Appendix

This appendix contains details on (A) the construction and sources of our data for the exogenous variables in our model (demography, rate of technical progress, world interest rate, and all fiscal policy and pension policy parameters), and (B) the first-order conditions of individuals in the baseline model. We would suggest that these appendices are not included if the paper were accepted. We can include the material in a supplementary online appendix, which can be made publicly available. We would then refer the reader to this online appendix. For details on the construction and sources of our data for the endogenous variables (employment, education, earnings) we refer to Devriendt and Heylen (2020, their Appendix C).

A Exogenous variables

This Appendix gives more details about the exogenous variables that we used in the calibration of the model in Section 3 and to obtain our baseline simulation in Section 4.

A.1 Demography

f_t: fertility rates

Data source: population by age since 1948 (Bevolkingsvooruitzichten 2015-2060 of the Belgian Federal Planning Bureau and Statistics Belgium)

Computation: We divided the population of age 18 to 20 during three years by the population of age 18 to 20 in the previous three years. The fertility rates are displayed in Figure 8. As to the impact of migration, both natives and immigrants of age 18 to 20 are included in the youngest generation. They affect population dynamics in our model. People who enter or leave the country after the age of 20 do not. Children of immigrants are included in the fertility rate when they become 18.

sr_j^t: conditional survival rates

Data sources: Statistics Belgium, Mortality rates before 1998 are by age category (sometimes 4 years, sometimes 5) and start from 1946. As of 1998 data are annual. Prospects were provided by the Belgian Federal Planning Bureau and Statistics Belgium (Bevolkingsvoorzichten 2015-2060).

Computation: Survival rates were calculated by substracting the mortality rate from 1. Figure 7 shows conditional survival rates at age 45, 60, 75 and 90 for individuals born in 1905, 1925, 1950, 1975 and 2014.

Figure 7: Age-specific conditional survival rates in Belgium

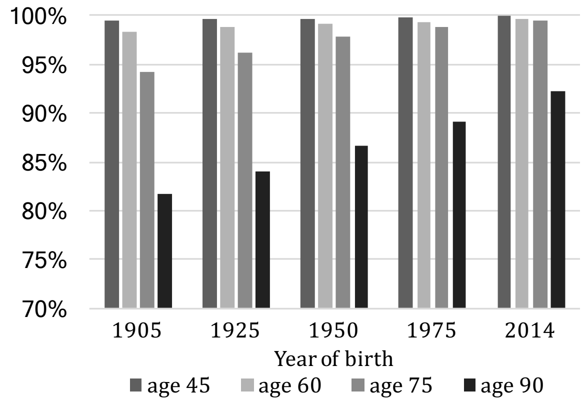
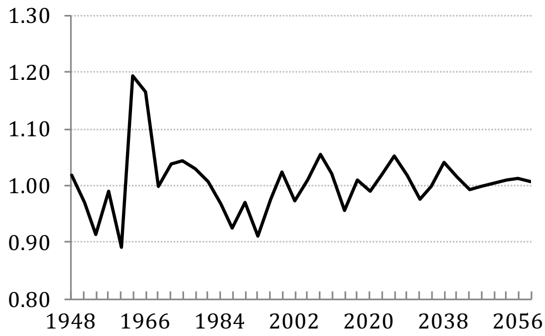


Figure 8: Fertility rate since 1948



A.2 World real interest rate

The assumption of an open economy with perfect capital mobility implies that the net after-tax rate of return on physical capital will always be equal to the (exogenous) world real interest rate r_t. It requires us to introduce data for this interest rate over a very long period of time. To the best of our knowledge, however, this is not readily available. Krueger and Ludwig (2007) and - more recently - Marchiori et al (2017) have computed highly relevant series using an OLG model and taking into account projections for future demography at the world or OECD level. Their results are fairly similar, but their data do not cover the whole period since 1950. To get data also for the earliest decades, we relied on the US stock market data from Shiller (2015). Figure 9 includes his cyclically-adjusted earnings/price ratio in %. We take it as a proxy for the return to physical capital in the world in the 20th century. Combining this proxy with the simulated real interest rate series for 2000-2050 from Marchiori et al (2017), and smoothing using a third degree polynomial, yields our world real interest rate.

