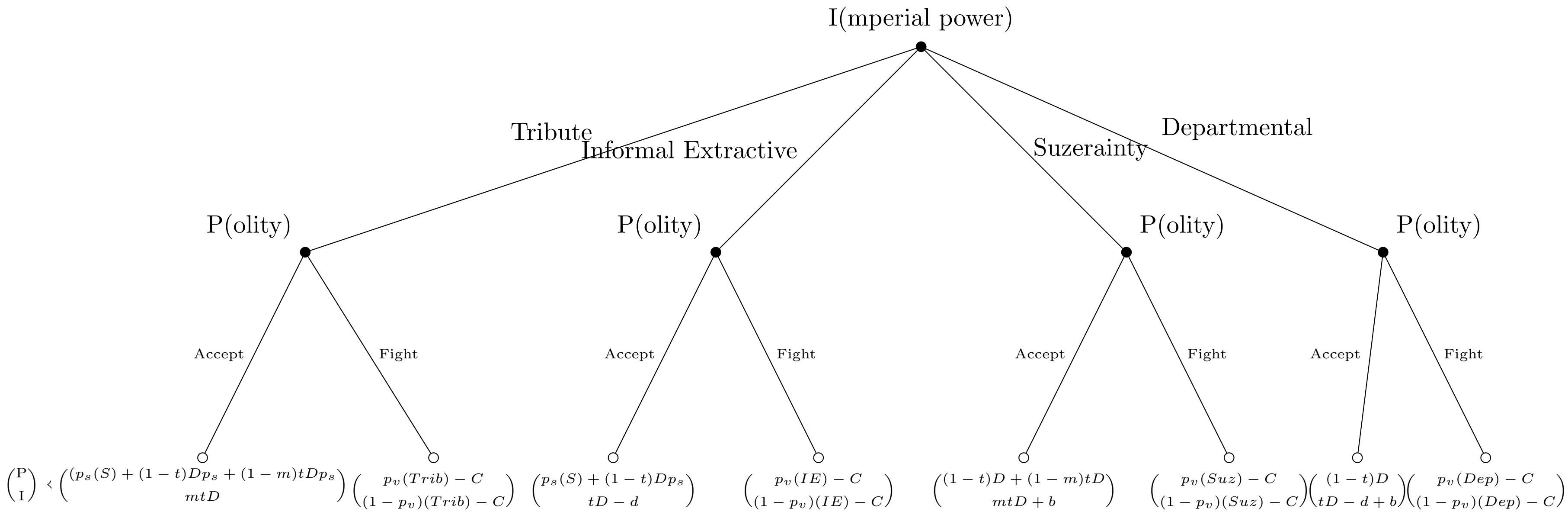
Proofs for War, Interaction Capacity and the Structures of State Systems

October 1, 2019

1 Proofs

1.1 Structure of the Game


I(mperial power)
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$\binom{P}{I} < \binom{p_s(S) + (1-t)Dp_s + (1-m)tDp_s}{mtD}$
$\binom{p_v(Trib) - C}{(1-p_v)(Trib) - C}$
$\binom{p_s(S) + (1-t)Dp_s}{tD - d}$
$\binom{p_v(IE) - C}{(1-p_v)(IE) - C}$
$\binom{(1-t)D + (1-m)tD}{mtD + b}$
$\binom{p_v(Suz) - C}{(1-p_v)(Suz) - C}$
$\binom{(1-t)D}{tD - d + b}$
$\binom{p_v(Dep) - C}{(1-p_v)(Dep) - C}$

1.2 Proofs for H1-H4 in the text

Proposition 1: *I's preference for extraction over transfers is independent of b and p_s*

Proposition 2: *I's preference for extraction over transfers is an increasing function of decreasing d and increasing D*

First, I always prefers bargains that involve controlling p's foreign policy (i.e Departmental or Suzerainty) because $mtD + b > mtD$ and $tD - d + b > tD - d$. Second, P always prefers transfers to extraction because $(1 - t)D + (1 - m)tD > (1 - t)D$ and $p_s(S) + (1 - t)Dp_s + (1 - m)tDp_s > p_s(S) + (1 - t)Dp_s$. When considering I's preferences for extraction over transfers we restrict the problem to a decision between suzerainty and departmental. When considering P's reservation point below, we restrict it to that determined by the value of tributary and suzerain relations.

