Appendix

A1. Proof of the proposition

Proposition. *In any local equilibrium, a) the platform mean of any party is located between the competition weighted mean and the decisive party member; b) the platform variance of any party is bound from above by $|x_{m_j} - z_j|/\sqrt{3} + \omega$.*

Proof. Suppose $\mathbf{z}$ is an equilibrium platform profile and there is at least one platform $\hat{z}_j = (z_j, \sigma_j)$ so that $z_j > x_{m_j} > EM_j$. Then there exist some $\delta_1, \delta_2 \geq 0$ with at least one of them strictly positive, so that for the platform $\hat{z}'_j$ with $z'_j = z_j - \delta_1$ and $\sigma'_j = \sigma_j - \delta_2$, it holds that $v'^*_j(\hat{z}'_j) > v'^*_j(\hat{z}_j)$ and thus $p_{2j}(\hat{z}'_j) > p_{2j}(\hat{z}_j)$. As $z_j > z'_j > EM_j$ or $\sigma'_j < \sigma_j$, hence $p_{1j}(\hat{z}'_j) > p_{1j}(\hat{z}_j)$. For this reason, $\mathbf{z}$ cannot be an equilibrium strategy. The argument for $z_j < x_{m_j} < EM_j$ is analogous which establishes the first claim. Next, suppose $\mathbf{z}$ is an equilibrium platform profile where for at least one platform $\hat{z}_j = (z_j, \sigma_j)$ with $x_{m_j} > z_j > EM_j$, it holds that $\sqrt{3}\sigma_j > |x_{m_j} - z_j| + \sqrt{3}\omega$. Consider a platform $\hat{z}'_j$ so that $z'_j = z_j$ and $|x_{m_j} - z_j|/\sqrt{3} + \omega \leq \sigma'_j < \sigma_j$. Then the posteriors of both $\hat{z}_j$ and $\hat{z}'_j$ are uniformly distributed over $[x_{m_j} - \sqrt{3}\omega, x_{m_j} + \sqrt{3}\omega]$ and thus $u_{m_j}(\hat{z}'_j) = u_{m_j}(\hat{z}_j)$. As $p_{1j}(\hat{z}_j) > p_{1j}(\hat{z}'_j)$, $\mathbf{z}$ cannot be an equilibrium. The argument for $z_j < x_{m_j} < EM_j$ is analogous which establishes the second claim. $\qquad\square$

A2. Proof of the theorem

We prove a theorem that establishes conditions for the existence and uniqueness of local (pure) Nash equilibria in open neighborhoods U of the equilibrium strategies. The proof is an application of the Banach fix point theorem that states that any contraction mapping $T : X \to X$ on a complete metric space X has a unique fix point $x^* = T(x^*)$. We first have to consider an appropriate metric space X and a mapping T that maps X into X and translates the equilibrium into a fix point problem.

Consider two reasonable large but finite real numbers M_1 and M_2 such that $M_1 > \max_{i,j}\{|x_i|, |x_{m_j}|\}$ and $M_2 > \max_{i,j}\{|x_i| + \omega, |x_{m_j}| + \omega\}$. We define the two intervals $I_1 = [-M_1, M_1]$ and $I_2 = [0, M_2]$ and consider the strategy space $\mathbf{I} = I_1^P \times I_2^P$ where P is the number of parties. Next, we use equation (6) from the main text

$$z_j = \frac{3\sum(\rho_{ij} - \rho_{ij}^2)x_i + \alpha n(p_{2j} - p_{2j}^2)(x_{m_j} \mp \sqrt{3}\sigma_j \pm \frac{1}{2}\sqrt{3}\omega)}{3\sum(\rho_{ij} - \rho_{ij}^2) + \alpha n(p_{2j} - p_{2j}^2)}$$

$$\sigma_j = \frac{\alpha n(p_{2j} - p_{2j}^2)(\pm\frac{1}{\sqrt{3}}(x_{m_j} - z_j) + \frac{1}{2}\omega)}{\sum(\rho_{ij} - \rho_{ij}^2) + \alpha n(p_{2j} - p_{2j}^2)}.$$

to define two vector-valued functions g and h on $\mathbf{I}$ with coordinate functions

