Popular Referendum and Electoral Accountability
Appendix

This appendix contains three sections. Appendix A provides characterizations (and the proofs for these characterizations) of the equilibria of the models described in the paper. More precisely, Appendix A provides a proof of Proposition A.1 which characterizes the equilibrium of the baseline model and a proof of Proposition A.2 which characterizes the equilibria of the model with the popular referendum. The characterizations are given for all values of holding office B, i.e. for the case of $B < 2$ discussed in the main body of the paper but also for the case of $B \geq 2$. Proposition 3 and corollary 4 in the paper follow immediately from Proposition A.2, while Propositions 5 and 6 follow immediately from a comparison of Propositions A.1 and A.2. Their proofs are therefore omitted. Finally, I prove Proposition A.3 which characterizes the equilibrium of the direct democracy game when $q_1 = 1$ and $q_2 \in [0, 1]$. Appendix B shows that the introduction of the popular referendum also improves congruence when the benefit of holding office B is greater than 2. Appendix C shows that the results derived in the main paper are robust to the introduction of semi-congruent types.

Appendix A: Equilibrium

Let T be the type space of the Incumbent with $T \equiv \{C_{1,\cdot}, C_{-1,\cdot}, N_{1,\cdot}, N_{-1,\cdot}\}$. $C_{1,\cdot}$ denotes the type of the Incumbent which is congruent and observed $\omega_1 = 1$ and so forth. In principle we would also have to distinguish types depending on what value of ω_2 they observed. However, as all the actors in the game observe ω_2 and the game is identical for each value of ω_2, I suppress this information. Also let P be the set of policies that can be chosen by the Incumbent for any pair of states of the world (ω_1, ω_2) with $P \equiv \{(p_1 = \omega_1, p_2 = \omega_2), (p_1 = \omega_1, p_2 \neq \omega_2), (p_1 \neq \omega_1, p_2 = \omega_2), (p_1 \neq \omega_1, p_2 \neq \omega_2)\}$. Let $\mathbf{p}, \mathbf{p}'$ be arbitrary elements in P. Moreover, denote $\eta_{\cdot,\cdot}(\mathbf{p})$ the probability that the non-congruent Incumbent which observes $(\cdot, \cdot)$ plays the policy vector $\mathbf{p} \in P$. Finally, let $\mu(\mathbf{p})$ denote the Voter's posterior belief that the Incumbent is congruent.

Note that, unless explicitly specified, we only consider strategy profiles in which the congruent Incumbent chooses $(p_1 = \omega_1, p_2 = \omega_2)$.

Baseline Model

Proposition A.1. *The following pair of strategies and beliefs constitute the unique perfect Bayesian equilibrium of the baseline model that satisfies criterion D1:*

1. *If $B < 2$ and $q_1 \in \left[\frac{B-1}{B}, 1\right]$, then congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$ for all $\boldsymbol{\omega}$, while non-congruent Incumbents choose $(p_1 \neq \omega_1, p_2 \neq \omega_2)$ for all $\boldsymbol{\omega}$. The Voter reelects with certainty upon observing $(p_1 = \omega_1, p_2 = \omega_2,)$, $(p_1 = 1, p_2 = \omega_2)$, or $(p_1 = -1, p_2 = \omega_2)$ and does not reelect otherwise. The Voter's beliefs satisfy $\mu(p_1 = \omega_1, p_2 = \omega_2) = \mu(p_1 = 1, p_2 = \omega_2) = \mu(p_1 = -1, p_2 = \omega_2) = 1$ and $\mu(p_1 \neq \omega_1, p_2 \neq \omega_2) = \mu(p_1 = 1, p_2 \neq \omega_2) = \mu(p_1 = -1, p_2 \neq \omega_2) = 0$. Out-of-equilibrium beliefs satisfy $\mu(p_1 = \omega_1, p_2 \neq \omega_2), \mu(p_1 \neq \omega_1, p_2 = \omega_2) \leq \pi$.*

2. *If $B < 2$ and $q_1 \in \left[0, \frac{B-1}{B}\right]$, then congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$ for all $\boldsymbol{\omega}$,*

   the non-congruent Incumbent who observes $\omega_1 = -1$ chooses $(p_1 = 1, p_2 = \omega_2)$,

   the non-congruent Incumbent who observes $\omega_1 = 1$ chooses $(p_1 = -1, p_2 = \omega_2)$ with *probability* $\frac{1}{\alpha} - 1$ *and* $(p_1 = -1, p_2 \neq \omega_2)$ *with probability* $2 - \frac{1}{\alpha}$,

the Voter reelects the Incumbent with certainty upon observing $(p_1 = \omega_1, p_2 = \omega_2)$ *or* $(p_1 = 1, p_2 = \omega_2)$, *with probability* $\frac{1}{(1-q_1)B}$ *upon observing* $(p_1 = -1, p_2 = \omega_2)$, *and does not reelect otherwise.*

The Voter's beliefs satisfy $\mu(p_1 = \omega_1, p_2 = \omega_2) = 1$, $\mu(p_1 = 1, p_2 = \omega_2) = \frac{\alpha\pi}{\alpha\pi + (1-\alpha)(1-\pi)} > \pi$, $\mu(p_1 = -1, p_2 = \omega_2) = \frac{(1-\alpha)\pi}{(1-\alpha)\pi + (1-\alpha)(1-\pi)} = \pi$, $\mu(p_1 \neq \omega_1, p_2 = \omega_2) = \mu(p_1 = -1, p_2 \neq \omega_2) = \mu(p_1 \neq \omega_1 = 1, p_2 \neq \omega_2) = 0$. *Out-of-equilibrium beliefs satisfy* $\mu(p_1 \neq \omega_1 = -1, p_2 \neq \omega_2), \mu(p_1 = \omega_1, p_2 \neq \omega_2), \mu(p_1 = 1, p_2 \neq \omega_2) \leq \pi$.

3. *If* $B \geq 2$ *and* $q_1 \in \left[\frac{1}{B}, 1\right]$, *then congruent and non-congruent Incumbents choose* $(p_1 = \omega_1, p_2 = \omega_2)$ *for all* $\boldsymbol{\omega}$. *The Voter may reelect with positive probability upon observing* $(p_1 = \omega_1, p_2 = \omega_2)$, $(p_1 = 1, p_2 = \omega_2)$ *or* $(p_1 = -1, p_2 = \omega_2)$ *and does not reelect otherwise. The Voter's beliefs satisfy* $\mu(p_1 = \omega_1, p_2 = \omega_2) = \mu(p_1 = 1, p_2 = \omega_2) = \mu(p_1 = -1, p_2 = \omega_2) = \pi$. *Out-of-equilibrium beliefs satisfy* $\mu(p_1 \neq \omega_1, p_2 = \omega_2)$, $\mu(p_1 = \cdot, p_2 \neq \omega_2) \leq \pi$.

4. *If* $B \geq 2$ *and* $q_1 \in \left[0, \frac{1}{B}\right]$, *then congruent Incumbents choose* $(p_1 = \omega_1, p_2 = \omega_2)$ *for all* $\boldsymbol{\omega}$;

   the non-congruent Incumbent who observes $\omega_1 = -1$ *chooses* $(p_1 = 1, p_2 = \omega_2)$;

   the non-congruent Incumbent who observes $\omega_1 = 1$ *chooses* $(p_1 = 1, p_2 = \omega_2)$ *with probability* $2 - \frac{1}{\alpha}$ *and* $(p_1 = -1, p_2 = \omega_2)$ *with probability* $\frac{1}{\alpha} - 1$;

   the Voter's reelection strategy is: $r^*(p_1 = \omega_1, p_2 = \omega_2) = 1$, $r^*(p_1 = 1, p_2 = \omega_2) = \frac{1}{(1-q_1)B} - \frac{q_1}{1-q_1} + r^*(p_1 = -1, p_2 = \omega_2)$ *with* $r^*(p_1 = -1, p_2 = \omega_2) \geq \frac{1}{(1-q_1)B}$, *and* $r^*(p_1 \neq \omega_1, p_2 = \omega_2) = r^*(p_1 = \cdot, p_2 \neq \omega_2) = 0$.

   The Voter's beliefs satisfy $\mu(p_1 = 1, p_2 = \omega_2) = \mu(p_1 = -1, p_2 = \omega_2) = \pi$, $\mu(p_1 = \omega_1 = 1, p_2 = \omega_2) > \pi$, $\mu(p_1 = \omega_1 = -1, p_2 = \omega_2) = 1$, $\mu(p_1 \neq \omega_1, p_2 = \omega_2) = 0$. *Out-of-equilibrium beliefs satisfy* $\mu(p_1 = \cdot, p_2 \neq \omega_2) \leq \pi$.

The proof proceeds in four steps. In Lemma 1, I show that the pair of strategies and beliefs identified in Proposition A.1 is indeed a perfect Bayesian equilibrium. In Lemma 2, I then show that this equilibrium satisfies a straightforward adaptation of criterion $D1$ (Cho and Kreps, 1987). Lemmata 3 and 4 then show that it is the unique equilibrium that satisfies $D1$.

Lemma 1. *The pairs of strategies and beliefs identified in proposition A.1 constitute a perfect Bayesian equilibrium.*

Proof.

1. Assume $B < 2$ and $q_1 \geq \frac{B-1}{B}$. Given the specified strategy for the Incumbent, Bayesian updating yields $\mu(p_1 = \omega_1, p_2 = \omega_2) = \mu(p_1 = 1, p_2 = \omega_2) = \mu(p_1 = -1, p_2 = \omega_2) = 1 > \pi$, $\mu(p_1 \neq \omega_1, p_2 \neq \omega_2) = \mu(p_1 = 1, p_2 \neq \omega_2) = \mu(p_1 = -1, p_2 \neq \omega_2) = 0$. Hence, if we let out-of-equilibrium beliefs satisfy $\mu(p_1 = \omega_1, p_2 \neq \omega_2), \mu(p_1 \neq \omega_1, p_2 = \omega_2) \leq \pi$ the Voter has no incentive to deviate from his reelection strategy. Given the retention behavior of the Voter, the congruent Incumbent always gets reelected if he chooses $(p_1 = \omega_1, p_2 = \omega_2)$. Hence, $U_C(p_1 = \omega_1, p_2 = \omega_2) = 2 + B$ which is the highest possible payoff that the congruent Incumbent can receive in the game and thus the congruent Incumbent has no incentive to deviate. Moreover, $U_N(p_1 \neq \omega_1, p_2 \neq \omega_2) = 2$, while $U_N(p_1 = \omega_1, p_2 \neq \omega_2) = 1 < 2$, $U_N(p_1 \neq \omega_1, p_2 = \omega_2) = 1 + (1 - q_1)B \leq 2$ because $q_1 \geq \frac{B-1}{B}$, and $U_N(p_1 = \omega_1, p_2 = \omega_2) = B < 2$. Hence, the non-congruent Incumbent has no incentive to deviate.

2. Assume $B < 2$ and $q_1 < \frac{B-1}{B}$. Given the specified strategy for the Incumbent, Bayesian updating yields $\mu(p_1 = \omega_1, p_2 = \omega_2) = 1$, $\mu(p_1 = 1, p_2 = \omega_2) = \frac{\alpha\pi}{\alpha\pi + (1-\alpha)(1-\pi)} > \pi$ because $\alpha > 1/2$, $\mu(p_1 = -1, p_2 = \omega_2) = \frac{(1-\alpha)\pi}{(1-\alpha)\pi + \alpha(\frac{1}{\alpha} - 1)(1-\pi)} = \pi$ and $\mu(p_1 \neq \omega_1, p_2 = \omega_2) = \mu(p_1 = -1, p_2 \neq \omega_2) = \mu(p_1 \neq \omega_1 = -1, p_2 \neq \omega_2) = 0$. Hence, if we let out-of-equilibrium beliefs satisfy $\mu(p_1 \neq \omega_1 = -1, p_2 \neq \omega_2), \mu(p_1 = \omega_1, p_2 \neq \omega_2), \mu(p_1 = 1, p_2 \neq \omega_2) \leq \pi$ the Voter has no incentive to deviate from his reelection strategy. Given the reelection behavior of the Voter, $C_{1,\cdot}$ receives a payoff of $2 + B$ when choosing

$(p_1 = \omega_1, p_2 = \omega_2)$ and thus does not have an incentive to deviate. $C_{-1,\cdot}$ receives a payoff of $3 + q_1 B$ when choosing $(p_1 = \omega_1, p_2 = \omega_2)$. Deviating to $(p_1 = 1, p_2 = \omega_2)$ yields $1 + (1 - q_1)B < 3$, while deviating to $p_2 \neq \omega_2$ yields at most 1. Hence, $C_{-1,\cdot}$ has no incentive to deviate. Moreover, $U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) = 1 + (1 - q_1)B$, while $U_{N_{-1,\cdot}}(p_1 = \cdot, p_2 \neq \omega_2) \leq 2 < 1 + (1 - q_1)B$ because $q_1 < \frac{B-1}{B}$, and $U_{N_{-1,\cdot}}(p_1 = -1, p_2 = \omega_2) = q_1 B + 1$ which is lower than 2 given that $q_1 < \frac{B-1}{B}$. Hence, $N_{-1,\cdot}$ has no incentive to deviate. $N_{1,\cdot}$ receives a payoff of 2 from choosing $(p_1 = -1, p_2 = \omega_2)$ as well as from choosing $(p_1 = -1, p_2 \neq \omega_2)$. Deviating to $(p_1 = 1, p_2 = \omega_2)$ yields $B < 2$, while deviating to $(p_1 = 1, p_2 \neq \omega_2)$ yields $1 < 2$. Hence, $N_{1,\cdot}$ has no incentive to deviate.