A.3 Rate of technical progress

Figure 10 displays the exogenous rate of labour augmenting technical change $g_{a,t}$ since 1951. Our main source is Feenstra et al (2015, Penn World Table 8.1). We used their data for TFP growth until 2011, after a double adjustment. First, a correction was necessary for the different treatment of hours worked[1]. Second, we HP-filtered the corrected data to obtain the trend rate of technical change and to exclude cyclical effects. For the years until 2021, we approximate $g_{a,t}$ by productivity per hour worked as projected by the Federal Planning Bureau (2016). Missing data in between both periods are determined by linear interpolation. As of 2022, we use productivity per worker as advanced by the Belgian Studiecommissie voor de Vergrijzing (2016) as a proxy. The projected 1.5% annual growth rate after 2034 also corresponds to the projection for the rate of technical progress of the 2015 European Commission's Working Group on Ageing.

Figure 9: Annual world interest rate

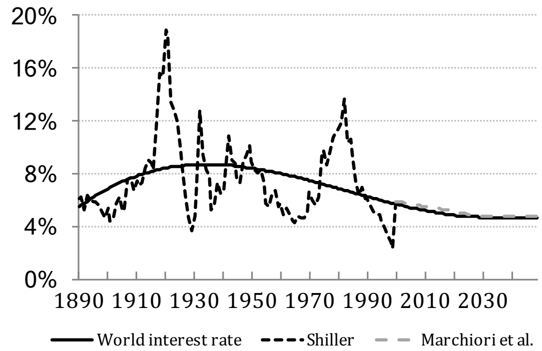
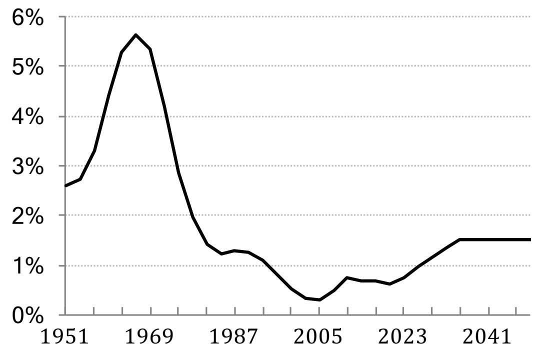


Figure 10: Average annual rate of technical progress



[1]The Penn World Table 8.1 includes data for TFP (rtfpna) which correspond to the following production function: $Y = BK^{\alpha}(hc.L)^{1-\alpha}$, with B the level of TFP, K physical capital, hc human capital and L employment (in persons). Using comparable notation, our production function would be $Y = K^{\alpha}(A.hc.L.h)^{1-\alpha}$ with h hours worked per employed person. It then follows that $B = (A.h)^{1-\alpha}$. The relevant growth rate of A in our model can then be approximated as the growth rate of B (or rtfpna in PWT) divided by the labour share $(1-\alpha)$ minus the growth rate of hours worked per employed person.

A.4 Fiscal policy and pension parameters

We model labour income taxes paid by workers $\tau_{w,j,s}$ as progressive. We use as tax function

$$\tau_{w,j,s} = \Gamma\left(\frac{y_{j,s}}{\tilde{y}}\right)^{\xi} + cr_1 \text{ with } s = L,M,H, \xi \geq 0, 0 < \Gamma < 1$$

where $y_{j,s}$ is gross labour income of an individual of ability s and age j, and $\tilde{y}$ is average gross labour income. As in Guo and Lansing (1998) and Koyuncu (2011), ξ and Γ govern the level and slope of the tax schedule. The marginal tax rate $\tau_{w^m,j,s}$ is the rate applied to the last euro earned:

$$\tau_{w^m,j,s} = \frac{\partial\left(\tau_{w,j,s}y_{j,s}\right)}{\partial y_{j,s}} = (1+\xi)\Gamma\left(\frac{y_{j,s}}{\tilde{y}}\right)^{\xi} + cr_1$$

Individuals are aware of the progressive structure of the tax system when making decisions. Consequently, in the budget constraints average tax rates are used, while in the first-order conditions (cf. Appendix B) marginal tax rates are used.