Now consider the decision for I to change from *Suzerainty* to *Departmental*, summarized by:

$$U(Dep_i) > U(Suz_i) = tD - d + b > mtD + b$$

Which simplifies to:

$$tD - mtD > d \tag{1}$$

Informally, equation 1 captures the notion that extraction is preferred to transfers when the difference between could be extracted from D and what is actually taken after shirking is greater than the fixed costs that need to be paid to establish direct rule. Neither the RHS nor the LHS of Equation 1 are a function of b (Proposition 1). The LHS of equation 1 is a positive function of D and the right hand side a negative function of d (Propostion 2). Thus increasing potential revenues from D increase the value of extraction while decreasing fixed infrastructural costs reduce it, consistent with **Proposition 1** and **Proposition 2**.

Stated in terms of the hypotheses in the text, since P always prefers transfer based arrangements, this implies that situations where $tD - mtD < d$ (i.e low density systems) are characterized by transfer-arrangements or, composite states where P's retain wide-ranging autonomy (Nexon 2009), corresponding to $H1$ in the text. Decreasing d or increasing D are a cause of the shift to extraction, corresponding to $H2$ in the text. Since, in low density systems, equation 1 is independent of b and ps, and P always prefers transfers over extraction, international competition does not cause a shift in either P or I's preference for transfer over extraction. International competition does not cause a change to extraction in low-density systems ($H3$).

Proposition 3: *International competition makes extraction more likely as an equilibrium only when I has preferences for extraction and as S becomes more valuable*

Proposition 3 relates to $H4$ in the text. Now consider a situation where I has preferences for extraction (i.e $tD - mtD > d$). In the model explained in the main text, I gets its highest preference that does not trigger P's reservation price (i.e the most that it can get from war). International competition can make it more likely that I gets it's first preference (extraction in this case) by lowering P's reservation price, meaning that I has to compensate P less with shirking opportunities to avoid war.

P's reservation price is a function of international competition (p_s) only when $U(Trib_p) > U(Suz_p)$ (recalling that p always prefers transfers to extraction), or:

$$p_s(S) + (1-t)Dp_s + (1-m)tDp_s > (1-t)D + (1-m)tD$$

Which simplifies to:

$$p_s(S + D - mtD) > D - mtD \tag{2}$$

In general tributary arrangements are more likely to be preferred when p_s is high such that P gains little from accepting protection from I and when the value of S is high. If P already has preferences for suzerainty then international competition does not further change those preferences or lower P's reservation price further because $p_v(D - mtD) - C$ is not a function of p_s. If $S = 0$, $U(Suz_p) > U(Trib_p)$ because:

$$p_s(D - mtD) < D - mtD \qquad | \qquad 0 >= p_s <= 1$$

Thus a necessary condition for international competition to cause changes from transfers to extraction is that $S > 0$ and the bigger that S is, the greater impact negative changes in p_s have on lowering P's reservation price. This is because $(S + D + mtD)$ in Equation 2 is the gradient of the line $p_s(S + D - mtD)$ and so long as $U(Trib_p) > U(Suz_p)$, increasing values of S makes the line steeper and the effect of p_s on P's reservation price higher. Where S is very high, a decreasing p_s can lower P's reservation price without triggering a shift in p's preferences from tributary to suzerainty, and make it more likely that I gets it's first preference (extraction).

Therefore, international competition can make the transition from transfers to extraction more likely under at least one circumstance identified here, namely when:

(1) $tD - mtD > d$

(2) $S > 0$

And the impact of international competition is greater with higher values of S.