3. Assume $B \geq 2$ and $q_1 \in \left[\frac{1}{B}, 1\right]$ and that the choice of policy vectors is as specified in proposition A.1. Then, $\mu(p_1 = \omega_1, p_2 = \omega_2) = \mu(p_1 = 1, p_2 = \omega_2) = \mu(p_1 = -1, p_2 = \omega_2) = \pi$. Out-of-equilibrium beliefs satisfy $\mu(p) \leq \pi$ for all $\mathbf{p}$. Based on these beliefs the reelection strategy used by the Voter is indeed a best-response. In order for the non-congruent Incumbent not to deviate, the reelection probabilities used by the Voter need to satisfy the following inequalities:

$$
\begin{aligned}
U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2) = {} & q_1 r^*(p_1 = \omega_1 = 1, p_2 = \omega_2)B \\
& + (1 - q_1) r^*(p_1 = 1, p_2 = \omega_2)B \\
& \geq \begin{cases} 1 + (1 - q_1) r^*(p_1 = -1, p_2 = \omega_2)B \\ 2 \end{cases} \\
& \geq \begin{cases} U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) \\ U_{N_{1,\cdot}}(p_1 = \cdot, p_2 \neq \omega_2) \end{cases}
\end{aligned}
$$

and

$$U_{N_{-1,\cdot}}(p_1=-1,p_2=\omega_2) = q_1 r^*(p_1=\omega_1=-1,p_2=\omega_2)B$$
$$+ (1-q_1)r^*(p_1=-1,p_2=\omega_2)B$$
$$\geq \begin{cases} 1+(1-q_1)r^*(p_1=1,p_2=\omega_2)B \\ 2 \end{cases}$$
$$\geq \begin{cases} U_{N_{-1,\cdot}}(p_1=1,p_2=\omega_2) \\ U_{N_{-1,\cdot}}(p_1=\cdot,p_2\neq\omega_2). \end{cases}$$

Note that there always exists reelection probabilities $r^*(p_1=\omega_1=1,p_2=\omega_2)$, $r^*(p_1=\omega_1=-1,p_2=\omega_2)$, $r^*(p_1=1,p_2=\omega_2)$, and $r^*(p_1=-1,p_2=\omega_2)$ such that these inequalities are satisfied. Consider for example $r^*(p_1=\omega_1,p_2=\omega_2)=r^*(p_1=1,p_2=\omega_2)=r^*(p_1=-1,p_2=\omega_2)=1$. It is straightforward to show that for such reelection probabilities the congruent Incumbent has no incentive to deviate either.

4. Assume $B\geq 2$ and $q_1\in\left[0,\frac{1}{B}\right)$ and that the choice of policy vectors is as specified in proposition A.1. Then, $\mu(p_1=\omega_1=1,p_2=\omega_2)=\frac{\pi}{\pi+(2-\frac{1}{\alpha})(1-\pi)}>\pi$, $\mu(p_1=\omega_1=-1,p_2=\omega_2)=1$, $\mu(p_1=1,p_2=\omega_2)=\frac{\alpha\pi}{\alpha\pi+\left[(1-\alpha)+\alpha(2-\frac{1}{\alpha})\right](1-\pi)}=\pi$, $\mu(p_1=-1,p_2=\omega_2)=\frac{(1-\alpha)\pi}{(1-\alpha)\pi+\alpha(\frac{1}{\alpha}-1)(1-\pi)}=\pi$, and $\mu(p_1\neq\omega_1,p_2=\omega_2)=0$. Out-of-equilibrium beliefs satisfy $\mu(\mathbf{p})\leq\pi$ for all out-of-equilibrium policy vectors $\mathbf{p}$. Based on these beliefs the reelection strategy used by the Voter is indeed a best-response.

   Hence, $U_{N_{1,\cdot}}(p_1=1,p_2=\omega_2)=(1-q_1)r^*(p_1=1,p_2=\omega_2)B+q_1B$ and $U_{N_{1,\cdot}}(p_1=-1,p_2=\omega_2)=q_1+(1-q_1)(1+r^*(p_1=-1,p_2=\omega_2)B)$. In equilibrium, $N_{1,\cdot}$ is mixing between $(p_1=1,p_2=\omega_2)$ and $(p_1=-1,p_2=\omega_2)$ and hence is indifferent between these two policy vectors. Indifference is satisfied as $r^*(p_1=1,p_2=\omega_2)=\frac{1}{(1-q_1)B}-\frac{q_1}{1-q_1}+r^*(p_1=-1,p_2=\omega_2)$. Because $q_1<\frac{1}{B}$, this implies $1\geq r^*(p_1=1,p_2=\omega_2)>r^*(p_1=-1,p_2=\omega_2)$. Moreover, as $r^*(p_1=-1,p_2=\omega_2)\geq\frac{1}{(1-q_1)B}$, we have

$U_{N_{1,\cdot}}(p_1 = \cdot, p_2 \neq \omega_2) \leq 2 \leq q_1 + (1-q_1)(1 + r^*(p_1 = -1, p_2 = \omega_2)B) = U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2)$. Hence, $N_{1,\cdot}$ has no incentive to deviate.

To see that $N_{-1,\cdot}$ has no incentive to deviate, note that, because $r^*(p_1 = 1, p_2 = \omega_2) > r^*(p_1 = -1, p_2 = \omega_2) \geq \frac{1}{(1-q_1)B}$ and because $q_1 < \frac{1}{B}$, we have $U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) = q_1 + (1-q_1)(1 + r^*(p_1 = 1, p_2 = \omega_2)B) > 2 \geq U_{N_{-1,\cdot}}(p_1 = \cdot, p_2 \neq \omega_2)$ and $U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) > (1-q_1)r^*(p_1 = -1, p_2 = \omega_2)B + q_1 B = U_{N_{-1,\cdot}}(p_1 = -1, p_2 = \omega_2)$.

Moreover, we have $U_{C_{-1,\cdot}}(p_1 = -1, p_2 = \omega_2) = 2 + q_1 B + (1-q_1)r^*(p_1 = -1, p_2 = \omega_2)B > 1 + (1-q_1)r^*(p_1 = 1, p_2 = \omega_2)B = U_{C_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2)$ and $U_{C_{1,\cdot}}(p_1 = 1, p_2 = \omega_2) = 2 + (1-q_1)r^*(p_1 = 1, p_2 = \omega_2)B + q_1 B > 1 + (1-q_1)r^*(p_1 = -1, p_2 = \omega_2)B = U_{C_{1,\cdot}}(p_1 = -1, p_2 = \omega_2)$ because $r^*(p_1 = 1, p_2 = \omega_2) = \frac{1}{(1-q_1)B} - \frac{q_1}{1-q_1} + r^*(p_1 = -1, p_2 = \omega_2)$. Because deviating to $p_2 \neq \omega_2$ yields at most a payoff of $1 < 2$, congruent Incumbents have no incentive to deviate.

□

Lemma 2. *The pair of strategies and beliefs identified in proposition A.1 satisfy an adaptation to this game of criterion* $D1$ *(Cho and Kreps, 1987).*

Proof. Note that for any policy vector $\mathbf{p} \in P$, there are two information sets that could be reached: $(p_1, p_2; \cdot, \omega_2)$ and $(p_1, p_2; \omega_1, \omega_2)$. Note that for any retention probability used at any information set there exists a specification of beliefs of the Voter about the types of the Incumbent that makes this retention probability a best-response of the Voter at that information set. Let $\mathbf{r}$ denote the vector of reelection probabilities $(r(p_1, p_2; \cdot, \omega_2), r(p_1, p_2; \omega_1, \omega_2))$ used by the Voter. Now let $D(t, T, \mathbf{p})$ be the set of vectors of probabilities of retention $\mathbf{r}$ that make type t strictly prefer $\mathbf{p}$ to his equilibrium strategy policy vector and let $D^0(t, T, \mathbf{p})$ be the set of vectors of retention probabilities that make type t exactly indifferent. Finally, let $D^1(t, T, \mathbf{p}) \equiv D(t, T, \mathbf{p}) \cup D^0(t, T, \mathbf{p})$. Criterion D1 requires that if for some type t there exists a type t' such that $D^1(t, T, \mathbf{p}) \subset D(t', T, \mathbf{p})$, then the Voter should not believe that she is facing an Incumbent of type t when she observes $\mathbf{p}$.

Note that in any equilibrium identified in proposition A.1, the equilibrium payoff of any congruent type is strictly greater than the equilibrium payoff of any non-congruent type. Now consider any policy vector $\mathbf{p} \neq (p_1 = \omega_1, p_2 = \omega_2)$ and fix any vector of reelection probabilities $\mathbf{r}$. We have $U_N(\mathbf{p}, \mathbf{r}) \geq U_C(\mathbf{p}, \mathbf{r})$. It follows that $D^1(C, T, \mathbf{p}) \subset D(N, T, \mathbf{p})$ at any out-of-equilibrium information set $(\mathbf{p}; \boldsymbol{\omega})$. Therefore, $D1$ requires that the Voter believes she is facing a non-congruent type upon observing $(\mathbf{p}; \boldsymbol{\omega})$ and thus should not re-elect at these information sets. Note that so far we only have pinned down out-of-equilibrium beliefs at information sets where the Voter observes both the policy vector $\mathbf{p}$ and the vector of states of the world $\boldsymbol{\omega}$. To pin down beliefs at information sets of the kind $(p_1, p_2; \cdot, \omega_2)$, we use the reelection probabilities just specified for corresponding information sets $(p_1, p_2; \omega_1, \omega_2)$. Note that the only information sets to consider are of the kind $(p_1, p_2 \neq \omega_2)$ and that based on the previous step we have $r(p_1 \neq \omega_1, p_2 \neq \omega_2) = 0$. Hence, $U_N(p_1 \neq \omega_1, p_2 \neq \omega_2) = 2 + (1 - q_1) r(p_1, p_2 \neq \omega_2) B$, while the payoff to a congruent type of choosing the same policy vector $(p_1, p_2 \neq \omega_2)$ is at most $1 + (1 - q_1) r(p_1, p_2 \neq \omega_2) B$. Therefore, $D1$ requires that the Voter believes she is facing a non-congruent type upon observing $(p_1, p_2 \neq \omega_2)$ at any such out-of-equilibrium information set. □

Lemma 3. *The pair of strategies and beliefs characterized in proposition A.1 constitute the unique equilibrium in which congruent incumbents choose* $(p_1 = \omega_1, p_2 = \omega_2)$.

Proof.

1. Assume $B < 2$ and that congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$. As $U_N(p_1 \neq \omega_1, p_2 \neq \omega_2) \geq 2 > B \geq U_N(p_1 = \omega_1, p_2 = \omega_2)$, the non-congruent Incumbent never chooses $(p_1 = \omega_1, p_2 = \omega_2)$ in equilibrium. Moreover, if the non-congruent Incumbent chooses $p_i \neq \omega_i$ $(i \in \{1, 2\})$ with positive probability in equilibrium, the Voter learns that the Incumbent is non-congruent upon observing $p_i \neq \omega_i$ and thus does not reelect whenever $p_i \neq \omega_i$ is revealed to her. It follows that there is no equilibrium in which the non-congruent Incumbent chooses $(p_1 = \omega_1, p_2 \neq \omega_2)$ as then

$U_N(p_1 = \omega_1, p_2 \neq \omega_2) = 1 < 2 \leq U_N(p_1 \neq \omega_1, p_2 \neq \omega_2)$. Hence, in equilibrium the non-congruent Incumbent chooses between $(p_1 \neq \omega_1, p_2 = \omega_2)$ and $(p_1 \neq \omega_2, p_2 \neq \omega_2)$. Moreover, we have $U_N(p_1 \neq \omega_1, p_2 = \omega_2) \leq 1 + (1 - q_1)B$. Hence, if $q_1 > \frac{B-1}{B}$, $U_N(p_1 \neq \omega_1, p_2 = \omega_2) < 2$ and the non-congruent Incumbent chooses $(p_1 \neq \omega_1, p_2 \neq \omega_2)$ in equilibrium.

Because $N_{1,\cdot}$ never chooses $(p_1 = 1, p_2 = \omega_2)$ and because $\alpha > 1/2$ we have $\mu(p_1 = 1, p_2 = \omega_2) = \frac{\alpha\pi}{\alpha\pi + \eta_{-1,\cdot}(p_1=1,p_2=\omega_2)(1-\alpha)(1-\pi)} > \pi$ for all $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2)$ in any equilibrium. It follows that $U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) = 1 + (1 - q_1)B$. Hence, if $q_1 < \frac{B-1}{B}$, we have $U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) > 2$ and $N_{-1,\cdot}$ chooses $(p_1 = 1, p_2 = \omega_2)$ in equilibrium.

There is no equilibrium, however, in which $N_{1,\cdot}$ chooses $(p_1 = -1, p_2 = \omega_2)$ with certainty. Assume otherwise, then $\mu(p_1 = -1, p_2 = \omega_2) = \frac{(1-\alpha)\pi}{(1-\alpha)\pi + \alpha(1-\pi)} < \pi$ and thus $U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) = 1 < 2$. Similarly, if $q_1 < \frac{B-1}{B}$, there is no equilibrium in which $N_{1,\cdot}$ chooses $(p_1 = -1, p_2 \neq \omega_2)$ with certainty, as then $\mu(p_1 = -1, p_2 = \omega_2) = 1$ and thus $U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) = 1 + (1 - q_1)B > 2$. It follows that if $q_1 < \frac{B-1}{B}$ then $N_{1,\cdot}$ mixes between $(p_1 = -1, p_2 = \omega_2)$ and $(p_1 = -1, p_2 \neq \omega_2)$ in any equilibrium, which requires $U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) = 1 + (1 - q_1)r^*(p_1 = -1, p_2 = \omega_2)B = 2 = U_{N_{1,\cdot}}(p_1 \neq \omega_1, p_2 \neq \omega_2)$ and thus $r^*(p_1 = -1, p_2 = \omega_2) = \frac{1}{(1-q_1)B} \in (0, 1)$. For the Voter to be willing to re-elect with positive probability, we need $\mu(p_1 = -1, p_2 = \omega_2) = \frac{(1-\alpha)\pi}{(1-\alpha)\pi + \alpha\eta_{1,\cdot}(p_1=-1,p_2=\omega_2)(1-\pi)} = \pi$ which implies $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) = \frac{1}{\alpha} - 1$.

