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Shannon entropy applied to dynamical instability studies

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Introduction

In the past few years, some works ([1], [2], [3], [4]) have been published concerning the utilization of Shannon's formulation of entropy (S) in the study of chaotic diffusion in dynamical systems. Such studies were focused on deriving the relation between a (numerically) calculated diffusion coefficient (D_S) and a so-called instability time ($T_{\rm inst}$), a measure of the macroscopic timescale within which a dynamical system is able to evolve without the occurrence of any type of catastrophic event.

Basic idea

Fundamentally, the Shannon entropy can be calculated from the dynamical evolution of a set of trajectories that interact mutually in a conservative scenario. By evaluating how fast the trajectories fill up 2-dimensional cells of a partitioned phase space, we are able to compute the time derivative of S, and associate it with a proper diffusion coefficient (D_S) which, ultimately, gives a timescale $T_D \sim D_S^{-1}$ for the diffusion time of the system. Using geometrical arguments, T_D may be correlated to $T_{\rm inst}(S)$, the instability timescale within which the system would remain isolated from catastrophic events (collisions and crossing orbits, or ejections). The basic relation reads:

$$T_{\rm inst} = K\frac{\Delta^2}{D_S}, \quad (1)$$

where Δ^2 denotes a given mean-square displacement, the squared distance between the initial and boundary values of actions-like variables, whereas K is a numerical factor of the order of unity.

Applications

(I) Use as a complementary numerical tool to analyze and characterize the phase space of initial conditions (ICs) of a dynamical system, by accessing information about the instability timescales of each IC applying short to medium-term integration timespans.

