# The Perverse Consequences of Policy Restrictions in the Presence of Asymmetric Information

## SUPPLEMENTARY MATERIAL - ONLINE APPENDIX

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# 1. Appendix A: proof of Proposition 1

The optimal choice of leisure for an informed voter with productivity $w$ given the equilibrium size of government $(T_i, t_i)$ is:

$$l_i = \arg\max_{l\in[0,1]} U_w\left(l, t_i, T_i\right) = \alpha \ln\left(\left(1-t_i\right) w \left(1-l\right) + T_i\right) + \left(1-\alpha\right)\ln l_i.$$

Assuming that $l_i$ is an interior solution, it can be uniquely characterized by the first order condition (second order conditions ensure that this is indeed a maximum):

$$0 = \frac{-\alpha\left(1-t_i\right)w}{\left(1-t_i\right)w\left(1-l_i\right)+T_i} + \frac{1-\alpha}{l_i} \Leftrightarrow \frac{\left(1-t_i\right)wl_i}{\left(1-t_i\right)w\left(1-l_i\right)+T_i} = \frac{1-\alpha}{\alpha}.$$

The optimal choice of an uninformed individual that rationally anticipates the equilibrium size of government is:

$$\begin{aligned} l_u &= \arg\max_{l\in[0,1]} p\left(\alpha\ln\left(\left(1-t_1\right)w\left(1-l\right)+T_1\right)+\left(1-\alpha\right)\ln l\right)+ \\ &\quad +\left(1-p\right)\left(\alpha\ln\left(\left(1-t_2\right)w\left(1-l\right)+T_2\right)+\left(1-\alpha\right)\ln l\right) \end{aligned}$$

Assuming that $l_u$ is an interior solution, it can be uniquely characterized by the first order condition:

$$\begin{aligned} 0 &= p\left(\frac{-\alpha\left(1-t_1\right)w}{\left(1-t_1\right)w\left(1-l_u\right)+T_1}+\frac{1-\alpha}{l_u}\right)+\left(1-p\right)\left(\frac{-\alpha\left(1-t_2\right)w}{\left(1-t_2\right)w\left(1-l_u\right)+T_2}+\frac{1-\alpha}{l_u}\right) \Leftrightarrow \\ \Leftrightarrow &\quad \frac{p\left(1-t_1\right)wl_u}{\left(1-t_1\right)w\left(1-l_u\right)+T_1}+\frac{\left(1-p\right)\left(1-t_2\right)wl_u}{\left(1-t_2\right)w\left(1-l_u\right)+T_2} = \frac{1-\alpha}{\alpha}. \end{aligned}$$

Using the first order condition for the informed citizens we can rewrite the first order condition of the uninformed as follows:

$$p\frac{l_u-l_1}{l_1-\left(1-\alpha\right)l_u}+\left(1-p\right)\frac{l_u-l_2}{l_2-\left(1-\alpha\right)l_u}=0.$$

We need to show that $\frac{\partial\Delta(w)}{\partial w} > 0$. Whenever our solutions are interior we can apply the envelope theorem (i.e. we can disregard the effect of a change of $w$ in the optimal solutions $l_1, l_2$, and $l_u$). It follows that:

$$\begin{aligned} \frac{\partial\Delta\left(w\right)}{\partial w} &= p\alpha\left(\frac{\left(1-t_1\right)\left(1-l_1\right)}{\left(1-t_1\right)w\left(1-l_1\right)+T_1}-\frac{\left(1-t_1\right)\left(1-l_u\right)}{\left(1-t_1\right)w\left(1-l_u\right)+T_1}\right)+ \\ &\quad +\left(1-p\right)\alpha\left(\frac{\left(1-t_2\right)\left(1-l_2\right)}{\left(1-t_2\right)w\left(1-l_2\right)+T_2}-\frac{\left(1-t_2\right)\left(1-l_u\right)}{\left(1-t_2\right)w\left(1-l_u\right)+T_2}\right) \end{aligned}$$

Using the first order conditions for the informed and uninformed citizens we can rewrite the expression above as follows:

$$\frac{1-\alpha}{wl_u}\left(p\frac{l_u-l_1}{l_1}+\left(1-p\right)\frac{l_u-l_2}{l_2}\right) \tag{1}$$

We define $\hat{l}$ as the value of $l_u$ for which expression (1) is equal to 0. This value is uniquely determined because the expression is increasing in $l_u$ and when evaluated at $l_u = l_1$ or at $l_u = l_2$ expression (1) takes different signs. We now know that for any $l > \hat{l}$, expression (1) is greater than 0.