2. Assume $B \geq 2$ and that congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$. I first show that there then is no equilibrium in which non-congruent Incumbents choose $(p_1 = \cdot, p_2 \neq \omega_2)$ with positive probability. WLOG assume $N_{1,\cdot}$ plays $(p_1 = \cdot, p_2 \neq \omega_2)$ with positive probability. Then, $\mu(p_1 = \cdot, p_2 \neq \omega_2) = 0$ and thus $U_{N_{1,\cdot}}(p_1 = \cdot, p_2 \neq \omega_2) \leq 2$. Moreover, $\mu(p_1 = \omega_1 = 1, p_2 = \omega_2) > \pi$ and $r^*(p_1 = \omega_1 = 1, p_2 = \omega_2) = 1$. Finally, $\mu(p_1 = 1, p_2 = \omega_2) > \pi$ or $\mu(p_1 = -1, p_2 = \omega_2) > \pi$. If $\mu(p_1 = 1, p_2 = \omega_2) > \pi$,

then $U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2) = B \geq 2$ and $N_{1,\cdot}$ deviates to $(p_1 = 1, p_2 = \omega_2)$. If $\mu(p_1 = -1, p_2 = \omega_2) > \pi$, then $\eta_{-1,\cdot}(p_1 = -1, p_2 = \omega_2) < 1$ in equilibrium. Note that $\mu(p_1 = -1, p_2 = \omega_2) > \pi$ implies $\mu(p_1 = \omega_1 = -1, p_2 = \omega_2) > \pi$. Thus, if $q_1 \leq \frac{1}{B}$, $U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) = 1 + (1 - q_1)B \geq B \geq 2$ and $N_{1,\cdot}$ wants to deviate to $(p_1 = -1, p_2 = \omega_2)$. Moreover, $U_{N_{-1,\cdot}}(p_1 = -1, p_2 = \omega_2) = B \geq 2 \geq U_{N_{-1,\cdot}}(p_1 = \cdot, p_2 \neq \omega_2)$. Hence, $\eta_{-1,\cdot}(p_1 = -1, p_2 = \omega_2) < 1$ implies that $U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) = 1 + (1 - q_1)r^*(p_1 = 1, p_2 = \omega_2)B \geq B$ which implies $r^*(p_1 = 1, p_2 = \omega_2) \geq \frac{B}{(1-q_1)B}$. If $q_1 > \frac{1}{B}$, this is impossible, however, as then $\frac{B}{(1-q_1)B} > 1$. It follows that $N_{1,\cdot}$ and $N_{-1,\cdot}$ choose between $(p_1 = \omega_1, p_2 = \omega_2)$ and $(p_1 \neq \omega_1, p_2 = \omega_2)$.

Note moreover, that there is no equilibrium in which $\mu(p_1 = 1, p_2 = \omega_2) < \pi$. Assume otherwise. Then it must be the case that $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2)$ and $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2)$ are superior to 0. But then, $U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) = 1 < 2 = U_{N_{-1,\cdot}}(p_1 = 1, p_2 \neq \omega_2)$ which contradicts $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2) > 0$. By a similar argument, there is no equilibrium in which $\mu(p_1 = -1, p_2 = \omega_2) < \pi$. This implies that in any equilibrium $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) \leq \frac{1}{\alpha} - 1$ as otherwise $\mu(p_1 = -1, p_2 = \omega_2) < \pi$. The fact that in equilibrium $\mu(p_1 = 1, p_2 = \omega_2)$ and $\mu(p_1 = -1, p_2 = \omega_2)$ are superior or equal to π implies that $N_{1,\cdot}$ chooses $(p_1 = \omega_1, p_2 = \omega_2)$ deterministically if, and only if $N_{-1,\cdot}$ does so as well.

I now show that if $q_1 > \frac{1}{B}$, then N chooses $(p_1 = \omega_1, p_2 = \omega_2)$ deterministically. To derive a contradiction assume that $N_{1,\cdot}$ is mixing between $(p_1 = 1, p_2 = \omega_2)$ and $(p_1 = -1, p_2 = \omega_2)$. Then, $U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2) = q_1 B + (1 - q_1)r^*(p_1 = 1, p_2 = \omega_2)B = 1 + (1 - q_1)r^*(p_1 = -1, p_2 = \omega_2)B = U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2)$. Because $q_1 > \frac{1}{B}$ we have $r^*(p_1 = -1, p_2 = \omega_2) > r^*(p_1 = 1, p_2 = \omega_2)$. But then $N_{-1,\cdot}$ chooses $(p_1 = -1, p_2 = \omega_2)$ deterministically, which by the argument made in the previous paragraph implies that $N_{1,\cdot}$ chooses $(p_1 = 1, p_2 = \omega_2)$ deterministically as well. To see this, assume otherwise. Then $U_{N_{-1,\cdot}}(p_1 = -1, p_2 = \omega_2) = q_1 B + (1 - q_1)r^*(p_1 = -1, p_2 = \omega_2)B > 1 + (1 - q_1)r^*(p_1 = 1, p_2 = \omega_2)B = U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2)$ and $N_{-1,\cdot}$

wants to deviate to $(p_1 = -1, p_2 = \omega_2)$.

Next I show that if $q_1 < \frac{1}{B}$, then $N_{1,\cdot}$ playing $(p_1 = 1, p_2 = \omega_2)$ with positive probability implies that $N_{-1,\cdot}$ chooses $(p_1 = 1, p_2 = \omega_2)$ deterministically. By arguments made above, it is the case in any equilibrium that $N_{1,\cdot}$ plays $(p_1 = 1, p_2 = \omega_2)$ with positive probability. This requires that $U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2) = q_1 r^*(p_1 = \omega_1 = 1, p_2 = \omega_2)B + (1-q_1)r^*(p_1 = 1, p_2 = \omega_2)B \geq 1 + (1-q_1)r^*(p_1 = -1, p_2 = \omega_2)B = U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2)$. Because $q_1 < \frac{1}{B}$, $q_1 B < 1$ and thus $r^*(p_1 = 1, p_2 = \omega_2) > r^*(p_1 = -1, p_2 = \omega_2)$. But then $U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) = 1 + (1-q_1)r^*(p_1 = 1, p_2 = \omega_2)B > q_1 B + (1-q_1)r^*(p_1 = -1, p_2 = \omega_2)B = U_{N_{-1,\cdot}}(p_1 = -1, p_2 = \omega_2)$ and $N_{-1,\cdot}$ chooses $(p_1 = 1, p_2 = \omega_2)$ deterministically. This in turn implies that $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) \leq 2 - \frac{1}{\alpha}$ as otherwise $\mu(p_1 = 1, p_2 = \omega_2) < \pi$. From $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) \leq 2 - \frac{1}{\alpha}$, $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) \leq \frac{1}{\alpha} - 1$, and $\eta_{1,\cdot}(p_1, p_2 \neq \omega_2) = 0$, we conclude that $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) = 2 - \frac{1}{\alpha}$, and $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) = \frac{1}{\alpha} - 1$ in equilibrium.

□

Lemma 4. *The pair of strategies and beliefs characterized in proposition A.1 constitute the unique equilibrium that satisfies criterion D1.*

Proof. Assume there exists an equilibrium in which $(p_1 = \omega_1, p_2 = \omega_2)$ is not played with positive probability for some state of the world $\boldsymbol{\omega}$. Fix a vector of reelection probabilities $\mathbf{r}$, then $U_C(p_1 = \omega_1, p_2 = \omega_2, \mathbf{r}) > U_N(p_1 = \omega_1, p_2 = \omega_2, \mathbf{r})$. Moreover, for any policy vector $\mathbf{p} \neq (p_1 = \omega_1, p_2 = \omega_2)$ and any vector of equilibrium reelection probabilities $\mathbf{r}^*$ we have $U_C(\mathbf{p}, \mathbf{r}^*) \leq U_N(\mathbf{p}, \mathbf{r}^*)$. In other words, in any equilibrium in which $(p_1 = \omega_1, p_2 = \omega_2)$ is not played with positive probability the equilibrium payoff of any non-congruent type is at least as high as the equilibrium payoff of any congruent type. It follows that $D^1(N, T, p_1 = \omega_1, p_2 = \omega_2) \subset D(C, T, p_1 = \omega_1, p_2 = \omega_2)$ and the Voter should believe she is facing a congruent Incumbent upon observing $(p_1 = \omega_1, p_2 = \omega_2)$. Similarly, $D^1(N_{1,\cdot}, T, p_1 = 1, p_2 =$

$\omega_2) \subset D^1(N_{-1,\cdot}, T, p_1 = 1, p_2 = \omega_2) \subset D(C_{1,\cdot}, T, p_1 = 1, p_2 = \omega_2)$ and the Voter should believe she is facing a congruent Incumbent upon observing $(p_1 = 1, p_2 = \omega_2)$. A similar argument holds for $(p_1 = -1, p_2 = \omega_2)$. But then the congruent Incumbent wants to deviate to $(p_1 = \omega_1, p_2 = \omega_2)$ as he then receives a payoff of $2 + B$ which is greater then $1 + B$, the highest payoff the congruent type can get in any equilibrium in which he does not play $(p_1 = \omega_1, p_2 = \omega_2)$. □

Model with Direct Democracy

Proposition A.2. *The following pairs of strategies and beliefs constitute the equilibrium of the direct democracy model when $q_1 \in [0, 1]$ and $q_2 = 1$.*

1. *If $B < 1$ and $q_1 \geq \frac{2B-1}{1+2B}$, then congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$ for all $\boldsymbol{\omega}$, while non-congruent Incumbents mix between $(p_1 = \omega_1, p_2 \neq \omega_2)$ and $(p_1 \neq \omega_1, p_2 \neq \omega_2)$.*

   The Voter holds a referendum to set $p_1 = \omega_1$ when $p_1 \neq \omega_1$. If ω_1 is not revealed, the Voter never holds a referendum when $p_2 = \omega_2$, and holds a referendum with non-degenerate probability whenever $p_2 \neq \omega_2$.

   The Voter reelects upon observing $(p_1 = \omega_1, p_2 = \omega_2)$, $(p_1 = 1, p_2 = \omega_2)$, or $(p_1 = -1, p_2 = \omega_2)$ and does not reelect otherwise.

   The Voter's beliefs satisfy $\mu(p_1 = \omega_1, p_2 = \omega_2) = \mu(p_1 = 1, p_2 = \omega_2) = \mu(p_1 = -1, p_2 = \omega_2) = 1$, $\mu(p_1 = \omega_1, p_2 \neq \omega_2) = \mu(p_1 \neq \omega_1, p_2 \neq \omega_2) = \mu(p_1 = 1, p_2 \neq \omega_2) = \mu(p_1 = -1, p_2 \neq \omega_2) = 0$. Out-of-equilibrium beliefs satisfy $\mu(p_1 \neq \omega_1, p_2 = \omega_2) \leq \pi$.

2. *If $B < 1$ and $q_1 < \frac{2B-1}{1+2B}$ or if $B \geq 1$ and $q_1 \in \left[\frac{2B-2}{B}, 3 - 2B\right]$, there exists an infinity of equilibria. In all these equilibria, congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$ for all $\boldsymbol{\omega}$ and the non-congruent Incumbents play $p_1 = \omega_1$ and $p_2 \neq \omega_2$ with positive probability. If $B < 1$ and $q_1 < \frac{2B-1}{1+2B}$, the non-congruent Incumbent chooses $p_2 = \omega_2$ with*

positive probability in any equilibrium. If $B \geq 1$ *then for all* $q_1 \in \left[\frac{2B-2}{B}, 3-2B\right]$ *there exists equilibria in which the non-congruent Incumbents choose* $p_2 = \omega_2$ *with positive probability. If* $B \geq 1$ *and* $q_1 \in \left[\frac{2B-1}{2B+1}, 3-2B\right]$ *however, there also exist equilibria in which the non-congruent Incumbents play* $p_2 \neq \omega_2$ *with certainty.*

The Voter holds a referendum to set $p_1 = \omega_1$ *when* $p_1 \neq \omega_1$*. If* ω_1 *is not revealed, the Voter never holds a referendum when* $p_2 = \omega_2$*, and holds a referendum with non-degenerate probability whenever* $p_2 \neq \omega_2$*.*

The Voter always reelects with positive probability upon observing $(p_1 = \omega_1, p_2 = \omega_2)$*,* $(p_1 = 1, p_2 = \omega_2)$*, or* $(p_1 = -1, p_2 = \omega_2)$ *and never reelects when* $p_2 \neq \omega_2$*.*

3. *If* $q_1 \in \left[\max\{3-2B, \frac{1}{B+1}\}, 1\right]$*, then congruent and non-congruent Incumbents choose* $(p_1 = \omega_1, p_2 = \omega_2)$ *for all* $\boldsymbol{\omega}$*, the Voter holds a referendum to set* $p_1 = \omega_1$ *if* $p_1 \neq \omega_1$ *is revealed to her, may reelect with positive probability upon observing* $(p_1 = \omega_1, p_2 = \omega_2)$*,* $(p_1 = 1, p_2 = \omega_2)$*, and* $(p_1 = -1, p_2 = \omega_2)$ *and does not reelect otherwise. The Voter's beliefs satisfy* $\mu(p_1 = \omega_1, p_2 = \omega_2) = \mu(p_1 = 1, p_2 = \omega_2) = \mu(p_1 = -1, p_2 = \omega_2) = \pi$*. Out-of-equilibrium beliefs satisfy* $\mu(p_1 \neq \omega_1, p_2 = \omega_2)$*,* $\mu(p_1 = \cdot, p_2 \neq \omega_2) \leq \pi$*. If* $q_1 \in [3-2B, 2-B]$*, the Voter also holds, with probability* $R_1 \in \left[1 - \frac{B-1}{1-q_1}, \frac{B-1}{1-q_1}\right]$*, a referendum to set* $p_1 = 1$ *upon observing* $(p_1 = -1, p_2 \neq \omega_2)$ *and with probability* $R_{-1} \in \left[1 - \frac{B-1}{1-q_1}, \frac{B-1}{1-q_1}\right]$ *a referendum to set* $p_1 = -1$ *upon observing* $(p_1 = 1, p_2 \neq \omega_2)$*.*

4. *If* $q_1 \in \left[0, \min\{\frac{2B-2}{B}, \frac{1}{B+1}\}\right]$*, then congruent Incumbents choose* $(p_1 = \omega_1, p_2 = \omega_2)$*,*

   The non-congruent Incumbent who observes $\omega_1 = -1$ *chooses* $(p_1 = 1, p_2 = \omega_2)$*;*

   The non-congruent Incumbent who observes $\omega_1 = 1$ *chooses* $(p_1 = 1, p_2 = \omega_2)$ *with probability* $2 - \frac{1}{\alpha}$ *and* $(p_1 = -1, p_2 = \omega_2)$ *with probability* $\frac{1}{\alpha} - 1$*;*

   The Voter holds a referendum to set $p_1 = \omega_1$ *if* $p_1 \neq \omega_1$ *is revealed to her and to set* $p_1 = 1$ *upon observing* $(p_1 = -1, p_2 \neq \omega_2)$*;*

   The Voter's reelection strategy is: $r^*(p_1 = \omega_1, p_2 = \omega_2) = 1$*,* $r^*(p_1 = 1, p_2 = \omega_2) =$

$\frac{1}{B} - \frac{q_1}{1-q_1} + r^*(p_1 = -1, p_2 = \omega_2) \geq \frac{1}{(1-q_1)B}$ *and* $r^*(p_1 \neq \omega_1, p_2) = r^*(p_1, p_2 \neq \omega_2) = 0$.