$$g_j(\mathbf{z}, \boldsymbol{\sigma}) := \frac{3\sum(\rho_{ij} - \rho_{ij}^2)x_i + \alpha n(p_{2j} - p_{2j}^2)(x_{m_j} \mp \sqrt{3}\sigma \pm \frac{1}{2}\sqrt{3}\omega)}{3\sum(\rho_{ij} - \rho_{ij}^2) + \alpha n(p_{2j} - p_{2j}^2)}$$

$$\text{(A1)}$$

$$h_j(\mathbf{z}, \boldsymbol{\sigma}) := \frac{\alpha n(p_{2j} - p_{2j}^2)(\pm\frac{1}{\sqrt{3}}(x_{m_j} - z_j) + \frac{1}{2}\omega)}{\sum(\rho_{ij} - \rho_{ij}^2) + \alpha n(p_{2j} - p_{2j}^2)}.$$

Again, we save notation and write $\pm$ for the two cases $EM_j \lessgtr z_j$. We can then state the theorem:

Theorem. (Existence and uniqueness of local Nash Equilibria) *Let $T : \mathbf{I} \longrightarrow \mathbf{I}$ defined by $T(s_1, ..., s_{2p}) = (g_1(\mathbf{z}, \sigma), ..., g_p(\mathbf{z}, \sigma), h_1(\mathbf{z}, \sigma), ..., h_p(\mathbf{z}, \sigma))$, and $\{U_1, U_2, ...\}$ be a collection of environments in $\mathbf{I}$. For any such U_i, if there exists a real number $\lambda < 1$ so that for the supremum norm $||.||$ of the Jacobian DT of T it holds that $||DT|| \leq \lambda$ in U_i, then there exists a fixed point s^* in U_i. At least one these fixed points is a unique local Nash equilibrium $s^* = (\mathbf{z}^*, \sigma^*)$ of party leader strategies in some U.*

Proof. From the definition of ρ_{ij} and p_{2j} we know that $0 < (\rho_{ij} - \rho_{ij}^2) < 1$ and $0 < (p_{2j} - p_{2j}^2) < 1$. From equation (A1) it is clear that z_j is a convex combination of all voter ideal points x_i and $x_{m_j} \mp \sqrt{3}\sigma \pm \frac{1}{2}\sqrt{3}\omega$. Likewise, σ_j is a convex combination of zero and $\pm\frac{1}{\sqrt{3}}(x_{m_j} - z_j) + \frac{1}{2}\omega$. Therefore T maps into $\mathbf{I}$.

The possibility that unique local Nash equilibria exist in some environments U is an immediate consequence of the application of the Banach fix point theorem (see also Merrill and Adams, 2001). Using the mean value theorem, we can infer that T is a λ-contraction in some U so that there is a unique fix point s^* of T in U. This is also an extrema of u_j. We still need to show that at least one of the critical points (there could be many, one for each U) is a maximum. To this end, we note that $\frac{\partial u_j}{\partial z_j} < 0$ if $z_j > M_1$, and $\frac{\partial u_j}{\partial z_j} > 0$ if $z_j < -M_1$. Moreover, $\frac{\partial u_j}{\partial \sigma_j} < 0$ if $\sigma_j > M_2$ and $\frac{\partial u_j}{\partial \sigma_j}(0+) \geq 0$. Thus, there is at least one $\mathbf{s}^*$ that is a local Nash equilibrium. $\square$