Finally, we want to evaluate the first order condition of the uninformed citizen at $\hat{l}$ (instead of $l_u$). Our proof concludes by realizing that this evaluation (given the definition of $\hat{l}$) is always greater than 0: given that the first order condition defines a maximum, we can conclude that the value that satisfies the first order condition is larger than $\hat{l}$. In other words, $l_u > \hat{l}$ and expression (1) is greater than 0.

## 2. Appendix B: proof of Proposition 2

The simplest environment in which policy restrictions have perverse consequences for poor voters requires three groups of voters with low, medium and high productivity. When we introduce a policy restriction, poorer voters will no longer have incentives to acquire information so the median voter will be richer. A necessary condition for the introduction of binding pro-poor restrictions is that equilibrium policies are not optimal from the perspective of these voters. In other words, the median voter in the absence of restrictions cannot be a low productivity voter. In what follows, we characterize a situation in which everyone votes in the absence of restrictions and the median voter has medium productivity. When we impose a binding restriction in one state of the world, poorer voters no longer acquire information and the new median voter is one with high productivity.

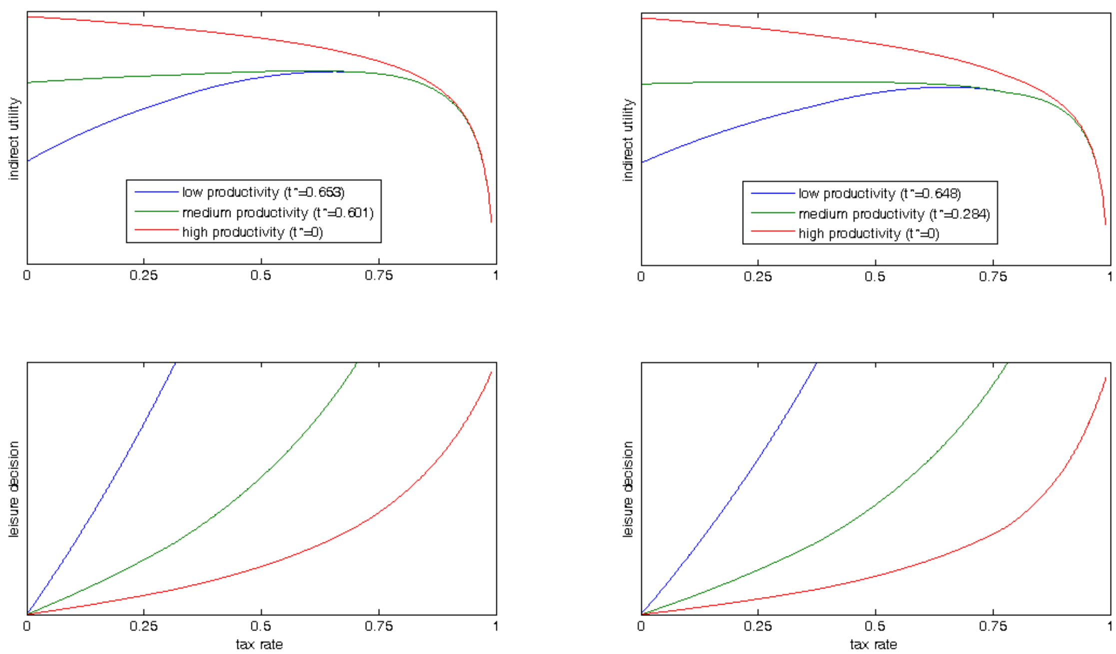
A simple way to achieve these previous conditions is to assume that low productivity voters are a 45% of the population, medium productivity voters are a $\lambda$% of the population and high productivity voters are $(100 - 45 - \lambda)$%, where $\lambda$ is the source of uncertainty in citizens' preferences.