The Voter's beliefs satisfy $\mu(p_1 = 1, p_2 = \omega_2) = \mu(p_1 = -1, p_2 = \omega_2) = \pi$, $\mu(p_1 = \omega_1 = 1, p_2 = \omega_2) > \pi$, $\mu(p_1 = \omega_1 = -1, p_2 = \omega_2) = 1$, $\mu(p_1 \neq \omega_1, p_2 = \omega_2) = 0$. *Out-of-equilibrium beliefs satisfy* $\mu(\cdot, p_2 \neq \omega_2) \leq \pi$.

The proof proceeds in the following steps. First, I show in Lemma 5 that the pairs of strategies and beliefs identified in proposition A.2 constitute a perfect Bayesian equilibrium. In Lemma 6, I then show that this equilibrium survives $D1$. Lemmas 7 to 14 prove that if $q_1 \in \left[0, \min\{\frac{2B-2}{B}, \frac{1}{B+1}\}\right]$ this equilibrium is the unique equilibrium in which congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$. Lemmas 7, 15, 16 then prove the same statement for $q_1 > \max\{3 - 2B, \frac{1}{B+1}\}$. Lemmas 17 and 18 then prove that when $B \geq 1$ and $q_1 \in \left[\frac{2B-2}{B}, 3 - 2B\right]$, then in any equilibrium in which congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$, the non-congruent Incumbents play $p_1 = \omega_1$ and $p_2 \neq \omega_2$ with positive probability. Lemma 19 then shows that if $B < 1$ and $q_1 < \frac{2B-1}{1+2B}$, there is no equilibrium in which non-congruent Incumbents never choose $p_2 = \omega_2$. Finally, Lemma 21 shows that an equilibrium satisfies $D1$ if, and only if, congruent Incumbents choose $p_2 = \omega_2$ in equilibrium.

Lemma 5. *The pairs of strategies and beliefs identified in proposition A.2 constitute a perfect Bayesian equilibrium.*

Proof.

1. A full statement and proof of the equilibria in that range is available upon request. In lemma 19 below I show, however, that there is no equilibrium in which non-congruent Incumbents choose $(p_1 \neq \omega_1, p_2 \neq \omega_2)$ deterministically. This implies that the non-congruent Incumbent is mixing between $(p_1 = \omega_1, p_2 \neq \omega_2)$ and $(p_1 \neq \omega_1, p_2 \neq \omega_2)$ in any equilibrium in which the non-congruent chooses $p_2 \neq \omega_2$ deterministically.

2. A full statement and proof of the equilibria in that range is available upon request. In lemmata 17 and 18 below, I show, however, that, if $B \geq 1$ and $q_1 \in \left[\frac{2B-2}{B}, 3 - 2B\right]$, there is neither an equilibrium in which non-congruent Incumbents never choose $p_2 \neq$

ω_2 nor an equilibrium in which they never choose $p_1 = \omega_1$ in that range. In lemma 20, I show, moreover, that, if $B < 1$ and $q_1 < \frac{2B-1}{1+2B}$, there is no equilibrium in which non-congruent Incumbents never choose $p_2 = \omega_2$.

3. Assume that $q_1 \in \left[\max\{3-2B, \frac{1}{B+1}\}, 1\right]$ and that the profile of policy vectors is as specified in proposition A.2. Then, $\mu(p_1 = \omega_1, p_2 = \omega_2) = \mu(p_1 = 1, p_2 = \omega_2) = \mu(p_1 = -1, p_2 = \omega_2) = \pi$. Out-of-equilibrium beliefs satisfy $\mu(p_1 \neq \omega_1, p_2 = \omega_2)$, $\mu(\cdot, p_2 \neq \omega_2) \leq \pi$. Based on these beliefs the reelection strategy used by the Voter is indeed a best-response. In order for the non-congruent Incumbent not to deviate, the reelection probabilities used by the Voter need to satisfy the following inequalities:

$$\begin{aligned}
U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2) &= q_1 r^*(p_1 = \omega_1 = 1, p_2 = \omega_2)B \\
&\quad + (1-q_1)r^*(p_1 = 1, p_2 = \omega_2)B \\
&\geq \begin{cases} (1-q_1)(1 + r^*(p_1 = -1, p_2 = \omega_2)B) \\ 2 - q_1 \end{cases} \\
&\geq \begin{cases} U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) \\ U_{N_{1,\cdot}}(p_1 = \cdot, p_2 \neq \omega_2), \end{cases}
\end{aligned}$$

and

$$\begin{aligned}
U_{N_{-1,\cdot}}(p_1 = -1, p_2 = \omega_2) &= q_1 r^*(p_1 = \omega_1 = -1, p_2 = \omega_2)B \\
&\quad + (1-q_1)r^*(p_1 = -1, p_2 = \omega_2)B \\
&\geq \begin{cases} (1-q_1)(1 + r^*(p_1 = 1, p_2 = \omega_2)B) \\ 2 - q_1 \end{cases} \\
&\geq \begin{cases} U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) \\ U_{N_{-1,\cdot}}(p_1 = \cdot, p_2 \neq \omega_2). \end{cases}
\end{aligned}$$

Note that, if $q_1 \geq \max\{2-B, \frac{1}{B+1}\}$ there always exists reelection probabilities $r^*(p_1 = \omega_1 = 1, p_2 = \omega_2)$, $r^*(p_1 = \omega_1 = -1, p_2 = \omega_2)$, $r^*(p_1 = 1, p_2 = \omega_2)$, and $r^*(p_1 = -1, p_2 = \omega_2)$ such that these inequalities are satisfied. In any case $r^*(p_1 = \omega_1, p_2 = \omega_2) = r^*(p_1 = 1, p_2 = \omega_2) = r^*(p_1 = -1, p_2 = \omega_2) = 1$ satisfies the conditions. It is straightforward to show that for such reelection probabilities congruent Incumbents have no incentive to deviate either. If $q_1 \in \big[\max\{3-2B, \frac{1}{B+1}\}, 2-B\big)$ then $B < 2 - q_1$, however. If the Voter holds, with probability $R_{-1} \in \left[1 - \frac{B-1}{1-q_1}, \frac{B-1}{1-q_1}\right]$, a referendum to set $p_1 = -1$ upon observing $(p_1 = 1, p_2 \neq \omega_2)$, then we have $B \geq 1 + (1-q_1)R_{-1} = U_{N_{1,\cdot}}(p_1 = 1, p_2 \neq \omega_2)$ and $B \geq 1 + (1-q_1)(1-R_{-1}) = U_{N_{-1,\cdot}}(p_1 = 1, p_2 \neq \omega_2)$. Thus, there then always exists reelection probabilities $r^*(p_1 = \omega_1 = 1, p_2 = \omega_2)$, $r^*(p_1 = \omega_1 = -1, p_2 = \omega_2)$, $r^*(p_1 = 1, p_2 = \omega_2)$, and $r^*(p_1 = -1, p_2 = \omega_2)$ such that non-congruent Incumbents have no incentive to deviate to $(p_1 = 1, p_2 \neq \omega_2)$. A similar remark holds with respect to $(p_1 = -1, p_2 \neq \omega_2)$.

4. Assume that $q_1 \in \left[0, \min\{\frac{2B-2}{B}, \frac{1}{B+1}\}\right]$ and that the choice of policy vectors is as specified in proposition A.2. Then, $\mu(p_1 = \omega_1 = 1, p_2 = \omega_2) = \frac{\pi}{\pi + (2-\frac{1}{\alpha})(1-\pi)} > \pi$, $\mu(p_1 = \omega_1 = -1, p_2 = \omega_2) = 1$, $\mu(p_1 = 1, p_2 = \omega_2) = \frac{\alpha\pi}{\alpha\pi + \left[(1-\alpha)+\alpha(2-\frac{1}{\alpha})\right](1-\pi)} = \pi$, $\mu(p_1 = -1, p_2 = \omega_2) = \frac{(1-\alpha)\pi}{(1-\alpha)\pi + \alpha(\frac{1}{\alpha}-1)(1-\pi)} = \pi$, and $\mu(p_1 \neq \omega_1, p_2 = \omega_2) = 0$. Out-of-equilibrium beliefs satisfy $\mu(p_1 = \cdot, p_2 \neq \omega_2) \leq \pi$. Based on these beliefs the reelection strategy used by the Voter is indeed a best-response.

   By lemma 7 below, the Voter never holds a referendum upon observing $(p_1 = 1, p_2 = \omega_2)$. Moreover, we have $Pr(\omega_1 = -1 | p_1 = -1, p_2 = \omega_2) = \frac{\pi(1-\alpha)}{\pi(1-\alpha) + (\frac{1}{\alpha}-1)(1-\pi)\alpha} = \pi > 1/2$ and the Voter does not hold a referendum upon observing $(p_1 = -1, p_2 = \omega_2)$ either.

   Hence, $U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2) = (1-q_1)r^*(p_1 = 1, p_2 = \omega_2)B + q_1 B$ and $U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) = (1-q_1)(1 + r^*(p_1 = -1, p_2 = \omega_2)B)$. In equilibrium, $N_{1,\cdot}$ is mixing between $(p_1 = 1, p_2 = \omega_2)$ and $(p_1 = -1, p_2 = \omega_2)$ and hence is indifferent between these two policy vectors. Indifference is satisfied as $r^*(p_1 = 1, p_2 = \omega_2) = \frac{1}{B} - \frac{q_1}{1-q_1} + r^*(p_1 =$

$-1, p_2 = \omega_2)$. As $q_1 < \frac{1}{B+1}$, this implies $1 \geq r^*(p_1 = 1, p_2 = \omega_2) > r^*(p_1 = -1, p_2 = \omega_2)$. Moreover, because the Voter holds a referendum to set $p_1 = 1$ upon observing $(p_1 = -1, p_2 \neq \omega_2)$ and because $r^*(p_1 = 1, p_2 = \omega_2) \geq \frac{1}{(1-q_1)B}$, we have $U_{N_{1,\cdot}}(p_1 = \cdot, p_2 \neq \omega_2) = 1 \leq (1-q_1)r^*(p_1 = 1, p_2 = \omega_2)B + q_1 B = U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2)$. Hence, $N_{1,\cdot}$ has no incentive to deviate.

To see that $N_{-1,\cdot}$ has no incentive to deviate, note that $U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) = (1-q_1)(1 + r^*(p_1 = 1, p_2 = \omega_2)B) \geq 2 - q_1 = U_{N_{-1,\cdot}}(p_1 = \cdot, p_2 \neq \omega_2)$ because $r^*(p_1 = 1, p_2 = \omega_2) \geq \frac{1}{(1-q_1)B}$. Moreover, because $r^*(p_1 = 1, p_2 = \omega_2) > r^*(p_1 = -1, p_2 = \omega_2)$ and $q_1 < \frac{1}{B+1}$, we have $U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) > (1-q_1)r^*(p_1 = -1, p_2 = \omega_2)B + q_1 B = U_{N_{-1,\cdot}}(p_1 = -1, p_2 = \omega_2)$.

Moreover, we have $U_{C_{1,\cdot}}(p_1 = 1, p_2 = \omega_2) = 2 + (1-q_1)r^*(p_1 = 1, p_2 = \omega_2)B + q_1 B > 1 + q_1 + (1-q_1)r^*(p_1 = -1, p_2 = \omega_2)B = U_{C_{1,\cdot}}(p_1 = -1, p_2 = \omega_2)$, and $U_{C_{-1,\cdot}}(p_1 = -1, p_2 = \omega_2) = 2 + (1-q_1)r^*(p_1 = -1, p_2 = \omega_2)B + q_1 B > 1 + q_1 + (1-q_1)r^*(p_1 = 1, p_2 = \omega_2)B = U_{C_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2)$. As deviating to $(p_1 = \cdot, p_2 \neq \omega_2)$ yields at most a payoff of $1 < 2$, congruent Incumbents have no incentive to deviate.

$\square$

Lemma 6. *The perfect Bayesian equilibria identified in proposition A.2 survive criterion D1.*

Proof. Note that in any equilibrium identified in proposition A.2, the equilibrium payoff of any congruent type is strictly greater than the equilibrium payoff of any non-congruent type. Now remark that $U_C(p_1 = \omega_1, p_2 \neq \omega_2), U_C(p_1 \neq \omega_1, p_2 \neq \omega_2) \leq 1 + q_1 r(p_1, p_2; \omega_1, \omega_2)B + (1-q_1)r(p_1, p_2; \cdot, \omega_2)B$, while $U_N(p_1 = \omega_1, p_2 \neq \omega_2), U_N(p_1 \neq \omega_1, p_2 \neq \omega_2) \geq 1 + q_1 r(p_1, p_2; \omega_1, \omega_2)B + (1-q_1)r(p_1, p_2; \cdot, \omega_2)B$. It follows that the Voter should believe she is facing a non-congruent Incumbent upon observing $(p_1 = \omega_1, p_2 \neq \omega_2)$ or $(p_1 \neq \omega_1, p_2 \neq \omega_2)$. A similar argument shows that the Voter should believe she is facing a non-congruent Incumbent upon observing $(p_1 = \cdot, p_2 \neq \omega_2)$. Next suppose $(p_1 \neq \omega_1, p_2 = \omega_2)$ is out-of-equilibrium. As $U_C(p_1 \neq \omega_1, p_2 = \omega_2) \leq 2 + q_1 r(p_1 \neq \omega_1, p_2 = \omega_2)B + (1 - q_1)r(p_1, p_2 = \omega_2)B$ while $U_C(p_1 = \omega_1, p_2 = \omega_2) = 2 + q_1 r(p_1 = \omega_1, p_2 = \omega_2)B + (1 - q_1)r(p_1, p_2 = \omega_2)B$, it must be the case that $q_1 r(p_1 \neq \omega_1, p_2 = \omega_2)B + (1 - q_1)r(p_1, p_2 = \omega_2)B \geq q_1 r(p_1 = \omega_1, p_2 = \omega_2)B + (1 - q_1)r(p_1, p_2 = \omega_2)B$ for C to be willing to deviate to $(p_1 \neq \omega_1, p_2 = \omega_2)$. Moreover, $U_N(p_1 \neq \omega_1, p_2 = \omega_2) \geq q_1 r(p_1 \neq \omega_1, p_2 = \omega_2)B + (1 - q_1)r(p_1, p_2 = \omega_2)B$ while $U_N(p_1 = \omega_1, p_2 = \omega_2) = q_1 r(p_1 = \omega_1, p_2 = \omega_2)B + (1 - q_1)r(p_1, p_2 = \omega_2)B$. It follows that if $q_1 r(p_1 \neq \omega_1, p_2 = \omega_2)B + (1-q_1)r(p_1, p_2 = \omega_2)B = q_1 r(p_1 = \omega_1, p_2 = \omega_2)B + (1-q_1)r(p_1, p_2 = \omega_2)B$ N is willing to deviate to $(p_1 \neq \omega_1, p_2 = \omega_2)$. Hence, if in equilibrium N chooses $(p_1 = \omega_1, p_2 = \omega_2)$, the Voter should believe she is facing a non-congruent Incumbent upon observing $(p_1 \neq \omega_1, p_2 = \omega_2)$. □

Lemmas 7 to 14 prove that if $B \geq 1$ and $q_1 \in \left[0, \min\{\frac{2B-2}{B}, \frac{1}{B+1}\}\right]$ the pair of strategies and beliefs characterized in proposition A.2 constitute the unique equilibrium in which congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$. Hence, in the following, I assume throughout that congruent Incumbents play $(p_1 = \omega_1, p_2 = \omega_2)$.