Supplementary materials

S1. Monte Carlo simulation

The theorem in the Appendix of the article relates the existence of local Nash equilibria z^* to the existence of some environment U of z^* where T is a contraction. Whether or not such an environment exists (or even many such environments) is difficult to establish in general terms. Fortunately, the Banach fix point theorem provides a constructive algorithm to find these fix points. Specifically, for any arbitrary starting point s_0 in U, the sequence s_n defined by $s_n = T(s_{n-1})$ converges to the fix point s^* of U. We here provide the R code of the simulation.

```
# MC simulation ambiguity model

rm(list = ls())
library(foreign)

# utility functions and voting probabilities
uij <- function(xi,zj,sigmaj) {
  if (length(xi) > 1 | length(zj) > 1) {
    n.x <- length(xi)
    n.z <- length(zj)
    x <- matrix(rep(xi,n.z),n.x,n.z,byrow=FALSE)
    z <- matrix(rep(zj,n.x),n.x,n.z,byrow=TRUE)
    sigma <- matrix(rep(sigmaj,n.x),n.x,n.z,byrow=TRUE)
    t <- -sigma^2 - (x-z)^2
  }
  else {
    t <- -sigmaj^2 - (xi-zj)^2
```

```r
  }
    t
}
rohij <- function(xi,zj,sigmaj,z,sigma) {
  exp(uij(xi,zj,sigmaj)) /  rowSums(exp(uij(xi,z,sigma)))
}
roh <- function(x,z,sigma) {
  exp(uij(x,z,sigma)) /  rowSums(exp(uij(x,z,sigma)))
}
u2j <- function(xmj,hetj,zj,sigmaj) {
  if (zj <= xmj) {
    lower <- xmj-sqrt(3)*hetj
    upper <- zj + sqrt(3)*sigmaj
    mu.post <- (upper+lower)/2
    sigma.post <- (upper-lower) / (2 * sqrt(3))
  }
  if (xmj < zj) {
    lower <- zj-sqrt(3)*sigmaj
    upper <- xmj + sqrt(3)*hetj
    mu.post <- (upper+lower)/2
    sigma.post <- (upper-lower) / (2 * sqrt(3))
  }
  if (lower > upper) {
    u <- NA
  }
  else {
    u <- -sigma.post^2 - (xmj-mu.post)^2
  }
  u
}
roh2j <- function(xmj,hetj,zj,sigmaj) {
  exp(u2j(xmj,hetj,zj,sigmaj)) / ( exp(u2j(xmj,hetj,zj,sigmaj)) + exp(uc) )
}
p2 <- function(z,sigma) {
  t <- rep(0,length(z))
  for (j in 1:length(z)) t[j] <- roh2j(xm[j],hetj[j],z[j],sigma[j])
  t
}
# contraction mapping T
g <- function(x,z,sigma,EM) {
  t <- (EM <= z)*2-1
  zstar <- (  6 * colSums( (roh(x,z,sigma)-roh(x,z,sigma)^2) * x) +
      alpha * length(x) * (p2(z,sigma)-p2(z,sigma)^2) *
      ( (-t) * 2* sqrt(3) * sigma + t * sqrt(3) * hetj + 2* xm ) ) /
      ( 2 * alpha * length(x) * (p2(z,sigma)-p2(z,sigma)^2) +
      6 * colSums(roh(x,z,sigma)-roh(x,z,sigma)^2) )
list("z"=zstar)
```

```
}
h <- function(x,z,sigma,EM) {
  sstar <- (    alpha * length(x) * (p2(z,sigma)-p2(z,sigma)^2) *
       ( 3 * hetj + 2 * sqrt(3) * abs(xm -z)) ) /
       ( 6 * (alpha * length(x) * (p2(z,sigma)-p2(z,sigma)^2) +
       colSums(roh(x,z,sigma)-roh(x,z,sigma)^2) ) ) )
list("sigma"=sstar)
}
# Nash equilibrium
ne <- function(x,start.z,start.sigma,precision) {
  i <- 1
  z <- start.z
  sigma <- start.sigma
  d.z <- 1000
  d.sigma <- 1000
  h.z <- 0
  h.sigma <- 0
  while ((d.z > precision | d.sigma > precision) & i < 500) {
    i <- i+1
    z_1 <- z
    sigma_1 <- sigma
    okay <- FALSE
    ii <- 0
    while (okay == FALSE & ii < 100) {
      ii <- ii + 1
      EM <- colSums( (roh(x,z_1,sigma_1)-roh(x,z_1,sigma_1)^2)*x) /
        colSums((roh(x,z_1,sigma_1)-roh(x,z_1,sigma_1)^2) )
      z <- g(x,z_1,sigma_1,EM)$z
      okay <- all(!is.na(z))
      if (okay==FALSE) {
        sigma_1[is.na(z)] <- sigma_1[is.na(z)]*1.1
      }
    }
    hh <- h(x,z_1,sigma_1,EM)
    sigma <- hh$sigma
    d.z <- sum(abs(z-z_1))
    d.sigma <- sum(abs(sigma-sigma_1))
    h.z <- c(h.z,d.z)
    h.sigma <- c(h.sigma,d.sigma)
  }
  list("z"=z,"sigma"=sigma,"it"=i,"start.z"=start.z,"start.sigma"=start.sigma,"EM"=EM)
}


# simulation with 100 runs and 10 starting values
sims <- 100
n.start <- 10
# fixed parameters
```

```r
uc <- -1
n <- 1000
alpha <- 1
prec.pow <- 5
prec <- 10^(-prec.pow)
# matrices to collect results
mc <- array(list(NULL), c(sims,10))
mc.xm <- mc.x <- array(list(NULL), c(sims,1))
# Monte Carlo simulation, sims runs
for (ii in 1:sims) {
  cat(".")
  x <- rnorm(n,0,1)
  p <- sum(rmultinom(1,size=1,prob=c(1,1,1)) * c(1,2,3))*2 + 1
  distinct.xm <- p-1
  while (distinct.xm < p) {
    xm <- round(rnorm(p,0,1),digits=3)
    distinct.xm <- length(table(xm))
  }
  xm <- sort(xm)
  hetj <- rep(0.2,p)
  z.ne <- NULL
  sigma.ne <- NULL
  # run for n.start different starting values
  for (i in 1:n.start) {
    start.z <- runif(p,0,1)*xm
    start.sigma <- (1/sqrt(3)*abs(start.z-xm)+1/2*hetj)*runif(p,0,1)
    t1 <- ne(x,start.z,start.sigma,prec)
    z.ne <- rbind(z.ne,round(t1$z, digits=prec.pow))
    sigma.ne <- rbind(sigma.ne,round(t1$sigma, digits=prec.pow))
    mc[[ii,i]] <- t1
  }
  mc.xm[[ii]] <- list("xm"=xm)
  mc.x[[ii]] <- list("x"=x)
  # plot simulation for different starting values
  par(mfrow=c(2,1))
  par(mar=c(2,1,2,1))
  par(oma=c(1,1,1,1))
  dotchart(z.ne, labels="",main="z",xlim=c(-3,3))
  dotchart(sigma.ne,labels="",ylab="sigma",main="sigma",xlim=c(0,3))
}
```