The optimal leisure decision by a voter with productivity $w$ when the government policy is $(t, T)$ is $l(w) = \min\left\{1, (1-\alpha)\left(1 + \frac{T}{(1-t)w}\right)\right\}$. Note that we are ensuring that we select the optimal leisure decision when the solution is not interior. Having three types of citizens simplifies the computation for the level of redistribution given any tax rate $t$ if all leisure decisions are interior. In that case redistribution is equal to: $T = t \cdot (1-t)\frac{\alpha\overline{w}}{1-\alpha t}$, where $\overline{w}$ is the average productivity in society. We can then obtain a closed form solution for the indirect utility function of a citizen with marginal productivity $w$:

$$U_w(t) = \alpha \cdot \ln\left((1-t)\alpha\left(w + \frac{\overline{w}\alpha t}{1-\alpha t}\right)\right) + (1-\alpha) \cdot \ln\left((1-\alpha)\left(1 + \frac{\overline{w}\alpha t}{w(1-\alpha t)}\right)\right).$$

The previous expressions are usually wrong as our model generally displays less productive voters subsisting on welfare payments (i.e. $l = 1$). This complicates the analysis and rules out closed form solutions. Accordingly we show the possibility of perverse consequences of policy restrictions with a numerical simulation.[1] The figures below show the optimal leisure decisions and indirect utility functions of our three types of citizens when the citizens' productivities can take values in $\{1, 3.5, 10\}$, $\alpha = 0.5$, and there is uncertainty about the size of the 'middle class' as $\lambda$ can take values 10 or 25.

[1]The Matlab code is available from the authors upon request.




(a) $\lambda = 10$ (b) $\lambda = 25$

FIGURE 1. Leisure decisions and indirect utilities in two states of the world

When $\lambda = 10$ there are very few mid productivity citizens yet there are a large proportion of citizens that are high productivity (i.e. society is richer) than when $\lambda = 25$. A richer society implies that, at any given tax rate, citizens decide to work less hours than when $\lambda$ is larger. The upper figures display the indirect utilities when considering the optimal leisure decisions at each level of income tax -we also denote the preferred tax rate, $t^*$, of each type of citizen. Below we depict the optimal leisure decisions of each type of citizen. Overall we see that richer individuals always devote more time towards work and obtain higher utility levels. The most productive citizens are always net contributors and prefer to set taxes at the lowest possible level. We set the costs to acquire information so that everyone votes when there is no restriction, yet poorer voters abstain when we introduce the restriction. This implies that in the absence of restriction the median voter has medium productivity: these citizens like low taxes when the fraction of rich individuals is small ($t^* = 284$ when $\lambda = 25$) and a level of tax close to that desired by low productivity citizens when the fraction of high productivity citizens is large ($t^* = 601$ when $\lambda = 10$). Table 1 reports optimal leisure decisions when informed and uninformed for each type of citizen. It also reports the value of acquiring information when the posterior belief on $\lambda = 10$ is $p = 0.5$. We observe that the value of information is increasing in income and indeed all citizens have incentives to vote when the costs of acquiring information are smaller than 0.015.

| | $l_{\lambda=10}$ | $l_{\lambda=25}$ | $l_u$ | $\Delta(w)$ |
|---|---|---|---|---|
| *high productivity* | .629 | .536 | .575 | .032 |
| *medium productivity* | .869 | .602 | .690 | .016 |
| *low productivity* | 1 | .858 | .918 | .015 |

TABLE 1. Leisure decisions and value of information when $t^*_{\lambda=10} = .601$ and $t^*_{\lambda=25} = .284$

Given that the level of taxation (and redistribution) is very small when society is poorer, we are interested in analyzing the role of introducing a restriction that increases the level of taxation when society is poor: we impose that taxes cannot be smaller than 0.38. At this level of taxation, poorer voters subsist on government transfers and so cannot derive any private benefit from knowing the distribution of wages in society. These voters surely abstain and the median voter now has high-productivity (thus desires minimal taxes). In Table 2 we once again report optimal leisure decisions and value of information, this time in the presence of a policy restriction ($t \geq t_R$).

| | $l_{\lambda=10}$ | $l_{\lambda=25}$ | $l_u$ | $\Delta(w)$ |
|---|---|---|---|---|
| *high productivity* | .564 | .551 | .557 | .00066 |
| *medium productivity* | .681 | .645 | .662 | .00038 |
| *low productivity* | 1 | 1 | 1 | 0 |

TABLE 2. Leisure decisions and value of information when $t^*_{\lambda=10} = t^*_{\lambda=25} = t_R = 0.38$

Looking at the table above we can conclude that as long as the costs of acquiring information are greater than 0 and smaller than 0.00038, we have that in the absence of restrictions all voters acquire information and turnout to vote. However, when we introduce a restriction (tax needs to be at least 38%) poorer voters no longer vote and the median voter is a high productivity voter instead of a medium productivity voter. We can now compare the utility the voter receives when informed with no restrictions with the utility she receives when uninformed and with restrictions: poorer voters see a 15% reduction of utility when policy restrictions are introduced.