1.3 Allowing competition to affect p_v

If we allow international competition to decrease p_v (i.e the likelihood that P prevails over I in a conflict) such that competition favors I, then this could induce a change in I's preferences from transfers to extraction. Equation (1) shows that I's preferences for extraction are a positive function of t, and the value of t is determined by the likelihood that I prevails in a conflict. This helps explain why extraction occurs near to the centre of states in low-density systems – transfers can be large enough that, even with shirking opportunities, they exceed the (lower) fixed costs d. If international competition favors P's then it will have the opposite effect, making it less likely that what can be gained from P is enough to overcome the fixed costs of rule and making transfer-based strategies more preferable. While this is a potential connection between international competition and centralization, it is (1) not the mechanism described in the classic literature on war and state-making which emphasizes increasing demand for resources and (2) the impact of competition is constrained by the size of D and the size of d, having the largest likelihood of inducing a change to extraction in situations where D is large and d is small, i.e, high density systems. Put differently, proportionally bigger transfers from a small pie where fixed costs of direct rule are high are less likely to induce a change to extraction than bigger transfers from a big pie and/or where fixed costs are lower.

2 Second Solution

Here we show similar results if we allow I to choose to collect a proportion of tD as extraction and a proportion as transfers, collapsing the suzerainty and departmental dimensions in to a single dimension. Let the proportion of tD taken as extraction be p and the proportion taken as transfers be $(1 - p)$ where $p[0, 1]$. Let $tD = V$ or the value of the resources that are available through transfers or extraction. I's utility function can be defined as:

$$p(V - d) + (1 - p)mV + b$$

Which expands to:

$$U_i = pV - pd + mV - pmV + b \tag{3}$$

To see how changes in p (i.e the proportion of revenue taken as extraction) changes I's utility we take the partial derivative with respect to p:

$$\frac{\partial U_i}{\partial p} = V - d - mV \tag{4}$$

First, whether investing in p brings I greater or lesser utility is not a function of b (what it gets from controlling another polity's sovereignty).

Whether investments in p bring I greater utility or less utility depends on how valuable the potential transfer from the territory is (V), how much of V is lost through shirking and the fixed costs of extraction. Setting $\frac{\partial}{\partial p} = 0$, Equation 4 has a stationary point at $V = d + mV$, or when the value of the transfer is equal to the fixed costs of extraction and the amount lost through shirking. Where $V > d + mV$, more investments in p bring greater utility, where $V < d + mV$, more investments bring lower utility. Thus, the value of investing more in extraction for I is a function of V, d and m, but not b (Proposition 1 and Proposition 2).

Assuming I is otherwise unconstrained, the above also implies that when $V < d + mV$ then I should prefer to set p to 0 and take all the revenue as transfers because each addition unit of p brings negative utility. However, when $V > d + mV$, I should prefer to set p to 1 because each additional unit brings greater utility. This implies that the condition $V = d + mV$ is a threshold where I strictly prefers extraction to transfers or vice versa.

We can also show that P always prefers to give resources in transfers, not extraction. Consider the utility function, where some proportion p of the transfer tD is taken as extraction and $p - 1$ is taken as transfers. This can be expressed as:

$$U_P = p(D - tD) + (1 - p)(D - mtD)$$

Which simplifies to:

$$U_P = -ptD + D - mtD + pmtD \quad (5)$$

Taking the partial derivative with respect to p:

$$\frac{\partial U_P}{\partial p} = -tD + mtD \quad (6)$$

Because $m[0, 1]$, equation 6 will always be negative. Thus, increasing values of p (i.e more extraction) imply lower utility for P. P maximizes utility by setting p to 0.

Therefore, in situations where $V < d + mV$, P and I have a mutual interest in setting p to zero and transfers will be a stable equilibrium.

