Supplementary materials

"Forecasting age distribution of death counts: An application to annuity pricing"

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S1. Annuity price calculation for the Australian male data

In Table A, to provide an example of the annuity calculations, we compute the best estimate of the annuity prices for different ages and maturities for a male policyholder residing in Australia. We assume a constant interest rate $\eta = 3\%$ and hence zero-coupon bond is given as $B(0,\tau) = \exp^{-\eta\tau}$.

Table A: *Estimates of annuity prices with different ages and maturities (T) for a male policyholder residing in Australia. These estimates are based on forecast mortality rates from 2015 to 2064. We consider only contracts with maturity so that age + maturity $\leq$ 110. If age + maturity $>$ 110, NA will be shown in the table.*

Age	$T = 5$	$T = 10$	$T = 15$	$T = 20$	$T = 25$	$T = 30$
CPV						
60	4.4821	8.1694	11.1358	13.4387	15.0911	16.1213
65	4.4417	8.0150	10.7890	12.7794	14.0204	14.5930
70	4.3779	7.7766	10.2151	11.7356	12.4372	12.6353
75	4.2734	7.3396	9.2514	10.1336	10.3827	10.4173
80	4.0425	6.5631	7.7262	8.0546	8.1001	8.1027
85	3.6839	5.3838	5.8637	5.9303	5.9341	NA
90	3.0140	3.8651	3.9832	3.9899	NA	NA
95	2.2655	2.5799	2.5978	NA	NA	NA
100	1.5593	1.6481	NA	NA	NA	NA
105	1.0617	NA	NA	NA	NA	NA
$K = 6$						
60	4.4851	8.1866	11.1898	13.5540	15.2963	16.4257
65	4.4520	8.0641	10.9077	13.0033	14.3617	15.0298
70	4.4041	7.8712	10.4262	12.0825	12.8970	13.1469
75	4.3124	7.4903	9.5503	10.5634	10.8742	10.9212
80	4.1171	6.7858	8.0983	8.5010	8.5618	8.5655
85	3.7732	5.6287	6.1980	6.2840	6.2893	NA
90	3.1297	4.0899	4.2350	4.2438	NA	NA
95	2.3655	2.7229	2.7448	NA	NA	NA
100	1.6179	1.7171	NA	NA	NA	NA
105	1.0895	NA	NA	NA	NA	NA

To measure forecast uncertainty, we construct the bootstrapped age-specific life-table death counts, derive the survival probabilities and calculate the corresponding annuities associated with different ages and maturities. Given that we consider ages from 60 to 110, we construct 50-steps-ahead bootstrap forecasts of age-specific life-table death counts. In Table B, we present the 95% pointwise prediction intervals of annuities for different ages and maturities, where age + maturity ≤ 110.

Table B: *95% pointwise prediction intervals of annuity prices with different ages and maturities (T) for a male policyholder residing in Australia. These estimates are based on forecast mortality rates from 2015 to 2064. We consider only contracts with maturity so that age + maturity* ≤ 110*. If age + maturity* > 110*, NA will be shown in the table.*

Age	$T = 5$	$T = 10$	$T = 15$	$T = 20$	$T = 25$	$T = 30$
CPV						
60	(4.540, 4.572)	(8.384, 8.500)	(11.582, 11.867)	(14.188, 14.739)	(16.146, 17.136)	(17.437, 19.004)
65	(4.510, 4.570)	(8.263, 8.487)	(11.319, 11.828)	(13.624, 14.612)	(15.147, 16.786)	(15.866, 18.174)
70	(4.461, 4.563)	(8.083, 8.453)	(10.814, 11.698)	(12.610, 14.239)	(13.460, 15.845)	(13.711, 16.480)
75	(4.367, 4.547)	(7.656, 8.341)	(9.809, 11.306)	(10.845, 13.191)	(11.137, 13.959)	(11.179, 14.106)
80	(4.137, 4.480)	(6.825, 7.989)	(8.109, 10.208)	(8.457, 11.104)	(8.502, 11.278)	(8.506, 11.290)
85	(3.727, 4.301)	(5.500, 7.047)	(6.015, 8.135)	(6.088, 8.332)	(6.092, 8.347)	NA
90	(2.959, 3.761)	(3.806, 5.258)	(3.930, 5.557)	(3.938, 5.575)	NA	NA
95	(2.157, 2.980)	(2.451, 3.523)	(2.465, 3.568)	NA	NA	NA
100	(1.460, 2.000)	(1.538, 2.137)	NA	NA	NA	NA
105	(0.811, 1.434)	NA	NA	NA	NA	NA
$K = 6$						
60	(4.542, 4.573)	(8.396, 8.505)	(11.631, 11.882)	(14.302, 14.773)	(16.375, 17.216)	(17.828, 19.201)
65	(4.520, 4.572)	(8.319, 8.498)	(11.437, 11.858)	(13.862, 14.700)	(15.560, 17.008)	(16.512, 18.656)
70	(4.477, 4.568)	(8.166, 8.479)	(11.039, 11.785)	(13.042, 14.475)	(14.159, 16.395)	(14.592, 17.442)
75	(4.406, 4.560)	(7.835, 8.416)	(10.235, 11.547)	(11.567, 13.793)	(12.075, 15.025)	(12.198, 15.418)
80	(4.230, 4.518)	(7.192, 8.201)	(8.848, 10.857)	(9.446, 12.278)	(9.564, 12.697)	(9.573, 12.784)
85	(3.923, 4.426)	(6.083, 7.594)	(6.893, 9.353)	(7.082, 9.900)	(7.091, 9.948)	NA
90	(3.275, 4.077)	(4.460, 6.292)	(4.689, 6.994)	(4.713, 7.097)	NA	NA
95	(2.423, 3.494)	(2.907, 4.582)	(2.955, 4.740)	NA	NA	NA
100	(1.611, 2.651)	(1.739, 3.006)	NA	NA	NA	NA
105	(1.055, 1.840)	NA	NA	NA	NA	NA