## 3. Appendix C: Simulation algorithm

The matlab code for our simulations is available from the authors upon request . Below we describe our algorithm leaving out programming technicalities.

3.1. **Simulation part 1 - Equilibrium without restrictions.** The two states of the world are denoted by $i = 1, 2$. $f_i$ is a lognormal distribution with parameters $\mu_i$ and $\sigma_i$ thus the average productivity in society is $\bar{w}_i = exp\left(\mu_i + \frac{\sigma_i^2}{2}\right)$. We compute the level of redistribution $T_i(t)$ for any level of tax using the Laffer curve $T_i = t(1-t)(At+B)\bar{w}_i$.

In the absence of restrictions we assume that there is a proportion $abst_1$ of citizens abstaining when the wage distribution is $f_1$ (this allows us to determine the costs of acquiring information that makes the marginal citizen indifferent between acquiring information and voting or not acquiring information and abstaining). The wage of the marginal citizen satisfies: $w_{marg} = F_1^{-1}(abst_1)$ where $F_i^{-1}$ is the inverse lognormal cdf of $f_i$. This implies that the proportion of citizens abstaining when the wage distribution is $f_2$ is given by $abst_2 = F_2(w_{marg})$, where $F_i$ is the lognormal cdf of $f_i$. The wage of the median voter in each state of the world is $w_i^m = F_i^{-1}\left(\frac{x_i+1}{2}\right)$. We can now compute the medians' optimal leisure decisions $l_i(t)$ given any tax rate. This in turn allows us to compute the medians' preferred tax rates $t_i, i = 1, 2$.

Knowing the optimal tax rates under both distributions of wages allows us to compute the optimal leisure decisions of the marginal citizen when informed ($l_i^{marg}, i = 1, 2$) and when uninformed ($l_u$). This allows us to know the value of information for the marginal voter, $\Delta(w_{marg})$. Recall that we started the algorithm assuming a level of abstention under the wage distribution $f_1$. In order to close the model, we need to set the costs of acquiring information equal to the marginal voter's value of information. Note that the following parameters are needed to run this first part of the simulation: $\mu_1, \sigma_1, \mu_2, \sigma_2, p, A, B, \alpha$ and $abst_1$.

3.2. **Simulation part 2 - Introduce a policy restriction.** In order to make all iterations of our simulation meaningful we assume that the policy restriction lies in between the level of tax implemented in the two states of the world when there is no restriction. More precisely we assume that the strength of the restriction is captured by $y$ and $t_R = yt_1 + (1-y)t_2$ whenever $t_1 < t_2$. This restriction implies that the new level of taxes in the first state of the world is $t_1^R = t_R$ which in turn implies a higher level of abstention. In order to characterise the new equilibrium, we iteratively increase the wage of the marginal citizen by $\epsilon > 0$ until the new marginal citizen prefers voting (that is, until her value of information is greater or equal than the value of information of the marginal citizen in the absence of restrictions).

This iterative process entails computing (1) the wage of the new median voter when the wage distribution is $f_2$, (2) the new median's optimal leisure decisions and (3) the equilibrium tax rate. We then compute the optimal leisure decisions of the new marginal citizen when informed and uninformed and her value of information. We finish this iterative process when the value of information is above the costs of getting informed or when the restriction binds the tax rate in both states of the world (note that whenever the support of the citizens' productivities is unbound, there is always a high enough wage rate to ensure the private value of information is above the costs of acquiring information for some citizens; these (very rich) citizens indeed vote for the lowest possible tax rate). We finally ask whether citizens who subsist on government transfers are better off by the introduction of the restriction.

Only two extra parameters were needed to run this second part of our simulation: $y$ and $\epsilon$.