. Analogously, the model constraints in the first lines of (58) and (69) are expressed in terms of $\Sigma\beta_0$, so it is easier to obtain an estimate $(\widetilde{\Sigma\beta_0})$ of the combination $\Sigma\beta_0$ than to obtain a direct estimate $\tilde{\beta}_0$ of β_0 itself. Hence, we obtain $\tilde{\beta}_1$ and $\tilde{\beta}_0$ in two steps. First, we obtain all elements of $(\widetilde{\Sigma\beta_1})$ and $(\widetilde{\Sigma\beta_0})$. Then, we obtain $\tilde{\beta}_1$ and $\tilde{\beta}_0$ from

$$\tilde{\beta}_1 = \hat{\Sigma}^{-1}(\widetilde{\Sigma\beta_1}). \tag{72}$$

$$\tilde{\beta}_0 = \hat{\Sigma}^{-1}(\widetilde{\Sigma\beta_0}), \tag{73}$$

where $\hat{\Sigma}$ is obtained from the estimation of equation (6).

Let us move to the determination of $(\widetilde{\Sigma\beta_1})$ and $(\widetilde{\Sigma\beta_0})$. The elements in the $(\widetilde{\Sigma\beta_1})$ row corresponding to the excess returns are predetermined by the constraint (58).

$$e'_{xs}(\widetilde{\Sigma\beta_1}) = e'_{xs}\hat{\Gamma}, \tag{74}$$

Further, using the second line of the constraint (69) and the fact that we impose a zero real wage risk premium, we can set the elements in the row of $(\widetilde{\Sigma\beta_1})$ corresponding to inflation and real wage growth to zeroes. The element of $(\widetilde{\Sigma\beta_0})$ corresponding to the excess stock returns follows straight away from the first line of (58):

$$e'_{xs}(\Sigma\beta_0) = e'_{xs}(I - \hat{\Gamma})\hat{\mu} + \frac{1}{2}e'_{xs}\hat{\Sigma}e_{xs}, \tag{75}$$

where $\hat{\mu}$ is based on imputation from the SPF in the main analysis and is the direct estimate obtained from equation (6) in the robustness analysis. $\hat{\Gamma}$ is obtained from the estimation of equation (6). Finally, the element of $(\widetilde{\Sigma\beta_0})$ corresponding to real wage growth is set to zero, as we impose a zero real wage risk premium due to a lack of market information on it.

Our optimization procedure can be used to solve for the one-period risk premia associated with inflation and real wage growth. This amounts to solving for the third and fourth elements of $(\widetilde{\Sigma\beta_0})$, alongside the other elements of $(\widetilde{\Sigma\beta_1})$ and $(\widetilde{\Sigma\beta_0})$ that we need to optimize over below. The first lines of (69) and/or (70) then determine the third and fourth element of $(\widetilde{\Sigma\beta_0})$ directly. However, to the best of our knowledge real-wage indexed bonds do not exist in practice. Hence, it is not possible to match its model-implied term structure to one that is estimated on actual data. Therefore, we directly impose a real wage growth risk premium of zero. This seems reasonable in view of the fact that real wage growth exhibits limited volatility compared to the other elements of the state vector. However, the inflation risk premium we keep as a free parameter in the model.

We obtain the remaining elements of $(\widetilde{\Sigma\beta_1})$ and $(\widetilde{\Sigma\beta_0})$, i.e. the first row of $(\widetilde{\Sigma\beta_1})$ and the first and third element of $(\widetilde{\Sigma\beta_0})$, by minimizing over these elements a criterion function that is the sum of the following four components that will all receive an equal weight in the optimization procedure.

The first component tries to match the model-implied exposures H_n of the nominal yield to the state variables to their empirical values $\hat{H}_n$:

$$\sum_{n\in\tau}\left\|H_n - \hat{H}_n\right\|^2 = \sum_{n\in\tau}\left\|e_y + (\hat{\Gamma} - \Sigma\beta_1)'H_{n-1} - \hat{H}_n\right\|^2, \tag{76}$$

where $\tau = \{4, 8, 20, 40\}$. Note that H_{n-1} is a function of $\Sigma\beta_1$ through the recursion in the second line of (32).

