Supplemental Appendix for:

Boehmke, Frederick J.; Douglas Dion; and Charles R. Shipan. "A Duration Estimator for a Continuous Time War of Attrition Game." *Political Science Research and Methods*.

1 Applications of War of Attrition Models

Here we provide a list of examples of applications of the war of attrition model within political science and other fields. We provide full citations in the references section of this appendix.

1. Deterrence and bargaining (Kilgore and Zagare 1991; Fearon 1994, 1995; Tingley and Wang 2010).
2. Terrorism (Sànchez-Cuenca 2007).
3. Cabinets and coalitions in parliamentary democracies (Carmignani 2001; Padovano and Venturi 2001).
4. Budgetary negotiations (Klarner, Phillips and Muckler 2012; Andersen, Lassen and Nielsen 2012) .
5. Public goods provision (Bliss and Nalebuff 1984).
6. Vote buying (Dekel, Jackson, and Wolinsky 2008).
7. Jury deliberations (Meyer-ter-Vehn, Smith and Bognar 2018).
8. The drop in world oil prices in 1985 (Alt, Calvert, and Humes 1988).
9. The conflict between industry and environmentalists (Burton 2004).
10. The duration of labor strikes strikes (Kennan and Wilson 1989; Geraghty and Wiseman 2008).
11. Exit from a declining industry (Fudenberg and Tirole 1986; Ghemawat and Nalebuff 1985; Oprea, Wilson and Zillante 2013).
12. Economic reform (Alesina and Drazen 1991; Drazen and Grilli 1993).
13. The movie industry (Takahashi 2015).
14. Animal conflicts (Parker and Thompson 1980; Lorenzi, Araguas, Picchi and Ricci-Bonot 2019).

2 War of Attrition Likelihood with Right Censoring

Here we extend our estimator to account for right censoring. In the version presented in the paper's main text we assume that all failure times are observed and used c_i to denote whether an observation fails before the endogenous time horizon, t^*. Here we use the indicator r_i to indicate whether an observation is right censored in the sense that we do not observe its failure time. We assume t_i indicates the right-censoring point for such observations (rather than the failure time). Combining these terms leads to the following likelihood function.

$$\begin{aligned} L(\beta, \alpha, \pi_i, p_{SS}, p_W, t^*|t, X) \quad = \quad & \prod_{i=1}^{n} \left[\pi_i f_{wbl}(t_i|\lambda^{SS}) + (1-\pi_i) f_{twbl}(t_i|\lambda_i^W)\right]^{1-r_i} \\ & + \left[\pi_i S_{wbl}(t_i|\lambda^{SS}) + (1-\pi_i) S_{twbl}(t_i|\lambda_i^W)\right]^{r_i}. \end{aligned} \tag{1}$$

We then substitute the specifics of the probabilities and densities:

$$\begin{aligned} & L(\beta, \alpha, \pi, p_{SS}, p_W, t^*|t, X) \\ & \quad = \prod_{i=1}^{n} \Bigg(\pi_i \left[p_{SS} (\lambda_i^{SS})^{p_{SS}} (t_i)^{p_{SS}-1} \exp\left(-(\lambda_i^{SS} t_i)^{p_{SS}}\right)\right] && (2) \\ & \quad +(1-\pi_i) c_i \left[\frac{p_W (\lambda_i^W)^{p_W} (t_i)^{p_W - 1} \exp\left(-(\lambda_i^W t_i)^{p_W}\right)}{1 - \exp\left(-(\lambda_i^W t^*)^{p_W}\right)} \right] \Bigg)^{1-r_i} && (3) \\ & \quad + \Bigg(\pi_i \left[\exp\left(-(\lambda_i^{SS} t_i)^{p_{SS}}\right)\right] + (1-\pi_i) c_i \left[\frac{\exp\left(-(\lambda_i^W y_i)^{p_W}\right)}{1 - \exp\left(-(\lambda_i^W t^*)^{p_W}\right)} \right] \Bigg)^{1-r_i} && (4) \end{aligned}$$

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