The parameter ξ of the tax function is chosen so that it generates the right amount of progressivity during the calibration period. The data with which we compare the tax function's values concern the observed differences in average personal income tax rates between three income groups in Belgium (67%, 100% and 150% of the average wage, OECD Tax Database, Table I.5). Minimizing the average root mean squared error between data and function values results in a value for ξ of 0.332.

Γ_t: overall average tax rate on gross labour income (% of gross wage)

Data sources: OECD Government Revenue Statistics, Details of tax revenue - Belgium, and OECD Economic Outlook (available via OECD.Stat).

Computation: Total tax revenues of individuals on income and profits (code 1110) plus social security contributions (code 2100) are divided by the gross wage bill.

τ_p: employer social contribution rate (% of the gross wage)

Data sources: OECD Government Revenue Statistics, Details of tax revenue - Belgium, and OECD Economic Outlook (available via OECD.Stat).

Computation: we divide the social contributions paid by employers (code 2200) by the private gross wage bill (the gross wage bill minus government wages).

τ_c: Consumption tax rate (in %)
Data source: McDaniel (2007, updated 2014).

τ_k: Tax rate on capital returns
Data sources: after 1982: effective marginal corporate tax rates taken from Devereux et al (2002). The data for 1970-1981 were extrapolated based on the evolution of Belgium's statutory corporate income tax rates.

g: government spending on goods and services as a fraction of GDP
Data sources: The data include government consumption and fixed capital formation (OECD Economic Outlook No 98)

rr_L, rr_M, rr_H: net own-earnings related pension replacement rates
Data sources and description: OECD Pensions at a Glance (2005,2007,2009,2013) presents net pension replacement rates for individuals at various multiples of average individual earnings in the economy. Taking into account that relative to average earnings, earnings of the low (no upper secondary degree), medium (upper secondary degree) and high ability group (tertiary degree) in Belgium are 86%, 95% and 122% (OECD Education at a Glance, 2011), we consider the data for individuals at 87,5% of average earnings as representative for the low ability group, individuals with average earnings as representative for the medium ability group, and individuals with 125% of average earnings as representative for the high ability group. Country studies show the composition (sources) of this net replacement rate. Our proxy for rr_s includes all earnings-related pensions and mandatory occupational pensions when they depend on wages or hours worked. Data before 2002 are extrapolated using Scruggs (2007), Ebbinghaus and Gronwald (2009), and Cantillon et al (1987).

Other pension policy parameters:

Other policy variables are the pension weights p_j with $j = 1-15$ for the medium and high ability group and $j = 1-14$ for the low ability group. In correspondence with the Belgian public pension system which imposes equal weights, we set them all to 1/15 and 1/14 respectively. Both the revaluation factor applied to past labour income in the determination of the pension benefit of new retirees wg, and the revaluation factor applied to adapt pension benefits of existing retirees to increased living standards pg follow the Belgian reality. In Belgium, only labour income earned between 1955 and 1974 underwent real revaluations according to wg^n with $n = 1$ in 1974, $n = 2$ in 1973, ..., $n = 20$ in 1955 and $wg = 1.036$ in 1974-1996, $wg = 1.032$ in 1997, $wg = 1.028$ in 1998, ..., $wg = 1$ as of 2005 (Festjens, 1997). pg is set to 1 before 1969, 1.023 annually between 1969 and 1992, 0.993 between 1993 and 2013, 1.003 for 2014-15, 1.005 for 2016-21 and 1.002 afterwards. Data before 1984 are from Festjens (1997). Observations until 2015 and future values were taken from Studiecommissie voor de Vergrijzing (2016). The contribution rates of individuals and firms to the first pillar pension scheme cr_1 and cr_2 are 7.5% and 8.9% respectively (OECD Pensions at a Glance, 2013).