The second component is constructed analogously for the real yields:

$$\sum_{n\in\tau}\left\|H^r_n - \hat{H}^r_n\right\|^2 = \sum_{n\in\tau}\left\|e_y + (\hat{\Gamma} - \Sigma\beta_1)'H^r_{n-1} - e_\pi) - \hat{H}^r_n\right\|^2, \tag{77}$$

where $\tau = \{4, 8, 20, 40\}$, where H^r_{n-1} is a function of $\Sigma\beta_1$ through the recursion in the second line of (50).

For the remaining components we will make use of the affine structure of the yields:

$$y_q^{(n)} = \frac{1}{n} D_n + \frac{1}{n} H_n' X_q, \tag{78}$$

and

$$y_q^{r(n)} = \frac{1}{n} D_n^r + \frac{1}{n} H_n^{r\prime} X_q. \tag{79}$$

The third component of the objective function tries to match the intercept D_n that follows from the model to the difference between the last observation of the yields and the model prediction without the intercept, so that the model matches the last interest rates as well as possible:

$$\sum_{n\in\tau} \left\| \left(D_{n-1} + \tilde{H}_{n-1}'(I-\hat{\Gamma})\hat{\mu} - \frac{1}{2}\tilde{H}_{n-1}'\hat{\Sigma}\tilde{H}_{n-1} - \tilde{H}_{n-1}'(\Sigma\beta_0) \right) - \left((y_{q_{last}}^{(n)} - \frac{1}{n}\tilde{H}_n' X_{q_{last}})n \right) \right\|^2 \tag{80}$$

where $\tau = \{4, 8, 20, 40\}$, q_{last} indicates the quarter of the last observation and we have used the first line of (32).

The fourth component is constructed analogously for the real yields:

$$\sum_{n\in\tau} \left\| \left(D_{n-1}^r + (\tilde{H}_{n-1}^r - e_\pi)'(I-\hat{\Gamma})\hat{\mu} - \frac{1}{2}(\tilde{H}_{n-1}^r - e_\pi)'\hat{\Sigma}(\tilde{H}_{n-1}^r - e_\pi) - (\tilde{H}_{n-1}^r - e_\pi)(\Sigma\beta_0) \right) - \left((y_{q_{last}}^{r(n)} - \frac{1}{n}\tilde{H} \right. \right. \tag{81}$$

where $\tau = \{4, 8, 20, 40\}$ and we have used the first line of (50).

Using $H_0 = H_0^r = 0$ and the optimal value $\tilde{\beta}_1$ in combination with the second lines of (32) and (50) we can thus calculate $\tilde{H}_1$, $\tilde{H}_2$,... and $\tilde{H}_1^r$, $\tilde{H}_2^r$,..., which we will use further in the model. Using $D_0 = D_0^r = 0$, the optimal value of $\tilde{\beta}_0$ and the sequences $\tilde{H}_n$ and $\tilde{H}_n^r$ we can thus construct $\tilde{D}_n$ and $\tilde{D}_n^r$ using the first lines in (32) and (50).

We now have the nominal and real term structures fully constructed.

B.3 Calculation of the portfolio returns

The model generates scenarios for the term structure of interest rates. The fixed-income portfolio returns have to be extracted from this information. We assume that the fixed-income component of the fund's asset portfolio consists of zero-coupon bonds with a principal value of one unit of currency to be repaid at maturity, τ years from now.

$$1 = \left(1 + Y_0^{(4\tau)}\right)^{-4\tau},$$

where we have assumed that the bond is priced at par. The interest rate $Y_0^{(4t)}$ is the quarterly interest rate obtained from the construction of the term structure using the above affine structure model. Hence, the left-hand side is the bond's price at issuance date when it is sold at par value 1, while the right-hand side is the present discounted value of the cash flows associated with the bond, i.e. the repayment of the principal, discounted back to issuance date. The bond return is:

$$\left(1+Y_1^{(4\tau-1)}\right)^{-(4\tau-1)} - \left(1+Y_0^{(4\tau)}\right)^{-4\tau} = \left(1+Y_1^{(4\tau-1)}\right)^{-(4\tau-1)} - 1,$$

divided by the purchase price of the bond, which is one.

With the estimation of β_0 and β_1 we have constructed the term structure of interest rates, so that, given the simulated state vector, we can compute, as just laid out, the return on the fixed-income component of the fund's asset portfolio, which consists of 10-year zero-coupon bonds. Hence, $\tau = 10$. It is rebalanced at the beginning of every time period so that it again consists of 10-year maturity bonds. The return on the stock component of the fund's portfolio is obtained directly from the simulation of the state vector.