For the case where $V > d + mV$ and I prefers p to be set to 1, we consider a very simple bargaining situation where P and I negotiate over the size of $p[0, 1]$. For simplicity we assume that utilities are determined by the value of p for I and $(p - 1)$ of P. I has an outside option $\underline{U}_I = p - C$ (where C are the costs of conflict) and we use a modification of P's tributary option as its outside option $\underline{U}_P = 1 - p - C + ps(S)$, reflecting the additional utility that P gets as a sovereign. The Nash bargaining solution to this game is (McCarty Meirowitz 2007, p. 277):

$$U_I = p_I = \frac{1 + \underline{U}_I - \underline{U}_P}{2}$$

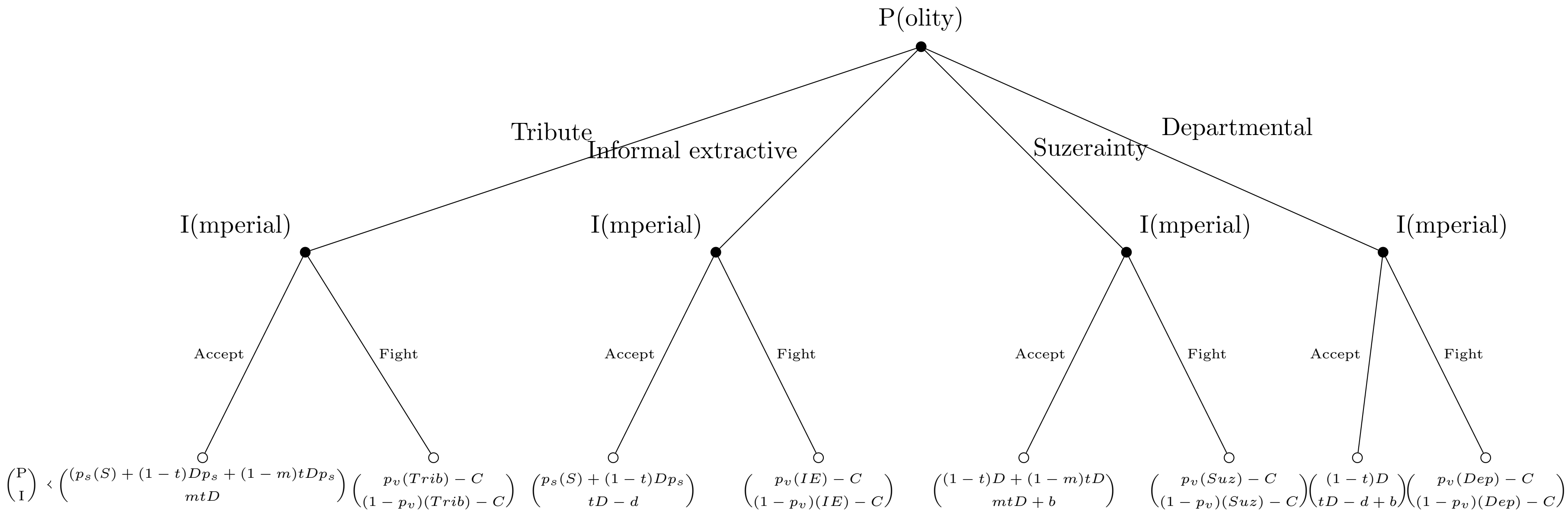
Substituting in and simplifying:

$$U_I = \frac{2p - p_s(S)}{2} \tag{7}$$

Therefore, if international competition reduces the value of p_s then the numerator will be larger and I will be able to set a higher p in equilibrium. Where this probability is larger, P's outside option improves and I must accept lower values of p. This only applies where S is greater than 0. Thus, it is only when sovereignty is valuable and when I has existing preferences for extraction that international competition can induce a change from transfers to extraction (Proposition 3).

3 Solutions if P moves first

3.1 Structure of the Game, P moves first



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$\binom{(1-t)D}{tD-d+b}$
$\binom{p_v(Dep)-C}{(1-p_v)(Dep)-C}$

We assumed in the main paper that I moves first, but show here that the main results from the model in relation to $H1 - H3$ do not change if we allow P to move first and I to respond. The results for $H4$ change slightly. In this scenario, P makes an offer to I and I either accepts that offer or rejects it. All other aspects of the game remain the same.