S2. List of parties

Sweden: CP Centre Party (1998), FP Liberal People's Party (1998), Green Ecology Party (1998), KdS Christian Democratic Community (1998), MSP Moderate Coalition Party (1998), SdaP Social Democratic Labour Party (1998), Vp Left Party (1998)

Denmark: CD Centre Democrats (1981, 1984, 1987, 1988, 1990, 1994, 1998), EL Red-Green Unity List (1994, 1998), FP Progress Party (1981, 1984, 1987, 1988, 1990, 1994, 1998), KF Conservative People's Party (1981, 1984, 1987, 1988, 1990, 1994, 1998), KrF Christian People's Party (1981, 1984, 1987, 1988, 1990, 1994, 1998), RV Radical Party (1981, 1984, 1987, 1988, 1990, 1994, 1998), SD Social Democratic Party (1981, 1984, 1987, 1988, 1990, 1994, 1998), SF Socialist People's Party (1981, 1984, 1987, 1988, 1990, 1994, 1998), V Liberals (1981, 1984, 1987, 1988, 1990, 1994, 1998)

Finland: KK National Coalition (1995, 1999), RKP/SFP Swedish People's Party (1995), SK Finnish Centre (1995, 1999), SKL Christian Union (1995), SSDP Social Democrats (1995), VL Green Union (1995), VL Left Wing Alliance (1995, 1999)

Netherlands: CDA Christian Democratic Appeal (1981, 1982, 1986, 1989, 1994, 1998), Communistische Partij Nederland (1981, 1982, 1986), D'66 Democrats 66 (1981, 1982, 1986, 1989, 1994, 1998), Gereformeerd Politiek Verbond (1986, 1989, 1994, 1998), GL Green Left (1989, 1994, 1998), Pacifistisch Socialistische Partij (1981, 1986), PPR Radical Political Party (1981, 1982, 1986), PvdA Labour Party (1981, 1982, 1986, 1989, 1994, 1998), ReformatorischePolitieke Federatie (1981, 1986, 1989, 1994, 1998), SP Socialist Party (1994, 1998), Staatkundig Gereformeerde Partij (1981, 1982, 1986, 1989, 1994, 1998), VVD People's Party for Freedom and Democracy (1981, 1982, 1986, 1989, 1994, 1998)