C Risk-neutral sampling

The price of a derivative paying a cash flow Z (which is a function of the path $X_1, X_2, ...X_\tau$ of the state vector) at time τ is

$$P_0 = \mathrm{E}_0 \left[Z_\tau \exp \sum_{q=1}^{\tau} m_q \right].$$

Using (23) this equation can be rewritten as

$$P_0 = \int Z(\varepsilon_1, \varepsilon_2, ..., \varepsilon_\tau) \exp \left[-\sum_{q=1}^{\tau} e_y' X_{q-1} \right]$$
$$\exp \left[-\sum_{q=1}^{\tau} \left(\frac{1}{2} (\beta_0 + \beta_1 X_{q-1})' \Sigma (\beta_0 + \beta_1 X_{q-1}) + (\beta_0 + \beta_1 X_{q-1})' \varepsilon_q \right) \right]$$
$$\frac{1}{(2\pi)^{k\tau/2} |\Sigma|^{\tau/2}} \exp \left[-\sum_{q=1}^{\tau} \frac{1}{2} \varepsilon_q' \Sigma^{-1} \varepsilon_q \right] d\varepsilon_1 d\varepsilon_2 ... d\varepsilon_\tau.$$

where k is the dimension of the state vector (in our case, 4). Hence,

$$P_0 = \int Z(\varepsilon_1, \varepsilon_2, ..., \varepsilon_\tau) \exp \left[-\sum_{q=1}^{\tau} y_{q-1}^{(1)} \right] \frac{1}{(2\pi)^{k\tau/2} |\Sigma|^{\tau/2}}$$
$$\exp \left[-\sum_{q=1}^{\tau} \left(\frac{1}{2} \left(\varepsilon_q + \Sigma(\beta_0 + \beta_1 X_{q-1}) \right)' \Sigma^{-1} \left(\varepsilon_q + \Sigma(\beta_0 + \beta_1 X_{q-1}) \right) \right) \right] d\varepsilon_1 d\varepsilon_2 ... d\varepsilon_\tau.$$

Here π is the number "pi" (to be distinguished from our symbol for inflation). This integral can be evaluated numerically in a Monte Carlo simulation by drawing a number of time series

$\{\varepsilon_1, \varepsilon_2, ..., \varepsilon_\tau\}$ from the multivariate normal density function

$$f = \frac{1}{(2\pi)^{k\tau/2}|\Sigma|^{\tau/2}} \exp\left[-\sum_{q=1}^{\tau}\left(\frac{1}{2}\left(\varepsilon_q + \Sigma(\beta_0 + \beta_1 X_{q-1})\right)'^{-1}\left(\varepsilon_q + \Sigma(\beta_0 + \beta_1 X_{q-1})\right)\right)\right], \quad (82)$$

calculating for each time series the derivative pay-off discounted at the risk-free rate

$$\exp\left[-\sum_{q=1}^{\tau} y_{q-1}^{(1)}\right] \cdot Z(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, ..., \tilde{\varepsilon}_\tau),$$

and taking a simple average over the drawings. Hence, we have transformed the original problem in which we draw from the multivariate normal density function of ε_q and discount cash flows at the stochastic discount factor into a problem in which we draw from the multivariate normal density function of $\tilde{\varepsilon}_q$ with mean $-\Sigma(\beta_0 + \beta_1 X_{q-1})$ and the same original variance-covariance matrix Σ, but discount cash flows at the risk-free rate.

We can draw $\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, ..., \tilde{\varepsilon}_\tau$ from the distribution $f(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, ..., \tilde{\varepsilon}_\tau)$ by first drawing $\tilde{\varepsilon}_1$ from its marginal distribution, then drawing $\tilde{\varepsilon}_2$ from the conditional distribution $f(\tilde{\varepsilon}_2|\tilde{\varepsilon}_1)$, and so on. The conditional density $f(\tilde{\varepsilon}_q|\tilde{\varepsilon}_{q-1}, ..., \tilde{\varepsilon}_1)$ is obtained using the Bayes' formula:

$$f(\tilde{\varepsilon}_q|\tilde{\varepsilon}_{q-1}, ..., \tilde{\varepsilon}_1) = \frac{f(\tilde{\varepsilon}_q, ..., \tilde{\varepsilon}_1)}{f(\tilde{\varepsilon}_{q-1}, ..., \tilde{\varepsilon}_1)}.$$