Lemma 7. *If the congruent Incumbent chooses $(p_1 = \omega_1, p_2 = \omega_2)$, then the Voter never holds a referendum upon observing $(p_1 = 1, p_2 = \omega_2)$.*

Proof. $Pr(\omega_1 = 1 | p_1 = 1, p_2 = \omega_2) = \frac{(\pi + \eta_{1,\cdot}(p_1=1,p_2=\omega_2)(1-\pi))\alpha}{(\pi+\eta_{1,\cdot}(p_1=1,p_2=\omega_2)(1-\pi))\alpha + \eta_{-1,\cdot}(p_1=1,p_2=\omega_2)(1-\pi)(1-\alpha)}$ which is increasing in $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2)$ and decreasing in $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2)$. Let $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) = 0$ and $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2) = 1$. Then, $Pr(\omega_1 = 1 | p_1 = 1, p_2 = \omega_2) = \frac{\pi\alpha}{\pi\alpha + (1-\pi)(1-\alpha)} > \alpha$ because $\pi > 1/2$. Because $\alpha > 1/2$, we have $Pr(\omega_1 = 1 | p_1 = 1, p_2 = \omega_2) > Pr(\omega_1 = -1 | p_1 = 1, p_2 = \omega_2)$ and the Voter has no incentive to hold a referendum upon observing $(p_1 = 1, p_2 = \omega_2)$. □

Lemma 8. *There does not exist an equilibrium in which $N_{1,\cdot}$ chooses $(p_1 = -1, p_2 = \omega_2)$ with probability superior to $\frac{1}{\alpha} - 1$.*

Proof. Assume otherwise. Then

$\mu(p_1 = -1, p_2 = \omega_2) = \frac{(1-\alpha)\pi}{(1-\alpha)\pi + (\alpha\eta_{1,\cdot}(p_1=-1,p_2=\omega_2)+(1-\alpha)\eta_{-1,\cdot}(p_1=-1,p_2=\omega_2))(1-\pi)} < \pi$ for all $\eta_{-1,\cdot}(p_1 = -1, p_2 = \omega_2)$ whenever $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) > \frac{1}{\alpha} - 1$. Thus, $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) > \frac{1}{\alpha} - 1$ implies $r^*(p_1 = -1, p_2 = \omega_2) = 0$. But then $U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) \leq 1 - q_1 < 1 \leq U_{N_{1,\cdot}}(p_1 = \cdot, p_2 \neq \omega_2)$. □

Lemma 9. *If the congruent Incumbent chooses $(p_1 = \omega_1, p_2 = \omega_2)$, then the Voter never holds a referendum upon observing $(p_1 = -1, p_2 = \omega_2)$ in equilibrium.*

Proof. $Pr(\omega_1 = -1|p_1 = -1, p_2 = \omega_2) = \frac{(\pi+\eta_{-1,\cdot}(p_1=-1,p_2=\omega_2)(1-\pi))(1-\alpha)}{(\pi+\eta_{-1,\cdot}(p_1=-1,p_2=\omega_2)(1-\pi))(1-\alpha)+\eta_{1,\cdot}(p_1=-1,p_2=\omega_2)(1-\pi)\alpha}$ which is increasing in $\eta_{-1,\cdot}(p_1 = -1, p_2 = \omega_2)$ and decreasing in $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2)$. By Lemma 8, $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) \leq \frac{1}{\alpha} - 1$ in any equilibrium. Hence, let $\eta_{-1,\cdot}(p_1 = -1, p_2 = \omega_2) = 0$ and $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) = \frac{1}{\alpha} - 1$. Then, $Pr(\omega_1 = -1|p_1 = -1, p_2 = \omega_2) = \pi > 1/2$. Hence, we have $Pr(\omega_1 = -1|p_1 = -1, p_2 = \omega_2) > Pr(\omega_1 = 1|p_1 = -1, p_2 = \omega_2)$ and the Voter has no incentive to hold a referendum upon observing $(p_1 = -1, p_2 = \omega_2)$. □

Lemma 10. *If $q_1 < \frac{2B-2}{B}$, there does not exist an equilibrium in which $N_{1,\cdot}$ does not play $(p_1 = 1, p_2 = \omega_2)$.*

Proof. Assume otherwise, i.e. assume there exists an equilibrium in which $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) = 0$. Then, $\mu(p_1 = \omega_1 = 1, p_2 = \omega_2) = 1$ and $\mu(p_1 = 1, p_2 = \omega_2) > \pi$ for all $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2)$, which implies that $U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2) = B$ and $U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) = (1 - q_1)(1 + B)$. By the Lemma 8, there is no equilibrium in which $N_{1,\cdot}$ chooses $(p_1 = -1, p_2 = \omega_2)$ with certainty. Hence, if $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) = 0$ in equilibrium then $N_{1,\cdot}$ must be playing $p_2 \neq \omega_2$ with positive probability. $N_{1,\cdot}$ is only willing to choose $(p_1 = \cdot, p_2 \neq \omega_2)$ with positive probability if $N_{-1,\cdot}$ does so as well, as otherwise the Voter infers that $\omega_1 = 1$ when observing $(p_1 = \cdot, p_2 \neq \omega_2)$ and sets $p_1 = 1$ via referendum if needed. But then, $U_{N_{1\cdot}}(p_1 = \cdot, p_2 \neq \omega_2) = 1 < B = U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2)$. So suppose $N_{1,\cdot}$ and $N_{-1\cdot}$ play

$(p_1 = 1, p_2 \neq \omega_2)$ with positive probability. For such a behavior to be a best-response, it has to be the case that

$$U_{N_{-1,\cdot}}(p_1 = 1, p_2 \neq \omega_2) = 1 + (1 - q_1)(1 - R_{-1}) \geq (1 - q_1)(1 + B), \text{ and}$$
$$U_{N_{1,\cdot}}(p_1 = 1, p_2 \neq \omega_2) = 1 + (1 - q_1)R_{-1} \geq B,$$

because otherwise $N_{-1,\cdot}$ deviates to $(p_1 = 1, p_2 = \omega_2)$ and $N_{1,\cdot}$ deviates to $(p_1 = 1, p_2 = \omega_2)$ respectively. This implies that $\frac{B-1}{1-q_1} \leq R_{-1} \leq \frac{1}{1-q_1} - B$. But this is impossible as $q_1 < \frac{2B-2}{B}$ implies $\frac{B-1}{1-q_1} > \frac{1}{1-q_1} - B$. A similar argument shows that there is no equilibrium in which $N_{1,\cdot}$ and $N_{-1,\cdot}$ choose $(p_1 = -1, p_2 \neq \omega_2)$ with positive probability when $q_1 < \frac{2B-2}{B}$. □

Lemma 11. *If $q_1 < \frac{1}{B+1}$, then there does not exist an equilibrium in which $N_{1,\cdot}$ plays $(p_1 = 1, p_2 = \omega_2)$ deterministically.*

Proof. Assume otherwise, i.e. assume there exists an equilibrium in which $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) = 1$. Then, $\mu(p_1 = 1, p_2 = \omega_2) \leq \pi$ and because by lemma 7 the Voter does not hold a referendum upon observing $(p_1 = 1, p_2 = \omega_2)$, we have $U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2) = (1 - q_1)r^*(p_1 = 1, p_2 = \omega_2)B + q_1 r^*(p_1 = \omega_1 = 1, p_2 = \omega_2)B$. Because $q_1 < \frac{1}{B+1}$, $q_1 r^*(p_1 = \omega_1 = 1, p_2 = \omega_2)B < 1$ and thus $U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2) \geq U_{N_{1,\cdot}}(p_1 = \cdot, p_2 \neq \omega_2) \geq 1$ implies that $r^*(p_1 = 1, p_2 = \omega_2) > 0$ in equilibrium. This requires $\mu(p_1 = 1, p_2 = \omega_2) \geq \pi$ and thus $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2) = 0$ because $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) = 1$ implies $\mu(p_1 = 1, p_2 = \omega_2) \leq \pi$. Moreover, $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) = 1$ implies $\mu(p_1 = -1, p_2 = \omega_2) \geq \pi$ and $Pr(\omega_1 = -1 | p_1 = -1, p_2 = \omega_2) = 1$ which, in turn, implies that the Voter does not hold a referendum upon observing $(p_1 = -1, p_2 = \omega_2)$. If $\eta_{-1,\cdot}(p_1 = -1, p_2 = \omega_2) < 1$ in equilibrium, then $\mu(p_1 = -1, p_2 = \omega_2) > \pi$ and $r^*(p_1 = -1, p_2 = \omega_2) = 1$. But then, $U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) = (1 - q_1)(1 + B) > B \geq U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2)$ because $q_1 < \frac{1}{B+1}$ which contradicts our starting assumption that $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) = 1$. Hence, if $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) = 1$ in equilibrium we have $\eta_{-1,\cdot}(p_1 = -1, p_2 = \omega_2) = 1$ which requires that

$$\begin{aligned} U_{N_{-1,\cdot}}(p_1 = -1, p_2 = \omega_2) &= (1 - q_1)r^*(p_1 = -1, p_2 = \omega_2)B \\ &+ q_1 r^*(p_1 = \omega_1 = -1, p_2 = \omega_2)B \\ &\geq (1 - q_1)(1 + r^*(p_1 = 1, p_2 = \omega_2)B) \\ &= U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) \end{aligned}$$

and

$$\begin{aligned} U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2) &= (1 - q_1)r^*(p_1 = 1, p_2 = \omega_2)B \\ &+ q_1 r^*(p_1 = \omega_1 = 1, p_2 = \omega_2)B \\ &\geq (1 - q_1)(1 + r^*(p_1 = -1, p_2 = \omega_2)B) \\ &= U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) \end{aligned}$$

Because $q_1 < \frac{1}{B+1}$ and thus $q_1 r(p_1 = \omega_1, p_2 = \omega_2)B < 1 - q_1$, the first inequality implies that $r^*(p_1 = -1, p_2 = \omega_2) > r^*(p_1 = 1, p_2 = \omega_2)$, whereas the second implies $r^*(p_1 = -1, p_2 = \omega_2) < r^*(p_1 = 1, p_2 = \omega_2)$. Contradiction. $\square$

Lemma 12. *If $q_1 < \frac{1}{B+1}$, there does not exist an equilibrium in which $N_{1,\cdot}$ mixes between $(p_1 = 1, p_2 = \omega_2)$ and $(p_1 = \cdot, p_2 \neq \omega_2)$ and does not play $(p_1 = -1, p_2 = \omega_2)$.*

Proof. Assume otherwise, i.e. assume there exists an equilibrium in which $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) > 0$, $\eta_{1,\cdot}(p_1 = \cdot, p_2 \neq \omega_2) > 0$ and $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) = 0$. Case 1: $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2) = 0$. Then, $\mu(p_1 = 1, p_2 = \omega_2) = \frac{\alpha\pi}{\alpha\pi + \eta_{1,\cdot}(p_1=1,p_2=\omega_2)\alpha(1-\pi)} > \pi$ because $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) < 1$. Thus, $r^*(p_1 = 1, p_2 = \omega_2) = 1$ and $U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) = (1-q_1)(1+B)$. Because $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) = 0$, we have $Pr(\omega_1 = -1 | p_1 = -1, p_2 = \omega_2) = 1$ and thus the Voter does not hold a referendum upon observing $(p_1 = -1, p_2 = \omega_2)$. Hence, $U_{N_{-1,\cdot}}(p_1 = -1, p_2 = \omega_2) \leq B$. Because $q_1 < \frac{1}{B+1}$, we then have $U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) > U_{N_{-1,\cdot}}(p_1 = -1, p_2 = \omega_2)$

and thus $\eta_{-1,\cdot}(p_1 = -1, p_2 = \omega_2) = 0$. This implies that $\mu(p_1 = -1, p_2 = \omega_2) = 1$. But then, $U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) = (1-q_1)(1+B) > B \geq U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2)$ and $N_{1,\cdot}$ has incentive to deviate to $(p_1 = -1, p_2 = \omega_2)$.

Case 2: $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2) > 0$. Then $\mu(p_1 = -1, p_2 = \omega_2) > \pi$ because $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) = 0$ and thus $r^*(p_1 = -1, p_2 = \omega_2) = 1$. But then $U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) = (1 - q_1)(1 + B)$ which, because $q_1 < \frac{1}{B+1}$, is strictly greater than $B \geq U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2)$. Thus, $N_{1,\cdot}$ has incentive to deviate to $(p_1 = -1, p_2 = \omega_2)$. $\square$

Lemma 13. *If $q_1 < \min\{\frac{2B-2}{B}, \frac{1}{B+1}\}$ and $N_{1,\cdot}$ mixes between $(p_1 = 1, p_2 = \omega_2)$ and $(p_1 = -1, p_2 = \omega_2)$ in equilibrium, then $N_{-1,\cdot}$ plays $(p_1 = 1, p_2 = \omega_2)$ deterministically.*

Proof. Assume otherwise, i.e. assume there exists an equilibrium in which $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) > 0$, $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) > 0$, yet $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2) < 1$. Case 1: $\mu(p_1 = 1, p_2 = \omega_2) < \pi$ and thus $r^*(p_1 = 1, p_2 = \omega_2) = 0$. Then, $U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) = 1 - q_1 < 1$ and thus $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2) = 0$. Because $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) < 1$, we have $\mu(p_1 = 1, p_2 = \omega_2) > \pi$, contradicting the premise.