Luxembourg: PCL/KPL Communist Party (1984, 1989), PCS/CSV Christian Social People's Party (1984, 1989), PD/DP Democratic Party (1984, 1989), POSL/LSAP Socialist Workers' Party (1984, 1989)

France: FN National Front (1986, 1993, 1997), Greens (1993, 1997), PCF Communist Party (1981, 1986, 1993, 1997), PS Socialist Party (1981, 1986, 1993, 1997), RPR/UDF (1981, 1986, 1993)

Italy: AN National Alliance (1994, 1996), CCD Christian Democratic Centre (1996), DC Christan Democrats (1983, 1987, 1992), FdV Green Federation (1992, 1994, 1996), FI Go Italy (1994, 1996), LN Northern League (1994, 1996), PCI Communist Party (1983, 1987), PDS Democratic Party of the Left (1992, 1994), PLI Liberal Party (1983, 1987, 1992), PPI Popular Party (1994, 1996), PRI Republican Party (1983, 1987, 1992), PSDI Democratic Socialist Party (1983, 1987, 1992), PSI Socialist Party (1983, 1987, 1992), RC Newly Founded Communists (1992, 1994, 1996)

Spain: CDS Centre Democrats (1989, 1993), IU United Left (1989, 1993, 1996, 2000), PP Popular Party (1989, 1993, 1996, 2000), PSOE Socialist Workers' Party (1989, 1993, 1996, 2000) Portugal: PCP Communist Party (1985, 1999), PSD Social Democratic Party (1985, 1987, 1999), PSP Socialist Party (1985, 1987, 1999), UDP Popular Democratic Union (1985, 1987)

Germany: CDU/CSU Christian Democratic Union/Social Union (1980, 1983, 1987, 1990, 1994, 1998), FDP Free Democratic Party (1980, 1983, 1987, 1990, 1994, 1998), Greens (1980, 1983, 1987, 1990, 1994, 1998), PDS Party of Democratic Socialism (1990, 1994, 1998), SPD Social Democratic Party (1980, 1983, 1987, 1990, 1994, 1998)

Austria: Freedom Movement (1995, 1999), Green Party (1995, 1999), LF Liberal Forum (1995, 1999), ÖVP People's Party (1995, 1999), SPÖ Social Democratic Party (1995, 1999)

Great Britain: Conservative Party (1983, 1987), Labour Party (1983, 1987), Liberal Party (1983, 1987)

Ireland: DLP Democratic Left Party (1992, 1997), Fianna Fail (1982, 1987, 1989, 1992, 1997), Fine Gael (1982, 1987, 1989, 1992, 1997), Greens Ecology Party/Green Party (1987, 1989, 1997), LP Labour Party (1982, 1987, 1989, 1992, 1997), PD Progressive Democrats (1987, 1989, 1992, 1997), Sinn Fein Ourselves III (1997), WP Workers' Party (1987, 1989)

Belgium (Walloon): ECOLO Francophone Ecologists (1985, 1987, 1991, 1999), PRL Francophone Liberals (1985, 1987, 1991), PSC Christian Social Party (1985, 1987, 1991, 1999), PS Francophone Socialist Party (1985, 1987, 1991, 1999)

Belgium (Flanders): AGALEV Live Differently (1991, 1995, 1999), CVP Christian People's Party (1981, 1985, 1991, 1995, 1999), Flemish Christian People's Union (1981, 1985, 1999), PVV Party of Liberty and Progress (1981, 1985, 1987), SP Flemish Socialist Party (1981, 1985, 1987, 1991, 1995, 1999), VB Flemish Bloc (1991, 1995, 1999), VLD Flemish Liberals and Democrats (1999), VU People's Union (1987, 1991, 1995)

References

Merrill, Samuel III, and James Adams. 2001. "Computing Nash Equilibria in Probabilistic, Multiparty Spatial Models with Nonpolicy Components." *Political Analysis* 9(4): 347–361.