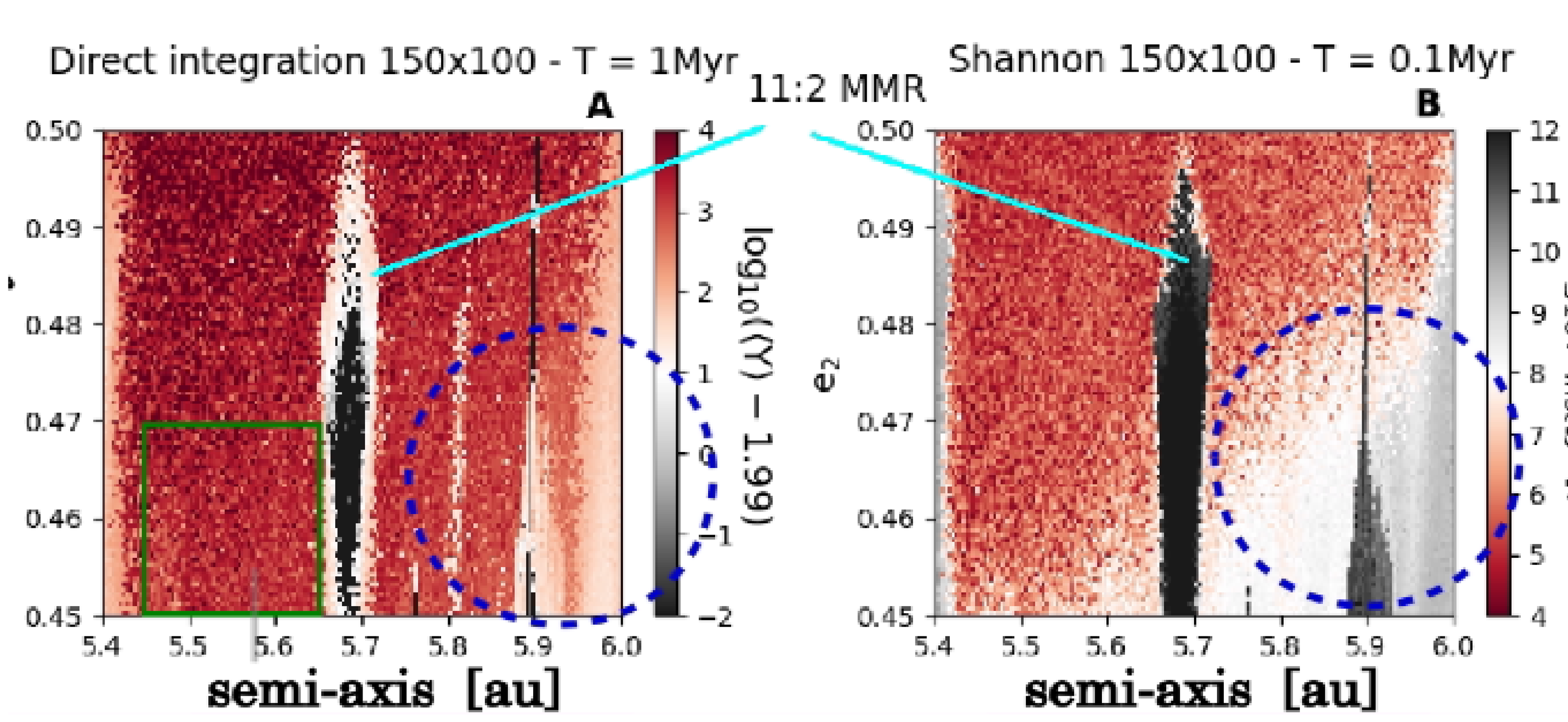

Figure 1: Dynamical maps of the (a, e)-phase plane of ICs for the outermost planet of HD 181433 exoplanetary system. *Left*: MEGNO map. *Right*: Instability map constructed from Shannon entropy evaluations.

(II) Discrimination of different types of long-term stability scenarios, e.g. regular domains, mildly chaotic domains and regions of hyperchaoticity (also referred to as *confined* or *stable* chaos).