Applying (82) to $\tau = q - 1$ and $\tau = q$, we obtain

$$f(\tilde{\varepsilon}_q|\tilde{\varepsilon}_{q-1}, ..., \tilde{\varepsilon}_1) = \frac{1}{(2\pi)^{k/2}|\Sigma|^{1/2}} \exp\left[-\frac{1}{2}\left(\tilde{\varepsilon}_q + \Sigma(\beta_0 + \beta_1 X_{q-1})\right)' \Sigma^{-1}\left(\tilde{\varepsilon}_q + \Sigma(\beta_0 + \beta_1 X_{q-1})\right)\right].$$

Hence,

$$\tilde{\varepsilon}_q|\tilde{\varepsilon}_{q-1}, ..., \tilde{\varepsilon}_1 \sim \mathbb{N}(-\Sigma(\beta_0 + \beta_1 X_{q-1}), \Sigma).$$

D Generating scenarios

The economic scenarios are generated using the parameter estimates $\hat{\alpha}$, $\hat{\Gamma}$, $\hat{\Sigma}$, $\tilde{\beta}_0$, $\tilde{\beta}_1$, $\tilde{D}_n$, $\tilde{H}_n$, $\tilde{D}_n^r$ and $\tilde{H}_n^r$.

First, setting the initial value of the state vector at the imputed or estimated value for μ and using (6) the path of the vector of state variables is simulated for the chosen horizon length. Then, at each quarter into the horizon, the term structure of the nominal interest rate is constructed using the above parameter estimates.

The same scenario-generating procedure is followed for both the real-world and risk-neutral scenarios. The only difference lies in the mean of the shock vector. For the real-world simulation the error terms are drawn from the mean-zero normal distribution $\varepsilon_{q+1} \sim \mathbb{N}(0, \hat{\Sigma})$. Under the risk-neutral scenarios, the error terms are normally distributed with $\tilde{\varepsilon}_{q+1} \sim \mathbb{N}(-\hat{\Sigma}(\tilde{\beta}_0 + \tilde{\beta}_1 X_q), \hat{\Sigma})$.

E Estimates of the VAR parameters

The parameter estimates of the VAR in equation (6) are:

$$\hat{\mu} = \begin{bmatrix} 0.0022747 \\ 0.0207877 \\ 0.0047909 \\ 0.0038911 \end{bmatrix}, \hat{\Sigma} = \begin{bmatrix} 0.0000011 & 0.0000212 & 0.0000011 & 0.0000025 \\ 0.0000212 & 0.0072488 & 0.0000434 & 0.0003188 \\ 0.0000011 & 0.0000434 & 0.0000670 & -0.0000548 \\ 0.0000025 & 0.0003188 & -0.0000548 & 0.0001418 \end{bmatrix}$$

After the imputation with the SPF data we have for the quarterly vector of means:

$$\hat{\mu} = \begin{bmatrix} 0.0066600 \\ 0.0031003 \\ 0.0051621 \\ 0.0041434 \end{bmatrix},$$

while $\hat{\Sigma}$ is kept at its originally estimated value.

The correlation matrix for the state variables is as follows:

	y	xs	cpi	w
y	1.00000000	−0.01038369	0.22675373	0.08041977
xs	−0.01038369	1.00000000	0.03858944	0.31787796
cpi	0.22675373	0.03858944	1.00000000	−0.53272535
w	0.08041977	0.31787796	−0.53272535	1.00000000

F Results of robustness analysis

Here, we provide the results of the simulations of the pension fund's performance for a set of 1000 economic scenarios in order to save the simulation time.

F.1 Exogenous inflation risk premium

Tables 8 - 10 report the results when an inflation risk premium of zero is imposed, while Tables 11-13 report the corresponding results for an annual inflation risk premium of 20 basis points.