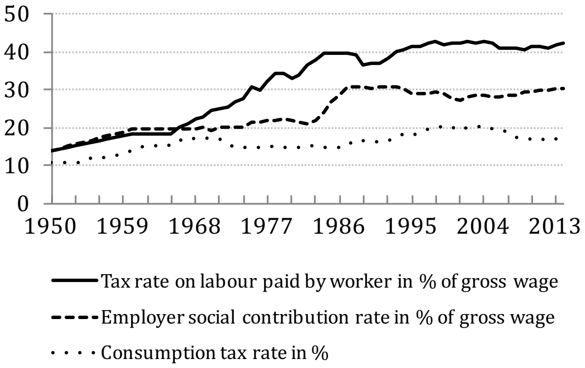
B: General government consolidated gross debt

Data source: EU Commission, AMECO, series UDGGL.

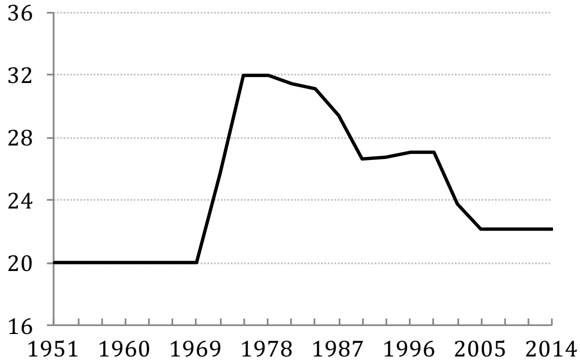
In the baseline simulation all policy parameters are kept constant at their 2014 level.

Figure 11: Fiscal and pension policy variables

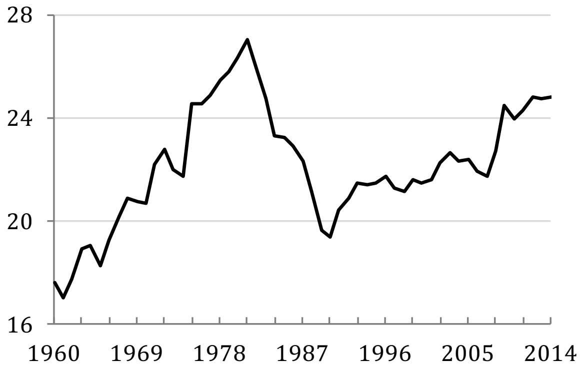
(a) Labour and consumption tax rates in %



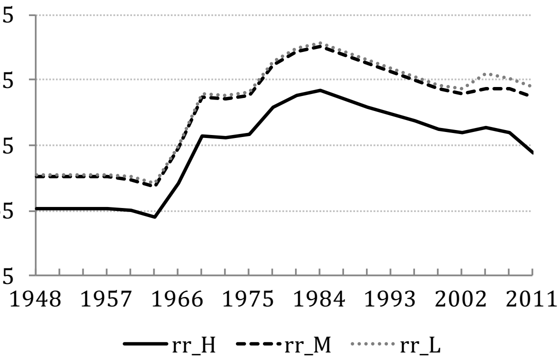
(b) Effective marginal corporate tax rate in %



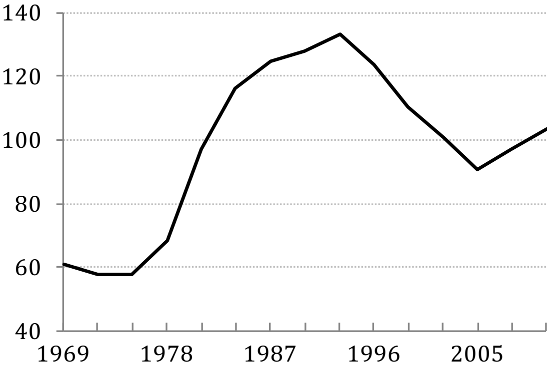
(c) Government spending per capita, in % of GDP per capita(1)



(d) Public pension replacement rates in %



(e) General government consolidated gross debt in % of GDP(1)



Note: (1) Earlier data are assumed to be equal to their level in the earliest available year.