Case 2: $\mu(p_1 = 1, p_2 = \omega_2) = \pi$ which implies that $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) = 1 - (\frac{1}{\alpha} - 1)\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2)$. Because $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2) < 1$ we have $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) > 2 - \frac{1}{\alpha}$ and $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) < \frac{1}{\alpha} - 1$. In turn, $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) < \frac{1}{\alpha} - 1$ implies $Pr(\omega_1 = -1 | p_1 = -1, p_2 = \omega_2) > \pi$ and thus the Voter does not hold a referendum upon observing $(p_1 = -1, p_2 = \omega_2)$. Hence, as $N_{1,\cdot}$ is mixing, we have

$$
\begin{aligned}
U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2) &= (1 - q_1)r^*(p_1 = 1, p_2 = \omega_2)B + q_1 B \\
&= (1 - q_1)(1 + r^*(p_1 = -1, p_2 = \omega_2)B) = U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2)
\end{aligned}
$$

Because $q_1 < \frac{1}{B+1}$, $q_1 B < 1 - q_1$ and thus $N_{1,\cdot}$ mixing implies $r^*(p_1 = 1, p_2 = \omega_2) > r^*(p_1 =$

$-1, p_2 = \omega_2)$. But then,

$$U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) = (1 - q_1)(1 + r^*(p_1 = 1, p_2 = \omega_2)B)$$
$$> (1 - q_1)r^*(p_1 = -1, p_2 = \omega_2)B + q_1 B$$
$$= U_{N_{-1,\cdot}}(p_1 = -1, p_2 = \omega_2)$$

and thus $\eta_{-1,\cdot}(p_1 = -1, p_2 = \omega_2) = 0$. Because $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) < \frac{1}{\alpha} - 1$, this implies that $\mu(p_1 = -1, p_2 = \omega_2) > \pi$ and thus $r^*(p_1 = -1, p_2 = \omega_2) = 1$ which contradicts $r^*(p_1 = 1, p_2 = \omega_2) > r^*(p_1 = -1, p_2 = \omega_2)$.

Case 3: $\mu(p_1 = 1, p_2 = \omega_2) > \pi$. Then, $r^*(p_1 = 1, p_2 = \omega_2) = 1$. By the arguments given under case 2 and because $q_1 < \frac{1}{B+1}$, $N_{-1,\cdot}$ never plays $(p_1 = -1, p_2 = \omega_2)$. Because $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2) < 1$, this implies that $N_{-1,\cdot}$ plays $p_2 \neq \omega_2$ with positive probability. In equilibrium, this requires $U_{N_{-1,\cdot}}(p_1 = -1, p_2 \neq \omega_2) = 1 + (1 - q_1)R_1 \geq (1 - q_1)(1 + B) = U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2)$ or $U_{N_{-1,\cdot}}(p_1 = 1, p_2 \neq \omega_2) = 1 + (1 - q_1)(1 - R_{-1}) \geq (1 - q_1)(1 + B) = U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2)$, which implies $R_1 \geq B + 1 - \frac{1}{1-q_1}$ or $R_{-1} \leq \frac{1}{1-q_1} - B$. In turn, because $N_{1,\cdot}$ plays $(p_1 = 1, p_2 = \omega_2)$ with positive probability, it must be the case that

$$B \geq 1 + (1 - q_1)R_{-1}$$
$$B \geq 1 + (1 - q_1)(1 - R_1)$$

which implies that $R_{-1} \leq \frac{B-1}{1-q_1} < 1$ and $R_1 \geq 1 - \frac{B-1}{1-q_1} > 0$. For the Voter to be willing to adopt such probabilities of holding a referendum, it must be the case that $N_{1,\cdot}$ plays $p_2 \neq \omega_2$ with positive probability. As $N_{1,\cdot}$ plays $(p_1 = 1, p_2 = \omega_2)$, this requires $R_{-1} = \frac{B-1}{1-q_1}$ and $R_1 = 1 - \frac{B-1}{1-q_1}$. Because $q_1 < \frac{2B-2}{B}$, we have $1 - \frac{B-1}{1-q_1} < B + 1 - \frac{1}{1-q_1}$ and $\frac{B-1}{1-q_1} > \frac{1}{1-q_1} - B$. But then, $N_{-1,\cdot}$ wants to deviate to $(p_1 = 1, p_2 = \omega_2)$. $\square$

Lemma 14. *In any equilibrium in which $N_{1,\cdot}$ mixes between $(p_1 = 1, p_2 = \omega_2)$ and $(p_1 = -1, p_2 = \omega_2)$ and $N_{-1,\cdot}$ plays $(p_1 = 1, p_2 = \omega_2)$ deterministically, we have $\eta_{1,\cdot}(p_1 = 1, p_2 =$*

$\omega_2) = 2 - \frac{1}{\alpha}$ *and* $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) = \frac{1}{\alpha} - 1$.

Proof. Assume there exists an equilibrium in which $N_{1,\cdot}$ mixes between $(p_1 = 1, p_2 = \omega_2)$ and $(p_1 = -1, p_2 = \omega_2)$ and $N_{-1,\cdot}$ plays $(p_1 = 1, p_2 = \omega_2)$ deterministically and assume that $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) > 2 - \frac{1}{\alpha}$. Then, $\mu(p_1 = 1, p_2 = \omega_2) < \pi$ and thus $r^*(p_1 = 1, p_2 = \omega_2) = 0$. Moreover, $\mu(p_1 = -1, p_2 = \omega_2) > \pi$ (and thus $r^*(p_1 = -1, p_2 = \omega_2) = 1$) and $Pr(\omega_1 = -1|p_1 = -1, p_2 = \omega_2) = \frac{\pi(1-\alpha)}{\pi(1-\alpha)+\eta_{1,\cdot}(p_1=-1,p_2=\omega_2)(1-\pi)\alpha} > \pi$ because $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) < \frac{1}{\alpha} - 1$. Hence, $U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2) = q_1 B < (1-q_1)(1+B) = U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2)$ because $q_1 < \frac{1}{B+1}$. But then $N_{1,\cdot}$ wants to deviate to $(p_1 = -1, p_2 = \omega_2)$.

So assume that $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) < 2 - \frac{1}{\alpha}$. Then, $\mu(p_1 = 1, p_2 = \omega_2) > \pi$ and thus $r^*(p_1 = 1, p_2 = \omega_2) = 1$, which implies that $U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2) = B$. If $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) \leq \frac{1}{\alpha} - 1$, then because $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) < 2 - \frac{1}{\alpha}$, we have $\eta_{1,\cdot}(p_1 = \cdot, p_2 \neq \omega_2) > 0$. As $N_{-1,\cdot}$ never plays $p_2 \neq \omega_2$, the Voter holds a referendum to set $p_1 = 1$ if needed upon observing $p_2 \neq \omega_2$. But then, $U_{N_{1,\cdot}}(p_2 \neq \omega_2) = 1 < B$ which implies that $\eta_{1,\cdot}(p_1 = \cdot, p_2 \neq \omega_2) = 0$. So assume that $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) > \frac{1}{\alpha} - 1$. Then, $\mu(p_1 = -1, p_2 = \omega_2) < \pi$ and thus $r^*(p_1 = -1, p_2 = \omega_2) = 0$. But then $U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) \leq 1 - q_1 < B$ and $N_{1,\cdot}$ deviates to $(p_1 = 1, p_2 = \omega_2)$. $\square$

The following two lemmas, together with lemma 7, prove that if $q_1 > \max\{3 - 2B, \frac{1}{B+1}\}$ the pair of strategies and beliefs characterized in proposition A.2 constitute the unique equilibrium in which congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$.

Lemma 15. *If* $q_1 > \max\{3 - 2B, \frac{1}{B+1}\}$*, there does not exist an equilibrium in which* $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) < 1$.

Proof. Assume otherwise, i.e. assume there exists an equilibrium in which $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) < 1$. Then, $\mu(p_1 = \omega_1 = 1, p_2 = \omega_2) > \pi$ and thus $r^*(p_1 = \omega_1 = 1, p_2 = \omega_2) = 1$. Case 1: $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) > 1 - (\frac{1}{\alpha} - 1)\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2)$. Then, $\mu(p_1 = 1, p_2 = \omega_2) < \pi$

and $r^*(p_1 = 1, p_2 = \omega_2) = 0$. But then $U_{N_{-1},\cdot}(p_1 = 1, p_2 = \omega_2) = 1 - q_1 < 1 \leq U_{N_{-1},\cdot}(p_1 = \cdot, p_2 \neq \omega_2)$ and thus $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2) = 0$. But then, $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) > 1$, which is impossible.

Case 2: $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) < 1 - (\frac{1}{\alpha} - 1)\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2)$. Then, $\mu(p_1 = 1, p_2 = \omega_2) > \pi$ and thus $r^*(p_1 = 1, p_2 = \omega_2) = 1$. Hence, $U_{N_1,\cdot}(p_1 = 1, p_2 = \omega_2) = B > (1 - q_1)(1 + B) \geq U_{N_1,\cdot}(p_1 = -1, p_2 = \omega_2)$ because $q_1 > \frac{1}{B+1}$ and thus $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) = 0$. In turn, $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) < 1$ and $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) = 0$ implies that $\eta_{1,\cdot}(p_1 = 1, p_2 \neq \omega_2) > 0$ or $\eta_{1,\cdot}(p_1 = -1, p_2 \neq \omega_2) > 0$ which requires $U_{N_1,\cdot}(p_1 = 1, p_2 \neq \omega_2) = 1 + (1 - q_1)R_{-1} \geq B = U_{N_1,\cdot}(p_1 = 1, p_2 = \omega_2)$ or $U_{N_1,\cdot}(p_1 = -1, p_2 \neq \omega_2) = 1 + (1 - q_1)(1 - R_1) \geq B = U_{N_1,\cdot}(p_1 = 1, p_2 = \omega_2)$. This, in turn implies that $R_{-1} \geq \frac{B-1}{1-q_1} > 0$ or $R_1 \leq 1 - \frac{B-1}{1-q_1} < 1$. This however requires $\eta_{-1,\cdot}(p_1 = 1, p_2 \neq \omega_2) > 0$ or $\eta_{-1,\cdot}(p_1 = -1, p_2 \neq \omega_2) > 0$ respectively because otherwise the Voter infers from $(p_1, p_2 \neq \omega_2)$ that $\omega_1 = 1$ and thus sets $R_{-1} = 0$ or $R_1 = 1$. It follows that $\eta_{-1,\cdot}(p_1 = -1, p_2 = \omega_2) < 1$ which implies that $\mu(p_1 = -1, p_2 = \omega_2) > \pi$ and $\mu(p_1 = \omega_1 = -1, p_2 = \omega_2) > \pi$. Thus, $U_{N_{-1},\cdot}(p_1 = -1, p_2 = \omega_2) = B$. Finally, $\eta_{-1,\cdot}(p_1 = 1, p_2 \neq \omega_2) > 0$ or $\eta_{-1,\cdot}(p_1 = -1, p_2 \neq \omega_2) > 0$ requires that $U_{N_{-1},\cdot}(p_1 = -1, p_2 \neq \omega_2) = 1 + (1 - q_1)R_1 \geq B = U_{N_{-1},\cdot}(p_1 = -1, p_2 = \omega_2)$ or $U_{N_{-1},\cdot}(p_1 = 1, p_2 \neq \omega_2) = 1 + (1 - q_1)(1 - R_{-1}) \geq B = U_{N_{-1},\cdot}(p_1 = -1, p_2 = \omega_2)$ which implies $R_1 \geq \frac{B-1}{1-q_1}$ or $R_{-1} \leq 1 - \frac{B-1}{1-q_1}$. Hence, in equilibrium, we need $1 - \frac{B-1}{1-q_1} \geq \frac{B-1}{1-q_1}$ which is impossible because $q_1 > 3 - 2B$.

Case 3: $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) = 1 - (\frac{1}{\alpha} - 1)\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2)$ and thus $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2) > 0$ because $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) < 1$. Moreover, $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) = 1 - (\frac{1}{\alpha} - 1)\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2)$ implies that $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) \leq \frac{1}{\alpha} - 1$ and thus $Pr(\omega_1 = -1 | p_1 = -1, p_2 = \omega_2) \geq \pi$. It follows that the Voter does not hold a referendum upon observing $(p_1 = -1, p_2 = \omega_2)$. Suppose first that $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) < 1 - \eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2)$. Then, $\mu(p_1 = -1, p_2 = \omega_2) > \pi$, and $\mu(p_1 = \omega_1 = -1, p_2 = \omega_2) > \pi$ and thus $U_{N_{-1},\cdot}(p_1 = -1, p_2 = \omega_2) = B > (1 - q_1)(B + 1) \geq U_{N_{-1},\cdot}(p_1 = 1, p_2 = \omega_2)$

because $q_1 > \frac{1}{B+1}$. But then, $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2) = 0$, contradiction. So suppose that $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) = 1 - \eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2)$ which implies $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) > 0$. It follows that $N_{1,\cdot}$ mixes between $(p_1 = 1, p_2 = \omega_2)$ and $(p_1 = -1, p_2 = \omega_2)$ which requires

$$\begin{aligned} U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) &= (1 - q_1)(1 + r^*(p_1 = -1, p_2 = \omega_2)B) \\ &= (1 - q_1)r^*(p_1 = 1, p_2 = \omega_2)B + q_1 B \\ &= U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2). \end{aligned}$$

As $q_1 > \frac{1}{B+1}$, $q_1 B > 1 - q_1$ and thus we have $r^*(p_1 = -1, p_2 = \omega_2) > r^*(p_1 = 1, p_2 = \omega_2)$. But then,

$$\begin{aligned} U_{N_{-1,\cdot}}(p_1 = -1, p_2 = \omega_2) &= (1 - q_1)r^*(p_1 = -1, p_2 = \omega_2)B + q_1 B \\ &> (1 - q_1)(1 + r^*(p_1 = 1, p_2 = \omega_2)B) \\ &= U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2) \end{aligned}$$

and thus $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2) = 0$. Contradiction. □

Lemma 16. *If $q_1 > \max\{3 - 2B, \frac{1}{B+1}\}$ and $N_{1,\cdot}$ plays $(p_1 = \omega_1, p_2 = \omega_2)$ deterministically, then so does $N_{-1,\cdot}$.*

Proof. Assume otherwise, i.e. assume $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) = 1$ and $\eta_{-1,\cdot}(p_1 = -1, p_2 = \omega_2) < 1$. Then, $\mu(p_1 = -1, p_2 = \omega_2) > \pi$ and $\mu(p_1 = \omega_1 = -1, p_2 = \omega_2) > \pi$ and thus $r^*(p_1 = -1, p_2 = \omega_2) = 1$ and $r^*(p_1 = \omega_1 = -1, p_2 = \omega_2) = 1$. But then, $U_{N_{-1,\cdot}}(p_1 = -1, p_2 = \omega_2) = B > (1 - q_1)(B + 1) \geq U_{N_{-1,\cdot}}(p_1 = 1, p_2 = \omega_2)$ because $q_1 > \frac{1}{B+1}$. Moreover, as $\eta_{1,\cdot}(p_1 = 1, p_2 = \omega_2) = 1$, if $\eta_{-1,\cdot}(p_2 \neq \omega_2) > 0$, the Voter infers that $\omega_1 = -1$ from $p_2 \neq \omega_2$ and sets $p_1 = -1$ via referendum whenever needed. Hence, $U_{N_{-1,\cdot}}(p_2 \neq \omega_2) = 1 < B$ and $N_{-1,\cdot}$ wants to deviate to $(p_1 = -1, p_2 = \omega_2)$. □

Lemma 17. *If $B \geq 1$ and $q_1 \in \left(\frac{2B-2}{B}, 3 - 2B\right)$, there does not exist an equilibrium s.t. non-congruent types never choose $p_2 \neq \omega_2$.*

Proof. Assume otherwise, i.e. assume there exists an equilibrium s.t. non-congruent types choose $p_2 = \omega_2$ deterministically. Case 1: $3 - 2B \geq q_1 \geq \frac{1}{B+1} \geq \frac{2B-2}{B}$. Then, the highest possible payoff that non-congruent incumbents can achieve when choosing $p_2 = \omega_2$ is B. In order for non-congruent types not to want to deviate to $(p_1 = 1, p_2 \neq \omega_2)$ we need $B \geq 1 + (1-q_1)R_{-1} = U_{N_{1,\cdot}}(p_1 = 1, p_2 \neq \omega_2)$ and $B \geq 1 + (1-q_1)(1-R_{-1}) = U_{N_{-1,\cdot}}(p_1 = 1, p_2 \neq \omega_2)$, which implies that $R_{-1} \leq \frac{B-1}{1-q_1}$ and $R_{-1} \geq 1 - \frac{B-1}{1-q_1}$. But $\frac{B-1}{1-q_1} \geq 1 - \frac{B-1}{1-q_1}$ implies $q_1 \geq 3 - 2B$.