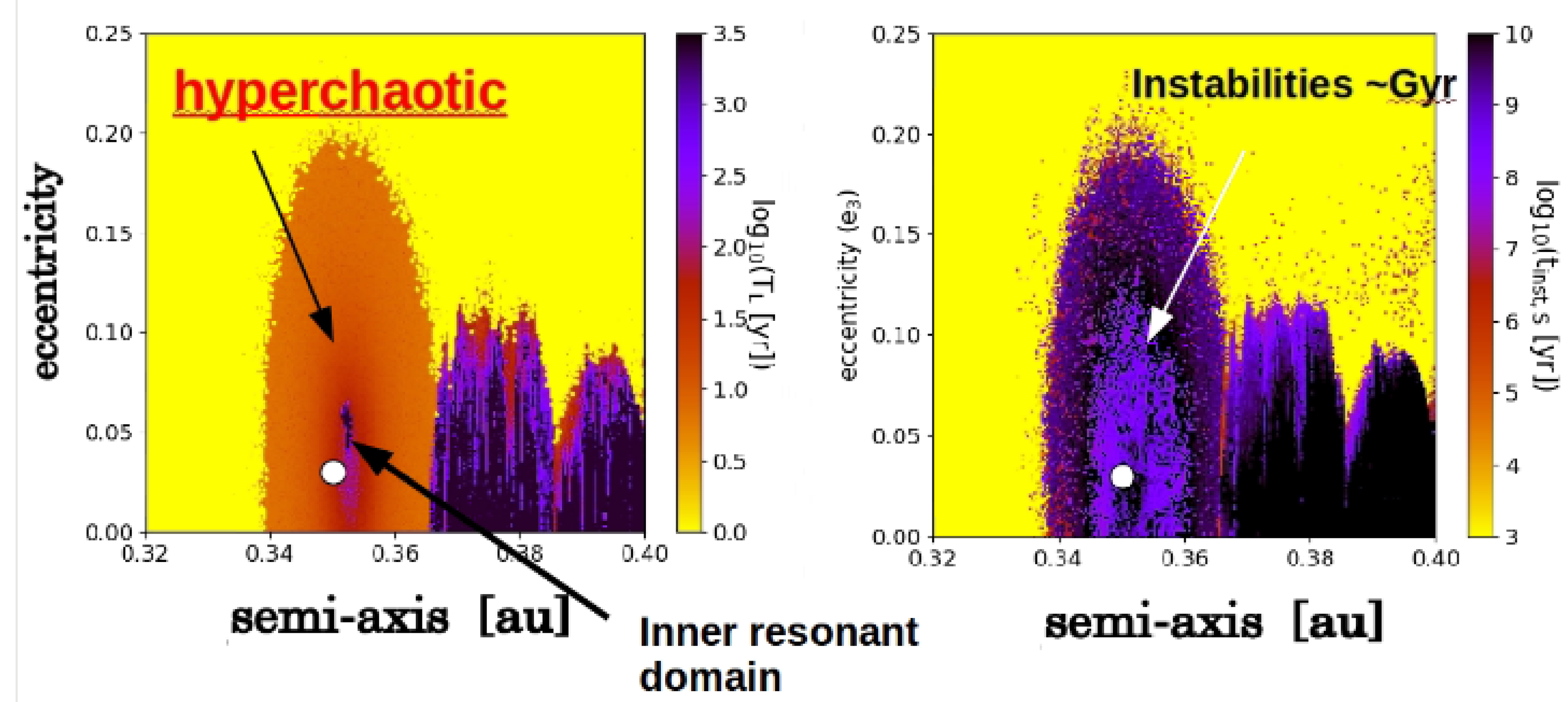

Figure 2: Dynamical maps of the phase space of ICs for the outermost planet of GJ 876 exoplanetary system. *Left*: MEGNO map, parametrized in terms of the logarithm of the Lyapunov time T_L. *Right*: Instability map constructed from Shannon entropy evaluations.

(III) Possibility of investigations on the correlation laws between the Lyapunov times and the instability timescales of sets of ICs in different types of dynamical systems.

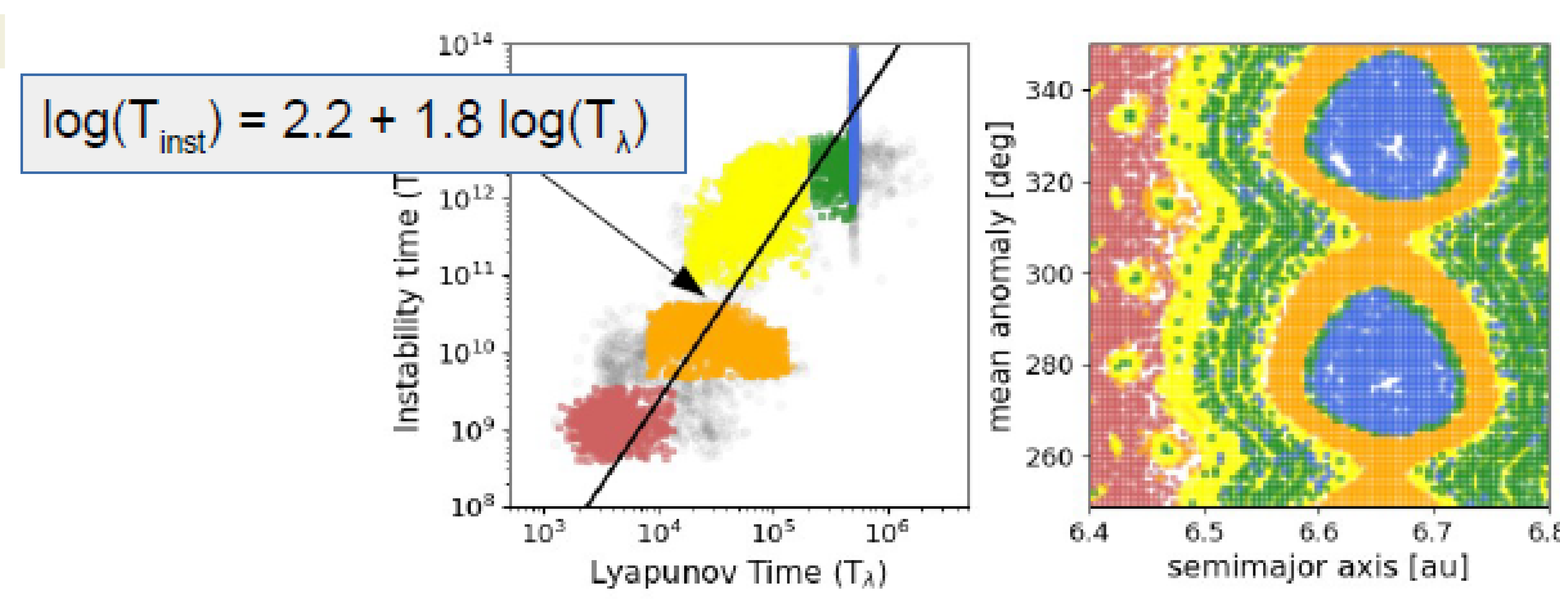

Figure 3: *Left*: Correlation diagram of Lyapunov times ($T_\lambda = T_L$) and instability times ($T_{\rm inst}$), considering the space of ICs surrounding the nominal solution of HD 181433 system, in the plane (a_2, M_2). *Right*: The corresponding map in the (a_2, M_2) plane of ICs.