As with the original form of the game, P demands from I the best bargain that it can get without triggering I to go to war. P always prefers transfer-based bargains, so will never demand and extraction based bargain unless coerced by I. This is because in either transfer-based bargain it never has incentives to prefer an extraction-based bargain. Consider if P was under *Tributary.* For P to want to defect to *Informal Extractive* the following equation would need to hold:

$$p_s(S) + (1-t)Dp_s + (1-m)tDp_s > p_s(S) + (1-t)Dp_s \tag{8}$$

Equation 8 never holds so long as $m > 0$ (i.e shirking opportunities exist). If we imagine that there were incentives for P to prefer *Departmental* over *Tribute* then it would always prefer *Suzerainty* over *Departmental* because:

$$(1-t)D + (1-m)tD > (1-t)D$$

always holds as long as $m > 1$. So P always has incentives to prefer transfer-based bargains that preserve resources from shirking.

Therefore, P will never have incentives to shift from transfers to extraction, unless coerced to do so by I. The preferences P can realise are therefore determined by the value of I's reservation price (or the highest utility from among I's *fight* options). We know from the original set-up of the game that I always prefers bargains where P cedes it's sovereignty to I, so I's reservation price (i.e what it would prefer to fight for over accepting lower-valued bargains from P) is determined by the suzerainty and departmental options.[1]. Whether I's reservation price is determined by the value of *Suzerainty* or *Departmental* is not a

[1]This is because, if I prefers transfers then $mtD + b > mtD$ and if I prefers extraction then $tD - d + b > tD - d$

function of the international competition variables in our model. This is because:

$$U(Dep_{fighti}) > U(Suz_{fighti}) = (1 - p_v)(tD - d + b) > (1 - p_v)(mtD + b)$$

Which simplifies to equation 1:

$$tD - mtD > d$$

As such, I's reservation price is only determined by *Departmental* when equation 1 holds, which is a positive function of D and a negative function of d and is independent of b and p_s. Thus, extraction based bargains are only a possible equilibrium when I is prepared to fight for them, i.e when the fixed costs of rule are higher than the size of the transfer minus what is taken by shirking. Equilibrium bargains will therefore be transfer based bargains and international competition does not change I's preferences for transfers. This is consistent with H_1, H_2 and H_3.

If equation 1 holds, when is I willing to fight for extraction over accepting a transfer-based bargain from P without fighting? P will only offer an extraction based-bargain (*Departmental*, *Informal extractive*) to I when it knows that I won't also accept a transfer-based bargain that doesn't incur the costs of fighting for I. This can be assessed by comparing the utility of fighting for extraction $U(Dep_{fighti})$ with the utility of accepting a suzerain arrangement without fighting $U(Suz_i)$.[2] It is only when I can credibly threaten P that it will fight for extraction that P can be forced to accept its least preferred options (*Departmental*, *Informal extractive*). I can credibly threaten to fight for *Departmental* when equation 9 holds:

$$(1 - p_v)(tD - d + b) - C > mtD + b \tag{9}$$

or:

[2]If I would prefer to fight for *Departmental* rather than accept *Suzerainty* then it would also prefer to fight for *Departmental* over *Tributary* because $U(Suz_i) > U(Trib_i)$.

$$(1 - p_v)(tD - d + b) > mtD + b + C \tag{10}$$

If the above condition holds, then P can do no better than offer I an extraction-based bargain. All other options will trigger I to fight for *Departmental*. Equation 10 implies that I is willing to fight for *Departmental* if the value of *Departmental*, discounted by the likelihood of winning the conflict, is greater than the transfer it would get under *Suzerainty*, plus the security benefits and the costs of war.

Equation 10 also implies that increasing d (i.e the fixed costs of extraction) makes the left hand side of the equation smaller, making I more likely to accept *Suzerainty*, in line with H_3.

However, increasing b (the security benefits from controlling P's external sovereignty), increases the right hand side of the equation faster than it increases the left hand side, entailing that the value of fighting for *Departmental* is less likely to exceed the offer of *Suzerainty* when b is high. This is because increases in b increase the value of the right hand side (*Suzerainty*) by b while they increase the value of the left hand side by $(1-p_v)(b)$ and because $p_v[0,1]$ this is always smaller than b. Put in less technical terms, competition makes the imperial power more willing to accept a secure offer of controlling P's external sovereignty than engage in a risky war that it may lose and this effect is bigger when the imperial power is unlikely to win a war. Therefore, in a situation where I prefers extraction, international competition makes extraction-based bargains less likely because it makes I more willing to accept suzerainty deals in exchange for the security benefits of controlling P's external sovereignty. Increasing b has a larger effect when the chances of I winning a war is small (most likely on the edges of it's effective military power).

