Accurate Analytical Calculation of Lunar/Solar/Planetary Perturbations in Motion of Earth Artificial Satellites

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ABSTRACT: We suggest an advanced algorithm for analytical calculation of the orbital perturbations of Earth satellites due to the gravity attraction of the Moon, the Sun and major planets. First, a new precision harmonic series for the relevant perturbation function is developed; it is based on the numerical DE/LE-406 planetary/lunar ephemeris. Then the series is used in the author's analytical theory of satellite motion where the lunar/solar/planetary perturbations up to the 2nd order are calculated. We compare the results of the motion prediction of several satellites obtained by means of the analytical theory and a numerical integration method.

I. INTRODUCTION

When predicting the motion of Earth satellites by analytical integration methods the first step is to represent all perturbation functions acting on the satellites by precision analytical series.

In order to get a development of the perturbation function caused by the "3rd bodies" (the Moon, the Sun and major planets) some known analytical motion models of the perturbing bodies are usually used. However, presently the accuracy of neither of such analytical model matches the accuracy of current numerical ephemerides of the lunar/planetary motion, like those of the DE-series (JPL, USA), INPOP-series (IMCCE, France), or EPM-series (IAA, Russia). Thus, a new precision analytical representation of the perturbation function due to the gravity attraction of the Moon, the Sun and major planets on a Earth satellite is an actual task.

II. THE «3rd-BODIES» PERTURBATION FUNCTION

The "3rd-bodies" perturbation function, R, is expanded as follows:

$$R=\sum_{l=2}^{\infty}\sum_{m=0}^{l}\sum_{p=0}^{l}\sum_{q=-\infty}^{\infty}\left(\frac{a}{R_s}\right)^l \bar{F}_{lmp}(i)X_{l-2p+q}^{l,l-2p}(e)\left[\bar{A}_{lm}\cos\psi_{lmpq}+\bar{B}_{lm}\sin\psi_{lmpq}\right],$$

where $a, e, i, \Omega, \pi, \lambda$ are Keplerian elements of the satellite orbit, $\bar{F}_{lmp}$ is an inclination function; $X_{l-2p+q}^{l,l-2p}$ is a Hansen coefficient, R_s is a scaling factor (assumed to be 43,000 km in this study),

$$\psi_{lmpq}=(l-2p+q)\lambda-q\pi+(m+2p-l)\Omega,$$

$$\bar{A}_{lm}=\begin{cases}\bar{C}_{lm}\text{ , if } l\text{-}m \text{ is even}\\ -\bar{S}_{lm}\text{ , if } l\text{-}m \text{ is odd ,}\end{cases}\qquad \bar{B}_{lm}=\begin{cases}\bar{S}_{lm}\text{ , if } l\text{-}m \text{ is even}\\ \bar{C}_{lm}\text{ , if } l\text{-}m \text{ is odd ,}\end{cases}$$

$$\bar{C}_{lm}(t)=\frac{1}{2l+1}\sum_j\frac{\mu_j}{R_s}\left(\frac{R_s}{r_j(t)}\right)^{l+1}\bar{P}_{lm}(\sin\delta_j(t))\cos m\alpha_j(t),$$

$$\bar{S}_{lm}(t)=\frac{1}{2l+1}\sum_j\frac{\mu_j}{R_s}\left(\frac{R_s}{r_j(t)}\right)^{l+1}\bar{P}_{lm}(\sin\delta_j(t))\sin m\alpha_j(t),$$

and $\mu_j, r_j, \alpha_j, \delta_j$ are the gravitational parameter, geocentric distance, right ascension and declination of the j^{th} perturbing body at epoch t, $\bar{P}_{lm}$ is a normalized associated Legendre function.

Coefficients $\bar{C}_{lm}(t)$, $\bar{S}_{lm}(t)$ accumulate the total information about the instantaneous positions of the perturbing bodies.

III. EXPANSION OF COEFFICIENTS TO HARMONIC SERIES

STEP 1: Numerical values of the $\bar{C}_{lm}(t)$, $\bar{S}_{lm}(t)$ coefficients are tabulated with a one day's step over a 2,000 years' time interval centered at epoch J2000. The Moon, the Sun, Mercury, Venus, Mars, Jupiter and Saturn are taken as the attracting bodies (the source: numerical ephemeris DE-406).

STEP 2: Thus tabulated values are expanded to harmonic series by our modification of the spectral analysis method (Kudryavtsev: *J. Geodesy*, **77**, 829, 2004; *Astron. Astroph.*, **471**, 1069, 2007). The expansion form is

$$\bar{C}_{lm}(t)\ [\text{or } \bar{S}_{lm}(t)]\approx A_0+A_1t+A_2t^2$$
$$+\sum_k\{[A_{k0}^c+A_{k1}^ct+A_{k2}^ct^2]\cos\omega_k(t)+[A_{k0}^s+A_{k1}^st+A_{k2}^st^2]\sin\omega_k(t)\},$$

where $\omega_k(t)=\nu_kt+\nu_{k2}t^2+\nu_{k3}t^3+\nu_{k4}t^4$ are some linear combinations of integer multipliers of Delaunay arguments & lunar/planetary mean mean longitudes, $A_0, A_1, A_2, A_{k0}^c, A_{k1}^c, A_{k2}^c, A_{k0}^s, A_{k1}^s, A_{k2}^s, \nu_k, \ldots, \nu_{k4}$ are constants.

Minimal amplitude of terms in the series for $\bar{C}_{lm}(t)$, $\bar{S}_{lm}(t)$ is 10^{-6} m²/s² (or ~ 10^{-8} of the maximum values of the coefficients). Maximum order l = 8.

The total number of terms in the expansion series is ~ 38,500.

IV. ANALYTICAL CALCULATION OF THE «3rd-BODIES» EFFECT

To calculate the "3rd-bodies" effect on satellites positions the algorithm of analytical integration of Lagrange motion equations by Kudryavtsev (*Cel. Mech. Dyn. Astron.*, **61**, 207, 1995) was used. The right-hand parts of the equations include the above presented development of the "3rd-bodies" perturbation function. The algorithm accuracy was estimated as follows. Positions of STARLETTE, LAGEOS and ETALON-1 satellites over 1 year by a numerical integration method were calculated. A complete model of the lunar/solar/planetary perturbations was used. Then the satellites positions were assumed as fictitious observations and processed by the analytical theory where the perturbations of up to the 2nd order from the "3rd bodies" were calculated. The obtained results are given in Table 1.

Table 1. Comparison of analytical and numerical calculation of the satellite orbital perturbations due to the Moon, the Sun and planets.

Satellite	Semi-major axis, km	The number of "3rd bodies" terms in analytical theory	RMS deviation, m (Time interval)
ETALON-1	25,500	22,216	8.8 (1 year)
LAGEOS	12,300	7,544	1.5 (1 year)
STARLETTE	7,330	3,040	0.2 (1 month)