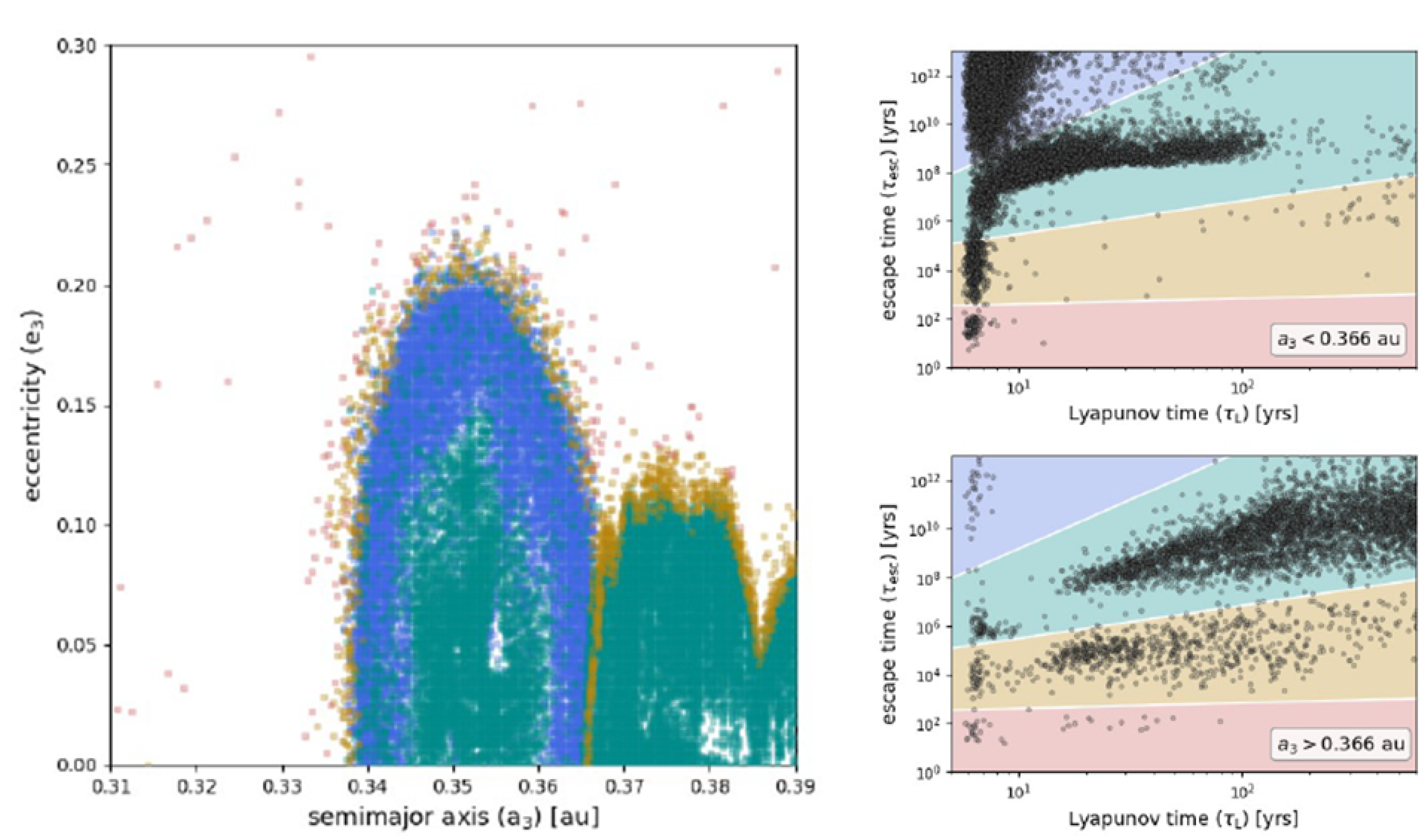

Figure 4: *Left*: The corresponding map in the (a, M)-plane of ICs of GJ 876 outermost planet. *Right*: Correlation diagrams of Lyapunov times ($T_\lambda = T_L$) and instability times ($T_{\rm inst}$), considering the space of ICs showed in the left panel.

References

[1] Beaugé, C. & Cincotta, C. M. 2019, CeMDA, 131,52.
[2] Cincotta, P. M., & Giordano, C. M. 2018, CeMDA, 130, 74.
[3] Cincotta, P. M., Giordano, C. M., Alves Silva, R. et al. 2021a, CeMDA, 133, 7.
[4] Alves Silva, R., Beaugé, C., Ferraz-Mello, et al., 2021, A&A. 652, A112.