Table 8: Classic ALM results under different policies after 75 years – zero inflation risk premium

(a) Contributions

Case	Description	c 5%	c 50%	c 95%	c^{NC} -	c^{Amort} 5%	c^{Amort} 50%	c^{Amort} 95%	c^{SS} 5%	c^{SS} 50%	c^{SS} 95%
Baseline											
0	baseline plan	26%	41%	47%	13%	12%	13%	14%	0%	15%	20%
Contribution											
1.1	0% amortization paid	36%	41%	47%	13%	0%	0%	0%	23%	28%	34%
1.2	100% amortization paid	23%	36%	44%	13%	10%	23%	28%	0%	0%	4%
1.3	amortization in 10 years	16%	35%	44%	13%	3%	21%	31%	0%	0%	0%
Indexation											
2.1	indexation is 0.5 CPI	20%	36%	42%	13%	7%	13%	14%	0%	10%	15%
2.2	conditional indexation	18%	31%	38%	13%	5%	12%	13%	0%	6%	12%
Portfolio composition											
3.1	100% stocks	15%	41%	47%	13%	2%	13%	14%	0%	15%	20%
3.2	0% stocks	36%	41%	47%	13%	13%	13%	14%	10%	15%	20%

(b) Funding ratios and pension result

Case	Description	FR^P 5%	FR^P 50%	FR^P 95%	FR^E 5%	FR^E 50%	FR^E 95%	PR 5%	PR 50%	PR 95%
Baseline										
0	baseline plan	0%	0%	1%	0%	0%	1%	102%	106%	112%
Contribution										
1.1	0% amortization paid	0%	0%	0%	0%	0%	0%	102%	106%	112%
1.2	100% amortization paid	0%	14%	61%	0%	11%	48%	102%	106%	112%
1.3	amortization in 10 years	26%	46%	95%	19%	36%	79%	102%	106%	112%
Indexation										
2.1	indexation is 0.5 CPI	0%	0%	44%	0%	0%	36%	41%	49%	59%
2.2	conditional indexation	0%	0%	56%	0%	0%	45%	19%	26%	45%
Portfolio composition										
3.1	100% stocks	0%	0%	80%	0%	0%	66%	102%	106%	112%
3.2	0% stocks	0%	0%	0%	0%	0%	0%	102%	106%	112%

Note: classic ALM results after 75 years for the 5%, 50% and 95% percentiles of (a) the total contribution rate (c), which is the sum of the normal cost (c^{NC}), amortization (c^{Amort}) and sponsor support payment (c^{SS}), all in percentages of the wage sum, and (b) the policy funding ratio (FR^P), the economic funding ratio (FR^E), and the pension result (PR) for the base Plan 0 and alternative contribution (Plans 1.1-1.3), indexation (Plans 2.1-2.2) and investment (Plans 3.1-3.2) policies. Note that the median values of the components of c do not necessarily add up to the median value of c. The policy funding ratio is defined as actuarial assets over actuarial liabilities, the economic funding ratio as market value of assets over economic liabilities and the pension result as the ratio of cumulative granted indexation to cumulative price inflation.

Table 9: Likelihood of full depletion of assets – zero inflation risk premium

Case	Description	Probability	Year, 5%	Year, 50%	Year, 95%
Baseline					
0	baseline plan	94.5%	29	43	67
Contribution					
1.1	0% amortization paid	98.8%	21	29	47
1.2	100% amortization paid	11.0%	52	66	74
1.3	amortization in 10 years	0.0%	-	-	-
Indexation					
2.1	indexation is 0.5 CPI	81.8%	32	51	70
2.2	conditional indexation	64.2%	37	58	73
Portfolio composition					
3.1	100% stocks	89.4%	21	35	65
3.2	0% stocks	100.0%	40	44	47

Note: the first column reports the probability that the fund's assets are fully depleted within the 75-year simulation horizon. The following columns show the quantiles for the distribution of the years of depletion, conditional on scenarios in which depletion takes place within the simulation horizon.

F.2 Extending the evaluation horizon to 100 years

Tables 14 - 15 report the results for a horizon of 100 years.

Table 10: Effects of plan changes on stakeholders – zero inflation risk premium

(a) Contract values to stakeholders

Case	Description	$V_0^{PP,Y}$	$V_0^{PP,O}$	$V_0^{TP,Y}$	$V_0^{TP,O}$
Baseline					
0	baseline plan	4671	9235	-8239	-2865
Contribution					
1.1	0% amortization paid	0	0	-384	381
1.2	100% amortization paid	0	0	676	-688
1.3	amortization in 10 years	0	0	1221	-1248
1.4	part. contr. rate doubled	-1631	-1007	1367	1270
Indexation					
2.1	indexation is 0.5 CPI	-512	-1231	1453	294
2.2	conditional indexation	-895	-2077	2522	444
Portfolio composition					
3.1	100% stocks	0	0	339	-129
3.2	0% stocks	0	0	-96	43