B First-order conditions of individuals in the baseline model

The law of motion of optimal consumption over time is denoted by the Euler equation (31).

$$\frac{c^t_{j+1,s}}{c^t_{j,s}} = \beta s r^t_j \left(1 + r_{t+j}\right), \text{ for } j = 1-27,\ s = L,M,H \tag{31}$$

Equation (32) describes the optimal labour-leisure choice for individuals of high and medium ability. In each period, they will supply labour up to the point where the marginal utility of leisure equals that of labour. Individuals of low ability face the same first-order condition (not shown), but they retire one year sooner.

$$\frac{\gamma_j}{\left(l^t_{j,s}\right)^\theta} \frac{-\partial l^t_{j,s}}{\partial n^t_{j,s}} = \frac{\left(w^s_{t+j-1}\varepsilon_j h^t_{j,s}\left(1-\tau_{w^m,j,s}\right)\right)}{c^t_{j,s}\left(1+\tau_c\right)} + \sum_{k=16}^{28} \frac{\beta^{k-j}(\pi^t_k/\pi^t_j)}{c^t_{k,s}\left(1+\tau_c\right)} \frac{\partial ppt^t_{k,s}}{\partial n^t_{j,s}}, \tag{32}$$

$$\text{for } j = 1-15,\ s = M,H$$

with

$$\frac{\partial ppt^t_{16,s}}{\partial n^t_{j,s}} = rr_s \frac{1}{15} w^s_{t+j-1}\varepsilon_j h^t_{j,s}\left(1-\tau_{w^m,j,s}\right) \prod_{l=j}^{15} wg_{t+l}$$

and

$$\frac{\partial ppt^t_{k,s}}{\partial n^t_{j,s}} = \frac{\partial ppt^t_{16,s}}{\partial n^t_{j,s}} \prod_{l=16}^{k} pg_{t+l-1}, \text{ for } k > 16$$

Equation (33) states that the marginal utility loss from investing in education in period j for $s = M,H$ must equal the (discounted) marginal utility gain over life.

$$\frac{\gamma_j}{\left(l^t_{j,s}\right)^\theta} \frac{-\partial l^t_{j,s}}{\partial e^t_j} = \sum_{k>j}^{15} \beta^{k-j} \frac{\pi^t_k}{\pi^t_j} \left(\frac{w^s_{t+k-1}\varepsilon_k n^t_{k,s}\left(1-\tau_{w^m,k,s}\right)}{c^t_{k,s}\left(1+\tau_c\right)} \frac{\partial h^t_{k,s}}{\partial e^t_j} \right)$$

$$+ \sum_{k=16}^{28} \beta^{k-j} \frac{\pi^t_k}{\pi^t_j} \left(\frac{1}{c^t_{k,s}\left(1+\tau_c\right)} \frac{\partial ppt^t_{k,s}}{\partial e^t_j} \right), \text{ for } j = 1-4,\ s = M,H \tag{33}$$

with

$$\frac{\partial h^t_{k,s}}{\partial e^t_j} = \frac{\partial \left[h^t_{1,s}\left(\prod_{i=2}^{k} x^t_{i,s}\right)\right]}{\partial e^t_j} = h^t_{1,s}\left(\frac{\partial x^t_{j+1,s}}{\partial e^t_j}\prod_{\substack{i=2\\ i\neq j+1}}^{k} x^t_{i,s}\right), \text{ for } k = 2-5$$

where

$$x^t_{i,s} = \left(1+\phi_s\left(e^t_{i-1,s}\right)^{\sigma}\right), \qquad \frac{\partial h^t_{k,s}}{\partial e^t_j} = \frac{\partial h^t_{5,s}}{\partial e^t_j}, \qquad \text{for } k > 5, \text{ and}$$

$$\frac{\partial ppt^t_{16,s}}{\partial e^t_j} = rr_s\frac{1}{15}\sum_{k>j}^{15}\left(w^s_{t+k-1}\varepsilon_k n^t_{k,s}\left(1-\tau_{w^m,k,s}\right)\frac{\partial h^t_{k,s}}{\partial e^t_j}\prod_{l=k}^{15} wg_{t+l}\right)$$

and

$$\frac{\partial ppt^t_{k,s}}{\partial e^t_j} = \frac{\partial ppt^t_{16,s}}{\partial e^t_j}\prod_{l=16}^{k} pg_{t+l-1}, \text{for } k > 16$$

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