Case 2: $q_1 \in \left(\frac{2B-2}{B}, \frac{1}{B+1}\right)$. Note first that if $q_1 < \frac{1}{B+1}$, the maximal payoff that $N_{1,\cdot}$ can achieve, in an equilibrium where non-congruent incumbents never choose $p_2 \neq \omega_2$, is B. To see this, assume there exists an equilibrium such that $U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) > B$. As non-congruent types never choose $p_2 \neq \omega_2$, we have $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) = 1$ which implies that $\mu(p_1 = -1, p_2 = \omega_2) < \pi$ and thus $r^*(p_1 = -1, p_2 = \omega_2) = 0$. But then, $U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) = 1 - q_1 < 1 \leq U_{N_{1,\cdot}}(p_2 \neq \omega_2)$. Contradiction. So assume there exists an equilibrium such that non-congruent Incumbents never choose $p_2 \neq \omega_2$. As the highest payoff that $N_{1,\cdot}$ can achieve is B and the highest payoff that $N_{-1,\cdot}$ can achieve is $(1-q_1)(B+1)$, in equilibrium, it has to be the case that $B \geq 1 + (1-q_1)(1-R_1) = U_{N_{1,\cdot}}(p_1 = -1, p_2 \neq \omega_2)$ and $(1-q_1)(B+1) \geq 1 + (1-q_1)R_1 = U_{N_{-1,\cdot}}(p_1 = -1, p_2 \neq \omega_2)$. This implies that $R_1 \geq 1 - \frac{B-1}{1-q_1}$ and $R_1 \leq B + 1 - \frac{1}{1-q_1}$ which is impossible as $q_1 > \frac{2B-2}{B}$. □

Lemma 18. *Suppose $B > 1$. Then, there does not exist an equilibrium such that $N_{1,\cdot}$ (or $N_{-1,\cdot}$) chooses $(p_1 = 1, p_2 \neq \omega_2)$ (or $(p_1 = -1, p_2 \neq \omega_2)$) yet $N_{-1,\cdot}$ ($N_{1,\cdot}$) does not.*

Proof. WLOG, assume there exists an equilibrium such that $N_{1,\cdot}$ chooses $(p_1 = 1, p_2 \neq \omega_2)$ with positive probability yet $N_{-1,\cdot}$ does not. Then, $Pr(\omega_1 = 1 | p_1 = 1, p_2 \neq \omega_2) = 1$ and the Voter does not hold a referendum upon observing $(p_1 = 1, p_2 \neq \omega_2)$. Hence, $U_{N_{1,\cdot}}(p_1 = 1, p_2 \neq \omega_2) = 1$. Moreover, as $N_{1,\cdot}$ does not play $p_2 = \omega_2$ deterministically, we have $\mu(p_1 = 1, p_2 = \omega_2) > \pi$ or $\mu(p_1 = -1, p_2 = \omega_2) > \pi$. In the first case, $U_{N_{1,\cdot}}(p_1 = 1, p_2 = \omega_2) = B > 1$ and $N_{1,\cdot}$ wants to deviate from $(p_1 = 1, p_2 \neq \omega_2)$. In the second case, we have $\eta_{-1,\cdot}(p_1 = -1, p_2 =$

$\omega_2) < 1$ and thus $\mu(p_1 = \omega_1 = -1, p_2 = \omega_2) > \pi$. Hence, $U_{N_{-1,\cdot}}(p_1 = -1, p_2 = \omega_2) = B$. If $q_1 > \frac{1}{B+1}$, then $B > (1-q_1)(B+1)$ and thus $\eta_{-1,\cdot}(p_1 = 1, p_2 = \omega_2) = 0$. But then $\mu(p_1 = 1, p_2 = \omega_2) > \pi$ and $N_{1,\cdot}$ deviates to $(p_1 = 1, p_2 = \omega_2)$. Moreover, $\mu(p_1 = -1, p_2 = \omega_2) > \pi$ implies $\eta_{1,\cdot}(p_1 = -1, p_2 = \omega_2) < \frac{1}{\alpha} - 1$ and thus $Pr(\omega_1 = -1 | p_1 = -1, p_2 = \omega_2) > 1/2$. Hence, if $q_1 \leq \frac{1}{B+1}$, then $U_{N_{1,\cdot}}(p_1 = -1, p_2 = \omega_2) = (1-q_1)(B+1) \geq B > 1$. But then, $N_{1,\cdot}$ deviates to $(p_1 = -1, p_2 = \omega_2)$. □

Lemma 19. *There does not exist an equilibrium such that N chooses $(p_1 \neq \omega_1, p_2 \neq \omega_2)$ deterministically.*

Proof. Assume otherwise. The Voter then infers upon observing $(p_1, p_2 \neq \omega_2)$ that $p_1 \neq \omega_1$ and therefore holds a referendum to overturn the policy decision p_1 made by the Incumbent. Consequently, the utility to N of choosing $(p_1 \neq \omega_1, p_2 \neq \omega_2)$ is 1. Deviating to $(p_1 = \omega_1, p_2 \neq \omega_2)$ in turn yields $1 + (1 - q_1) > 1$. □

Lemma 20. *If $B < 1$ and $q_1 < \frac{2B-1}{1+2B}$ there is no equilibrium in which N chooses $p_2 \neq \omega_2$ deterministically.*

Proof. Suppose N chooses $p_2 \neq \omega_2$ deterministically. Then, the Voter reelects upon observing $(p_1 = \omega_1, p_2 = \omega_2)$, $(p_1 = 1, p_2 = \omega_2)$, and $(p_1 = -1, p_2 = \omega_2)$. Moreover, by lemmata 7 and 9 the Voter does not hold a referendum upon observing $p_2 = \omega_2$. Consequently, choosing $(p_1 \neq \omega_1, p_2 = \omega_2)$ yields a policy payoff of $(1-q_1)(1+B)$ to the non-congruent Incumbent. WLOG suppose that $N_{1,\cdot}$ and $N_{-1,\cdot}$ choose $(p_1 = 1, p_2 \neq \omega_2)$ with positive probability in equilibrium. For this to be a best-response, it must be the case that

$$U_{N_{-1,\cdot}}(p_1 = 1, p_2 \neq \omega_2) = 1 + (1-q_1)(1 - R_{-1}) \geq (1-q_1)(1+B), \text{ and}$$
$$U_{N_{1,\cdot}}(p_1 = 1, p_2 \neq \omega_2) = 1 + (1-q_1)R_{-1} \geq (1-q_1)(1+B),$$

which is impossible because $q_1 < \frac{2B-1}{1+2B}$. □

Lemma 21. *The pair of strategies and beliefs characterized in proposition A.2 are the only equilibria to satisfy criterion $D1$.*

Proof. Assume there exists an equilibrium in which $p_2 = \omega_2$ is not played with positive probability for some $\omega_2 \in \{-1, 1\}$. Fix a vector of reelection probabilities $\mathbf{r}$ and probability R that the Voter holds a referendum to change p_1 when ω_1 is not revealed to her, then $U_C(p_1 = \cdot, p_2 = \omega_2, \mathbf{r}, R) > U_N(p_1 = \cdot, p_2 = \omega_2, \mathbf{r}, R)$. Moreover, for any policy vector $\mathbf{p}$ such that $p_2 \neq \omega_2$ and any vector of equilibrium reelection and referendum probabilities $(\mathbf{r}^*, R^*)$ we have $U_C(\mathbf{p}, \mathbf{r}^*, R^*) \leq U_N(\mathbf{p}, \mathbf{r}^*, R^*)$. In other words, in any equilibrium in which $p_2 = \omega_2$ is not played with positive probability the equilibrium payoff of any non-congruent type is at least as high as the equilibrium payoff of any congruent type. It follows that $D^1(N, T, p_1, p_2 = \omega_2) \subset D(C, T, p_1, p_2 = \omega_2)$ and the Voter should believe she is facing a congruent Incumbent upon observing $p_2 = \omega_2$. But then, if $q_1 > 0$ the congruent Incumbent wants to deviate to $p_2 = \omega_2$ as he then receives a payoff of at least $q_1 + 1 + B$ which is greater then $1 + B$, the highest payoff the congruent type can get in any equilibrium in which he does not play $p_2 = \omega_2$. Hence, to satisfy $D1$ it must be the case that C chooses $p_2 = \omega_2$ for all ω_2 in equilibrium. □

Proposition A.3. *The following pairs of strategies and beliefs constitute the equilibrium of the direct democracy model when $q_1 = 1$ and $q_2 \in [0, 1]$.*

1. *If $B < 1$ then congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$ for all $\boldsymbol{\omega}$, while non-congruent Incumbents choose $(p_1 = \omega_1, p_2 \neq \omega_2)$ for all $\boldsymbol{\omega}$.*

   The Voter holds a referendum to set $p_1 = \omega_1$ when $p_1 \neq \omega_1$.

   The Voter reelects upon observing $(p_1 = \omega_1, p_2 = \omega_2)$, or $(p_1 = \omega_1, p_2 = 1)$ and does not reelect otherwise.

   The Voter's beliefs satisfy $\mu(p_1 = \omega_1, p_2 = \omega_2) = 1$, $\mu(p_1 = \omega_1, p_2 = 1) > \pi$, $\mu(p_1 = \omega_1, p_2 = -1) < \pi$, $\mu(p_1 = \omega_1, p_2 \neq \omega_2) = 0$. Out-of-equilibrium beliefs satisfy $\mu(p_1 \neq$

$\omega_1, p_2 = \cdot) \leq \pi$.

2. *If $B \geq 1$ and $q_2 \geq 1/B$ then congruent and non-congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$ for all $\boldsymbol{\omega}$.*

   The Voter holds a referendum to set $p_1 = \omega_1$ when $p_1 \neq \omega_1$.

   The Voter reelects upon observing $(p_1 = \omega_1, p_2 = \omega_2)$, $(p_1 = \omega_1, p_2 = 1)$, and $(p_1 = \omega_1, p_2 = -1)$, and does not reelect otherwise.

   The Voter's beliefs satisfy $\mu(p_1 = \omega_1, p_2 = \omega_2) = \mu(p_1 = 1, p_2 = \omega_2) = \mu(p_1 = -1, p_2 = \omega_2) = \pi$. Out-of-equilibrium beliefs satisfy $\mu(p_1 = \omega_1, p_2 \neq \omega_2), \mu(p_1 \neq \omega_1, p_2 = \cdot) \leq \pi$.

3. *If $B \geq 1$ and $q_2 < 1/B$, then congruent Incumbents choose $(p_1 = \omega_1, p_2 = \omega_2)$,*

   The non-congruent Incumbent who observes $\omega_2 = -1$ chooses $(p_1 = \omega_1, p_2 = 1)$;

   The non-congruent Incumbent who observes $\omega_2 = 1$ chooses $(p_1 = \omega_1, p_2 = \omega_2)$ with probability $2 - \frac{1}{\alpha}$ and $(p_1 = \omega_1, p_2 \neq \omega_2)$ with probability $\frac{1}{\alpha} - 1$;

   The Voter holds a referendum to set $p_1 = \omega_1$ if $p_1 \neq \omega_1$.

   The Voter's reelection strategy satisfies: $r^*(p_1 = \omega_1, p_2 = \omega_2) = 1$, $r^*(p_1 = \omega_1, p_2 = 1) = \frac{1}{(1-q_2)B} - \frac{q_2}{1-q_2} + r^*(p_1 = \omega_1, p_2 = -1)$ and $r^*(p_1 \neq \omega_1, p_2 = \cdot) = r^*(p_1 = \cdot, p_2 \neq \omega_2) = 0$.

   The Voter's beliefs satisfy $\mu(p_1 = 1, p_2 = \omega_2) = \mu(p_1 = -1, p_2 = \omega_2) = \pi$, $\mu(p_1 = \omega_1 = 1, p_2 = \omega_2) > \pi$, $\mu(p_1 = \omega_1 = -1, p_2 = \omega_2) = 1$, $\mu(p_1 \neq \omega_1, p_2 = \omega_2) = 0$. Out-of-equilibrium beliefs satisfy $\mu(\cdot, p_2 \neq \omega_2) \leq \pi$.

Proof. 1. Assume $B < 1$. Given the specified strategy for the Incumbent, Bayesian updating yields $\mu(p_1 = \omega_1, p_2 = \omega_2) = 1$, $\mu(p_1 = \omega_1, p_2 = 1) = \frac{\alpha\pi}{\alpha\pi + (1-\alpha)(1-\pi)} > \pi$ because $\alpha > 1/2$, $\mu(p_1 = \omega_1, p_2 = -1) = \frac{(1-\alpha)\pi}{(1-\alpha)\pi + \alpha(1-\pi)} < \pi$, because $\alpha > 1/2$, and $\mu(p_1 = \omega_1, p_2 \neq \omega_2) = 0$. Hence, if we let out-of-equilibrium beliefs satisfy $\mu(p_1 \neq \omega_1, p_2 = \cdot) \leq \pi$ the Voter has no incentive to deviate from his reelection strategy.