(b) Relative effects

Plan	Description	$\Delta RV_0^{PP,Y}$	$\Delta RV_0^{PP,O}$	$\Delta RV_0^{TP,Y}$	$\Delta RV_0^{TP,O}$
Contribution					
1.1	0% amortization paid	0%	0%	-14%	14%
1.2	100% amortization paid	0%	0%	24%	-25%
1.3	amortization 10 years	0%	0%	44%	-45%
1.4	part. contr. rate doubled	-59%	-36%	49%	46%
Indexation					
2.1	indexation is 0.5 CPI	-18%	-44%	52%	11%
2.2	conditional indexation	-32%	-75%	91%	16%
Portfolio composition					
3.1	100% stocks	0%	0%	12%	-5%
3.2	0% stocks	0%	0%	-3%	2%

Note: the table reports the effects of switching from baseline Plan 0 to Plans 1.1-3.2 on future plan participants ($\Delta V_0^{PP,Y}$, $\Delta RV_0^{PP,Y}$), current plan participants ($\Delta V_0^{PP,O}$, $\Delta RV_0^{PP,O}$), future tax payers ($\Delta V_0^{TP,Y}$, $\Delta RV_0^{TP,Y}$) and current tax payers ($\Delta V_0^{TP,O}$, $\Delta RV_0^{TP,O}$). Panel (a) reports the value of the baseline plan and the change in value of switching from the baseline to an alternative plan in billions of dollars. Panel (b) reports relative changes as percentages of the fund's initial assets A_0. Negative numbers imply a deterioration of the value for that stakeholder.

Table 11: Classic ALM results under different policies after 75 years – inflation risk premium 20 bp

(a) Contributions

Case	Description	c			c^{NC}	c^{Amort}			c^{SS}		
		5%	50%	95%	-	5%	50%	95%	5%	50%	95%
Baseline											
0	baseline plan	25%	41%	47%	13%	12%	13%	14%	0%	15%	20%
Contribution											
1.1	0% amortization paid	36%	41%	47%	13%	0%	0%	0%	23%	28%	34%
1.2	100% amortization paid	23%	36%	44%	13%	10%	23%	28%	0%	0%	4%
1.3	amortization in 10 years	15%	34%	43%	13%	2%	21%	30%	0%	0%	0%
Indexation											
2.1	indexation is 0.5 CPI	19%	36%	42%	13%	6%	13%	14%	0%	10%	15%
2.2	conditional indexation	18%	31%	38%	13%	5%	12%	13%	0%	5%	12%
Portfolio composition											
3.1	100% stocks	15%	41%	47%	13%	2%	13%	14%	0%	15%	20%
3.2	0% stocks	36%	41%	47%	13%	13%	13%	14%	10%	15%	20%

(b) Funding ratios and pension result

Case	Description	FR^P			FR^E			PR		
		5%	50%	95%	5%	50%	95%	5%	50%	95%
Baseline										
0	baseline plan	0%	0%	4%	0%	0%	3%	102%	106%	112%
Contribution										
1.1	0% amortization paid	0%	0%	0%	0%	0%	0%	102%	106%	112%
1.2	100% amortization paid	0%	15%	63%	0%	12%	51%	102%	106%	112%
1.3	amortization in 10 years	27%	47%	97%	19%	37%	83%	102%	106%	112%
Indexation										
2.1	indexation is 0.5 CPI	0%	0%	48%	0%	0%	40%	41%	49%	59%
2.2	conditional indexation	0%	0%	59%	0%	0%	48%	19%	26%	46%
Portfolio composition										
3.1	100% stocks	0%	0%	80%	0%	0%	67%	102%	106%	112%
3.2	0% stocks	0%	0%	0%	0%	0%	0%	102%	106%	112%

Note: classic ALM results after 75 years for the 5%, 50% and 95% percentiles of (a) the total contribution rate (c), which is the sum of the normal cost (c^{NC}), amortization (c^{Amort}) and sponsor support payment (c^{SS}), all in percentages of the wage sum, and (b) the policy funding ratio (FR^P), the economic funding ratio (FR^E), and the pension result (PR) for the base Plan 0 and alternative contribution (Plans 1.1-1.3), indexation (Plans 2.1-2.2) and investment (Plans 3.1-3.2) policies. Note that the median values of the components of c do not necessarily add up to the median value of c. The policy funding ratio is defined as actuarial assets over actuarial liabilities, the economic funding ratio as market value of assets over economic liabilities and the pension result as the ratio of cumulative granted indexation to cumulative price inflation.