This is contrary to H_4 (competition in high density systems makes extraction more likely) where in this case international competition in high-density systems makes extraction-based bargains less likely. Thus H_4 depends on assumptions about whom moves first in this game. As a thought experiment, which of these effects is likely to be stronger if we ran the game multiple times, randomly assigning with equal probability who moves first? First, whether competition in high-density systems leads to more extraction or less also depends on assumptions about the relative impact of increases in international competition on p_s and b. Where competition makes P's very insecure but does not increase the value of b, more extraction will be the result. Where competition makes controlling P's external sovereignty very valuable but does not make P's insecure (i.e lower p_s) the less extraction is the result. It is also worth noting that increases in b make I prefer accepting suzerainty over fighting for departmental at a rate of $-p_v b$, assuming that I moves first and has preferences for extraction. If I moves first, increases in p_s improve P's reservation price at a rate of $p_s(S) + p_s D - p_s mtD$ (given that P reservation price is determined by U_{trib}, outlined in 2). Therefore international competition (in high-density systems) makes extraction less likely depending on how likely I is to win a war (if P moves first), while competition makes extraction based bargains more likely (if I moves first) for polities where sovereignty is valuable, the territory is valuable and where P can expect to retain a lot of resources through shirking (i.e m is small). We don't think it's possible to resolve this issue further here with this model, but it does suggest that the negative effect of competition on P's bargaining power is stronger in high density systems, while, once I has preferences for extraction, further increases in density don't affect the impact of b. This may point towards competition in higher-density systems favoring extraction on balance, but acknowledge that we are in very speculative territory here, and that assumptions about how competition affect b and p_s are likely much more important in resolving these issues than solving these equations further.

It's not central to our argument regarding the transition from transfers to extraction, but there are situations where *Informal extractive* can be an equilibrium solution when P moves first. If I will not accept a transfer-based bargain without fighting, then P can offer I *Informal extractive*, which preserves P's utility from external sovereignty. I will accept *Informal extractive* so long as 9 holds and $tD - d > (1 - p_v)(tD - d + b)$. This has the interesting implication that *Informal extractive* bargains may be most likely in high-density, low competition environments.

There is a final (albeit extreme) scenario whereby P would prefer to fight than offer I it's most favoured extraction based bargain *Departmental.* Assume that 10 holds and I would prefer to fight for extraction based bargains than transfer-based bargains. If $U_(Trib_{Fightp}) > U(Dep_{Acceptp})$ for example, then P will fight rather than offer I $Departmental$. This would obtain where:

$$p_v(p_s(S) + (1-t)Dp_s + (1-m)tDp_s - C > (1-t)D \tag{11}$$

The value of external sovereignty could be so high and P's so safe that they would never accept extraction from I. In this case, whether the bargain is $Departmental$ or $Tribute$ depends on the outcome of the conflict, determined by p_v. Under these conditions, international competition would make extraction-based bargains more likely by reducing the size of P's outside option and making it more likely that it compromises with I on extraction-based *Departmental* or *Informal extractive* based bargains. However, for this effect to be possible, the value of S must be high and I must already have preferences for extraction-based bargains. However, it's unlikely that both the conditions in equation 10 and equation 11 would hold because as p_v increases for P, making the condition in equation 11 more likely to hold it decreases the left hand side of equation 10, making this condition less likely to hold.

Overall, this suggests that I faces a trade-off during periods of international competition. On the one hand, competition reduces P's bargaining power by increasing the demand for protection, but on the other it strengthen P's bargaining hand by increasing the value of controlling P's external sovereignty to I. Readers should therefore be aware that H_4 in the paper depends on assumptions about whom moves first in the game.