Given the retention behavior of the Voter, and because $B < 1$, we have

$$
\begin{aligned}
U_{C\cdot,-1}(p_1 = \omega_1, p_2 = \omega_2) &= 2 + q_2 B \\
&\geq \begin{cases} 1 + (1 - q_2)B \\ 2 \end{cases}, \\
&\geq \begin{cases} U_{C\cdot,-1}(p_1 = \omega_1, p_2 \neq \omega_2) \\ U_{C\cdot,-1}(p_1 \neq \omega_1, p_2 = \cdot) \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
U_{N\cdot,-1}(p_1 = \omega_1, p_2 \neq \omega_2) &= 1 + (1 - q_2)B \\
&\geq \begin{cases} q_2 B \\ 1 \end{cases}, \\
&\geq \begin{cases} U_{N\cdot,-1}(p_1 = \omega_1, p_2 = \omega_2) \\ U_{N\cdot,-1}(p_1 \neq \omega_1, p_2 = \cdot) \end{cases}
\end{aligned}
$$

and

$$
\begin{aligned}
U_{N\cdot,1}(p_1 = \omega_1, p_2 \neq \omega_2) &= 1 \\
&\geq \begin{cases} B \\ 1 \end{cases}. \\
&\geq \begin{cases} U_{N\cdot,1}(p_1 = \omega_1, p_2 = \omega_2) \\ U_{N\cdot,1}(p_1 \neq \omega_1, p_2 = \cdot) \end{cases}
\end{aligned}
$$

Finally, we have $U_{C\cdot,1}(p_1 = \omega_1, p_2 = \omega_2) = 2 + B$ which is the highest possible payoff in the game and thus $C_{\cdot,1}$ has no incentive to deviate.

2. Assume $B \geq 1$ and $q_2 \geq 1/B$. Given the specified strategy for the Incumbent, Bayesian updating yields $\mu(p_1 = \omega_1, p_2 = \omega_2) = \mu(p_1 = \omega_1, p_2 = 1) = \mu(p_1 = \omega_1, p_2 = -1) = \pi$. Hence, if we let out-of-equilibrium beliefs satisfy $\mu(p_1 \neq \omega_1, p_2 = \cdot), \mu(p_1 = \omega_1, p_2 \neq \omega_2) \leq \pi$ the Voter has no incentive to deviate from his reelection strategy. Given the retention behavior of the Voter, we have $U_C(p_1 = \omega_1, p_2 = \omega_2) = 2 + B$ which is the highest possible payoff in the game and thus C has no incentive to deviate. Similarly, we have $U_N(p_1 = \omega_1, p_2 = \omega_2) = B$, while $U_N(p_1 = \omega_1, p_2 \neq \omega_2) = 1 + (1 - q_2)B$, and $U_N(p_1 \neq \omega_1, p_2 = \cdot) \leq 1$. Because, $B \geq 1$ and $q_2 \geq 1/B$, N has no incentive to deviate.

3. Assume $B \geq 1$ and $q_2 < 1/B$. Given the specified strategy for the Incumbent, Bayesian updating yields $\mu(p_1 = \omega_1, p_2 = \omega_2 = 1) = \frac{\pi}{\pi+\alpha(2-1/\alpha)(1-\pi)} > \pi$ because $\alpha > 1/2$, $\mu(p_1 = \omega_1, p_2 = \omega_2 = -1) = 1$, $\mu(p_1 = \omega_1, p_2 = 1) = \frac{\alpha\pi}{\alpha\pi+(1-\alpha+\alpha(2-1/\alpha))(1-\pi)} = \pi$, $\mu(p_1 = \omega_1, p_2 = -1) = \frac{(1-\alpha)\pi}{(1-\alpha)\pi+\alpha(1/\alpha-1)(1-\pi)} = \pi$, $\mu(p_1 = \omega_1, p_2 \neq \omega_2) = 0$. Hence, if we let out-of-equilibrium beliefs satisfy $\mu(p_1 \neq \omega_1, p_2 = \cdot) \leq \pi$ the Voter has no incentive to deviate from his reelection strategy.

   Given the retention behavior of the Voter, we have $U_{N_{\cdot,1}}(p_1 = \omega_1, p_2 = 1) = (1 - q_2)r^*(p_1 = \omega_1, p_2 = 1)B + q_2 B$ and $U_{N_{\cdot,1}}(p_1 = \omega_1, p_2 = -1) = 1 + (1-q_2)r^*(p_1 = \omega_1, p_2 = -1)B$. In equilibrium, $N_{\cdot,1}$ is mixing between $(p_1 = \omega_1, p_2 = 1)$ and $(p_1 = \omega_1, p_2 = -1)$ and hence is indifferent between these two policy vectors. Indifference is satisfied if, and only if, $r^*(p_1 = \omega_1, p_2 = 1) = \frac{1}{(1-q_2)B} - \frac{q_2}{1-q_2} + r^*(p_1 = \omega_1, p_2 = -1)$. Because $q_2 < \frac{1}{B}$, this implies $1 \geq r^*(p_1 = \omega_1, p_2 = 1) > r^*(p_1 = \omega_1, p_2 = -1)$. Moreover, we have $U_{N_{\cdot,1}}(p_1 \neq \omega_1\cdot, p_2 = \cdot) \leq 1 \leq 1 + (1 - q_2)r^*(p_1 = \omega_1, p_2 = -1)B = U_{N_{\cdot,1}}(p_1 = \omega_1, p_2 = -1)$. Hence, $N_{\cdot,1}$ has no incentive to deviate.

   To see that $N_{\cdot,-1}$ has no incentive to deviate, note that, because $r^*(p_1 = \omega_1, p_2 = 1) > r^*(p_1 = \omega_1, p_2 = -1) \geq 0$ and because $q_2 < \frac{1}{B}$, we have $U_{N_{\cdot,-1}}(p_1 = \omega_1, p_2 = 1) = 1 + (1 - q_2)r^*(p_1 = \omega_1, p_2 = 1)B > 1 \geq U_{N_{\cdot,-1}}(p_1 \neq \omega_1, p_2 = \cdot)$ and $U_{N_{\cdot,-1}}(p_1 = \omega_1, p_2 =$

$1) > (1-q_2)r^*(p_1 = \omega_1, p_2 = -1)B + q_2 B = U_{N.,-1}(p_1 = \omega_1, p_2 = -1)$.

Moreover, we have $U_{C.,-1}(p_1 = \omega_1, p_2 = -1) = 2 + q_2 B + (1-q_2)r^*(p_1 = \omega_1, p_2 = -1)B > 1 + (1-q_2)r^*(p_1 = \omega_1, p_2 = 1)B = U_{C.,-1}(p_1 = \omega_1, p_2 = 1)$ and $U_{C.,1}(p_1 = \omega_1, p_2 = 1) = 2 + (1-q_2)r^*(p_1 = \omega_1, p_2 = 1)B + q_2 B > 1 + (1-q_2)r^*(p_1 = \omega_1, p_2 = -1)B = U_{C.,-1}(p_1 = \omega_1, p_2 = -1)$ because $r^*(p_1 = \omega_1, p_2 = 1) = \frac{1}{(1-q_2)B} - \frac{q_2}{1-q_2} + r^*(p_1 = \omega_1, p_2 = -1)$. Because deviating to $p_2 \neq \omega_2$ yields at most a payoff of $1 < 2$, congruent Incumbents have no incentive to deviate.

□

Appendix B - Strong office-holding motive $B \geq 2$

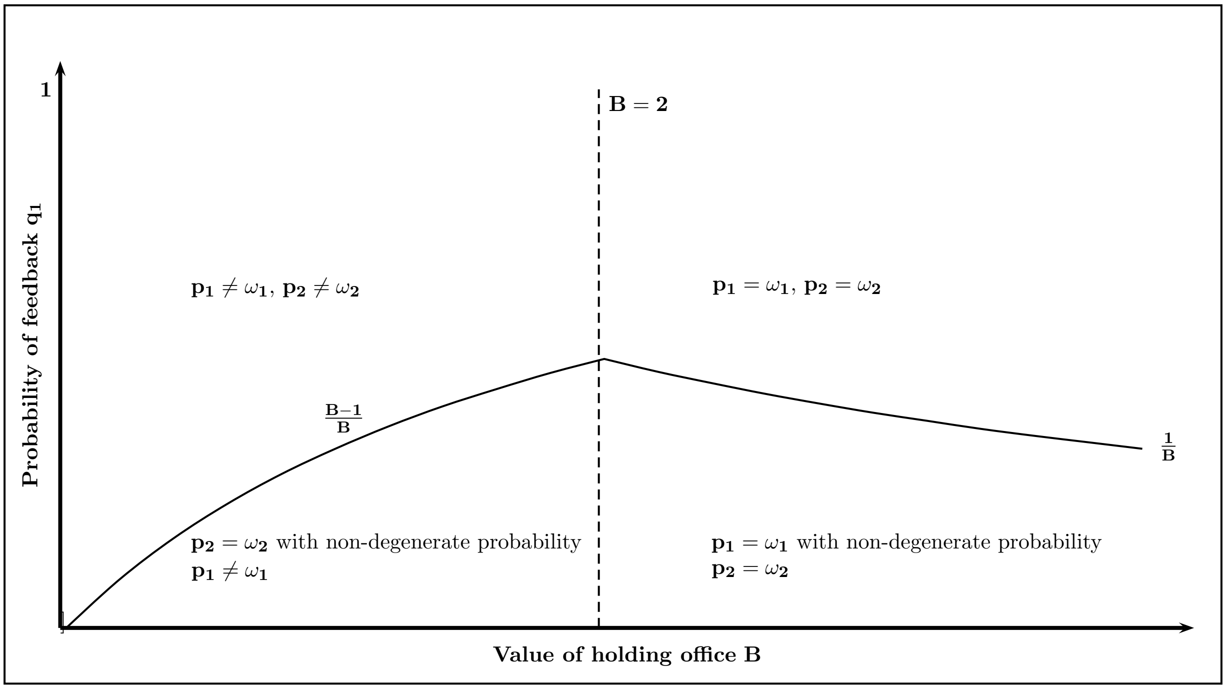
Appendix A provides the equilibrium of the baseline model as well as the equilibria of the direct democracy game for all values of holding office B. In this section I draw out the implications of a strong office-holding motive, i.e. when $B \geq 2$. Figure 1 illustrates the equilibrium of the baseline model in this case. As before, the equilibrium depends on the probability of feedback q_1 and the value of holding office B.

Proposition A.4. *Assume the value of holding office is high ($B \geq 2$). In equilibrium:*

1. *if the probability of feedback is high ($q_1 \geq \frac{1}{B}$), non-congruent incumbents choose the same policies as congruent ones ($p_1 = \omega_1, p_2 = \omega_2$),*

2. *if the probability of feedback is low ($q_1 < \frac{1}{B}$), non-congruent Incumbents choose $p_1 = \omega_1$ with non-degenerate probability and $p_2 = \omega_2$ with certainty.*

Figure 1: Equilibrium Behavior of non-congruent Incumbent in Baseline Model



Proposition A.5. *1. Increasing the value of holding office from $B < 2$ to $B \geq 2$ has a similar effect on the decision making of public officials as the introduction of the popular referendum.*

1. *When $B \geq 2$ and $q_1 \in \left(\frac{1}{B+1}, \frac{1}{B}\right)$, the introduction of the popular referendum strictly improves congruence with respect to p_1.*

Interestingly, increasing the value of holding office has a similar effect as the introduction of the popular referendum. First of all, the congruence with respect to policies p_1 and p_2 in the baseline model improves when $B \geq 2$ compared to a situation in which $B < 2$. Similarly, the control the Voter exerts over elected representatives increases with the level of information available to the Voter when $B \geq 2$. This is interesting in so far as it clarifies the mechanism through which the introduction of the referendum affects the decision-making of public officials. Indeed, by constraining the policy options of the Incumbent, the popular referendum essentially increases the value of retaining office, if only in relative terms.

Remark also that the introduction of the popular referendum still improves congruence even when Incumbents have a strong office-holding motive. In particular, when $q_1 \in \left[\frac{1}{B+1}, \frac{1}{B}\right]$, non-congruent Incumbents always choose $p_1 = \omega_1$ once the possibility of a popular referendum exists, whereas they choose $p_1 = \omega_1$ with non-degenerate probability in the absence of direct democracy. This stems from the fact that the costs a non-congruent Incumbent incurs when $p_1 \neq \omega_1$ is revealed to the Voter increase with the introduction of direct democracy, because the Voter now holds a referendum to set $p_1 = \omega_1$. As a result the Incumbent then not only loses reelection but also the policy benefit associated with $p_1 \neq \omega_1$. The probability of feedback above which the non-congruent Incumbent prefers to play $p_1 = \omega_1$ with certainty is thus lower under direct democracy than under representative democracy.

Appendix C - Semi-congruent Types

In the model, an Incumbent is either congruent or non-congruent. Note, however, that in principle an Incumbent may be congruent with respect to some policy dimensions and not others. In this section, I provide an argument for why including semi-congruent types does not alter the logic of the model. Assume that π now represents the probability that the incumbent is congruent with respect to policy dimension p_i. In such a setting, the incumbent is congruent with respect to both policies with probability π^2, congruent with respect to policy p_1 but not congruent with respect to policy p_2 with probability $\pi(1-\pi)$ and so forth. Suppose we make the following assumption.[1]

Assumption A.6. *If the Voter observes a policy vector which is never chosen by a congruent Incumbent in equilibrium, she does not reelect.*

[1]Note that this assumption can be easily made a proposition in a generalized version of the model with a second policy-making period after the election.

An implication of this assumption is that a non-congruent Incumbent, in order to get reelected, needs to enact a policy vector that is also enacted by a congruent type, even when semi-congruent types exist. In particular, this implies that a non-congruent Incumbent needs to choose $p_2 = \omega_2$ in order to get reelected. Similarly, when the probability of feedback q_1 is sufficiently high the non-congruent Incumbent essentially needs to choose $p_1 = \omega_1$ in order to get reelected with a substantial probability. Correspondingly, the incentives for non-congruent types to separate in the baseline model and to pool in the model with the popular referendum remain similar.

References

Cho, In-Koo and David M. Kreps. 1987. "Signaling Games and Stable Equilibria." *Quarterly Journal of Economics* 102(2):179–221.