Table 12: Likelihood of full depletion of assets – inflation risk premium 20 bp

Case	Description	Probability	Year, 5%	Year, 50%	Year, 95%
Baseline					
0	baseline plan	93.8%	29	44	67
Contribution					
1.1	0% amortization paid	98.7%	21	29	48
1.2	100% amortization paid	10.7%	52	67	75
1.3	amortization in 10 years	0.0%	-	-	-
Indexation					
2.1	indexation is 0.5 CPI	80.9%	33	51	70
2.2	conditional indexation	62.7%	37	58	73
Portfolio composition					
3.1	100% stocks	89.4%	21	35	65
3.2	0% stocks	100.0%	41	44	48

Note: the first column reports the probability that the fund's assets are fully depleted within the 75-year simulation horizon. The following columns show the quantiles for the distribution of the years of depletion, conditional on scenarios in which depletion takes place within the simulation horizon.

Table 13: Effects of plan changes on stakeholders – inflation risk premium 20 bp

(a) Contract values to stakeholders

Case	Description	$V_0^{PP,Y}$	$V_0^{PP,O}$	$V_0^{TP,Y}$	$V_0^{TP,O}$
Baseline					
0	baseline plan	5063	9456	-8781	-2936
Contribution					
1.1	0% amortization paid	0	0	-391	388
1.2	100% amortization paid	0	0	701	-713
1.3	amortization in 10 years	0	0	1265	-1294
1.4	part. contr. rate doubled	-1713	-1024	1441	1296
Indexation					
2.1	indexation is 0.5 CPI	-577	-1344	1610	316
2.2	conditional indexation	-1006	-2260	2788	471
Portfolio composition					
3.1	100% stocks	0	0	338	-127
3.2	0% stocks	0	0	-96	42

(b) Relative effects

Plan	Description	$\Delta RV_0^{PP,Y}$	$\Delta RV_0^{PP,O}$	$\Delta RV_0^{TP,Y}$	$\Delta RV_0^{TP,O}$
Contribution					
1.1	0% amortization paid	0%	0%	-14%	14%
1.2	100% amortization paid	0%	0%	25%	-26%
1.3	amortization in 10 years	0%	0%	46%	-47%
1.4	part. contr. rate doubled	-62%	-37%	52%	47%
Indexation					
2.1	indexation is 0.5 CPI	-21%	-49%	58%	11%
2.2	conditional indexation	-36%	-82%	101%	17%
Portfolio composition					
3.1	100% stocks	0%	0%	12%	-5%
3.2	0% stocks	0%	0%	-3%	2%

Note: the table reports the effects of switching from baseline Plan 0 to Plans 1.1-3.2 on future plan participants ($\Delta V_0^{PP,Y}$, $\Delta RV_0^{PP,Y}$), current plan participants ($\Delta V_0^{PP,O}$, $\Delta RV_0^{PP,O}$), future tax payers ($\Delta V_0^{TP,Y}$, $\Delta RV_0^{TP,Y}$) and current tax payers ($\Delta V_0^{TP,O}$, $\Delta RV_0^{TP,O}$). Panel (a) reports the value of the baseline plan and the change in value of switching from the baseline to an alternative plan in billions of dollars. Panel (b) reports relative changes as percentages of the fund's initial assets A_0. Negative numbers imply a deterioration of the value for that stakeholder.

Table 14: Classic ALM results under different policies – evaluation horizon 100 years

(a) Contributions

Case	Description	c			c^{NC}	c^{Amort}			c^{SS}		
		5%	50%	95%	-	5%	50%	95%	5%	50%	95%
Baseline											
0	baseline plan	37%	43%	50%	13%	13%	14%	15%	11%	16%	21%
Contribution											
1.1	0% amortization paid	38%	43%	50%	13%	0%	0%	0%	24%	30%	36%
1.2	100% amortization paid	33%	40%	49%	13%	19%	27%	30%	0%	0%	6%
1.3	amortization in 10 years	23%	38%	47%	13%	10%	25%	33%	0%	0%	0%
Indexation											
2.1	indexation is 0.5 CPI	32%	38%	44%	13%	13%	13%	14%	6%	11%	16%
2.2	conditional indexation	24%	34%	40%	13%	11%	13%	14%	0%	7%	13%
Portfolio composition											
3.1	100% stocks	27%	43%	49%	13%	13%	14%	15%	0%	15%	21%
3.2	0% stocks	38%	43%	50%	13%	13%	14%	15%	11%	16%	21%

(b) Funding ratios and pension result

Case	Description	FR^P			FR^E			PR		
		5%	50%	95%	5%	50%	95%	5%	50%	95%
Baseline										
0	baseline plan	0%	0%	0%	0%	0%	0%	103%	108%	115%
Contribution										
1.1	0% amortization paid	0%	0%	0%	0%	0%	0%	103%	108%	115%
1.2	100% amortization paid	0%	2%	30%	0%	2%	22%	103%	108%	115%
1.3	amortization in 10 years	24%	41%	77%	15%	28%	59%	103%	108%	115%
Indexation										
2.1	indexation is 0.5 CPI	0%	0%	0%	0%	0%	0%	31%	38%	47%
2.2	conditional indexation	0%	0%	14%	0%	0%	10%	10%	15%	25%
Portfolio composition										
3.1	100% stocks	0%	0%	1%	0%	0%	1%	103%	108%	115%
3.2	0% stocks	0%	0%	0%	0%	0%	0%	103%	108%	115%

Note: classic ALM results after 100 years for the 5%, 50% and 95% percentiles of (a) the total contribution rate (c), which is the sum of the normal cost (c^{NC}), amortization (c^{Amort}) and sponsor support payment (c^{SS}), all in percentages of the wage sum, and (b) the policy funding ratio (FR^P), the economic funding ratio (FR^E), and the pension result (PR) for the base Plan 0 and alternative contribution (Plans 1.1-1.3), indexation (Plans 2.1-2.2) and investment (Plans 3.1-3.2) policies. Note that the median values of the components of c do not necessarily add up to the median value of c. The policy funding ratio is defined as actuarial assets over actuarial liabilities, the economic funding ratio as market value of assets over economic liabilities and the pension result as the ratio of cumulative granted indexation to cumulative price inflation.

Table 15: Effects of plan changes on stakeholders – evaluation horizon 100 years

(a) Contract values to stakeholders

Case	Description	$V_0^{PP,Y}$	$V_0^{PP,O}$	$V_0^{TP,Y}$	$V_0^{TP,O}$
Benchmark					
0	baseline plan	5681	9209	-9241	-2853
Contribution					
1.1	0% amortization paid	0	0	-349	342
1.2	100% amortization paid	0	0	637	-630
1.3	amortization 10 years	0	0	1180	-1170
1.4	part. contr. rate doubled	-1902	-978	1642	1237
Indexation					
2.1	indexation is 0.5 CPI	-658	-955	1367	253
2.2	conditional indexation	-1195	-1656	2456	403
Portfolio composition					
3.1	100% stocks	0	0	247	-145
3.2	0% stocks	0	0	-71	51

(b) Relative effects

Plan	Description	$\Delta RV_0^{PP,Y}$	$\Delta RV_0^{PP,O}$	$\Delta RV_0^{TP,Y}$	$\Delta RV_0^{TP,O}$
Contribution					
1.1	0% amortization paid	0%	0%	-13%	12%
1.2	100% amortization paid	0%	0%	23%	-23%
1.3	amortization 10 years	0%	0%	43%	-42%
1.4	part. contr. rate doubled	-69%	-35%	59%	45%
Indexation					
2.1	indexation is 0.5 CPI	-24%	-34%	49%	9%
2.2	conditional indexation	-43%	-60%	89%	15%
Portfolio composition					
3.1	100% stocks	0%	0%	9%	-5%
3.2	0% stocks	0%	0%	-3%	2%

Note: the table reports the effects of switching from baseline Plan 0 to Plans 1.1-3.2 on future plan participants ($\Delta V_0^{PP,Y}$, $\Delta RV_0^{PP,Y}$), current plan participants ($\Delta V_0^{PP,O}$, $\Delta RV_0^{PP,O}$), future tax payers ($\Delta V_0^{TP,Y}$, $\Delta RV_0^{TP,Y}$) and current tax payers ($\Delta V_0^{TP,O}$, $\Delta RV_0^{TP,O}$). Panel (a) reports the value of the baseline plan and the change in value of switching from the baseline to an alternative plan in billions of dollars. Panel (b) reports relative changes as percentages of the fund's initial assets A_0. Negative numbers imply a deterioration of the value for that stakeholder.