A Comparison with Majority Rule for Case Hearings

Before proving our main results, we compare Rule of Four to case selection via majority rule, where the median justice is the sole hearing pivot.

Proposition 3. *The court hears a smaller set of cases under majority rule than under Rule of Four*

Proof. First, dispositional votes do not depend on the case selection rule. Second, judges use weakly undominated voting strategies for case selection. Thus, case selection strategies does not depend on the rule in the equilibria we study. Increasing the quota requirement for case selection therefore cannot expand the set of heard cases. ■

Changing the Rule of Four to majority rule would not change the final rulings of heard cases, which are determined collectively by the dispositional majority regardless. But the court would be less willing to hear moderate cases. Whether this is improves social welfare depends on how closely the median justice aligns with citizen preferences.

B Proofs

Recall that x_i^* denotes the final ruling's location if $d_j = 0$ for all $j \leq i$ and $d_j = 1$ for all $j > i$.

Lemma B.1. *If dispositional motivation is sufficiently strong, then the dispositional voting subgame has a PSNE with monotonic dispositional majorities in which* $d_i = 1$ *if and only if* $x_{sq} \leq \check{x}_i$*, where:* $\hat{x}_i \leq \check{x}_i$ *for all* $i < \frac{n+1}{2}$*, and* $\hat{x}_i \geq \check{x}_i$ *for all* $i > \frac{n+1}{2}$.

Proof. First, we define $\check{x}_i$ for $i \leq \frac{n+1}{2}$. Let x_i' be the unique $x_{sq} \geq \hat{x}_i$ solving $u_i(x_i^*) = u_i(x_{i-1}^*) + l(x_{sq} - \hat{x}_i)$, and similarly let x_i'' be the unique $x_{sq} \geq \hat{x}_i$ solving $u_i(x_i^*) = u_i(x_{sq}) + l(x_{sq} - \hat{x}_i)$. Note that $x_i' = x_i''$ if $x_i' = x_{i-1}^*$, and $x_i'' \leq x_{i-1}^*$ if $x_i' \leq x_{i-1}^*$. Define

$$\check{x}_i = \begin{cases} x_i' & \text{if } x_i' \leq x_{i-1}^* \\ x_i'' & \text{else.} \end{cases} \tag{2}$$

For $i > \frac{n+1}{2}$, define $\check{x}_i$ in a symmetric way. If dispositional motivation is sufficiently strong, i.e., l decreases fast enough as x_{sq} shifts away from $\hat{x}_i$, then: $\check{x}_i \in [\hat{x}_i, \hat{x}_{i+1})$ for $i < \frac{n+1}{2}$, $\check{x}_i \in (\hat{x}_{i-1}, \hat{x}_i]$ for $i > \frac{n+1}{2}$, and $\check{x}_{\frac{n+1}{2}} < \check{x}_{P_R}$. Thus, $\check{x}_i \leq \check{x}_{i+1}$ for all i.

Construct a profile of dispositional voting strategies such that $d_i = 1$ if and only if $x_{sq} < \check{x}_i$. In this profile, dispositional majorities are always monotonic because $\check{x}_i \leq \check{x}_{i+1}$ for all i.

It remains to check that no justice has a profitable deviation. Consider justice $j \leq \frac{n+1}{2}$, as symmetric arguments apply to $j > \frac{n+1}{2}$. First, suppose $x_{sq} \leq \hat{x}_j$. Voting 0 is not a profitable deviation because $d_j = 1$ matches j's dispositional motivation and also includes her in the dispositional majority, which shifts the ruling towards $\hat{x}_j$ by Property 2. Second, suppose $x_{sq} \in (\hat{x}_j, \min\{x^*_{j-1}, \hat{x}_{j+1}\})$. Then j strictly prefers voting 0 iff $u_j(x^*_j) > u_j(x^*_{j-1}) + l(x_{sq} - \hat{x}_j)$, which is equivalent to $x_{sq} > x'_j$. Next, suppose $x_{sq} \in (\max\{\hat{x}_j, x^*_{j-1}\}, \hat{x}_{j+1})$. Then j strictly prefers voting 0 iff $u_j(x^*_j) > u_j(x_{sq}) + l(x_{sq} - \hat{x}_j)$, equivalently $x_{sq} > x''_j$. By properties of x''_j and x'_j, the previous two cases imply that j strictly prefers voting 0 for $x_{sq} \in (\hat{x}_j, \hat{x}_{j+1})$ iff $x_{sq} > \check{x}_j$. Finally, suppose $x_{sq} > \hat{x}_{j+1}$. If $l(\hat{x}_{j+1} - \hat{x}_j)$ is sufficiently negative, then voting 1 is not a profitable deviation. ■

Henceforth, we assume dispositional motivation is strong enough to apply Lemma B.1.

Proof of Lemma 1. In the PSNE of the dispositional voting subgame characterized in Lemma B.1, the final ruling is worse than x_{sq} for each minority justice. Thus, it is never strictly optimal for any justice to support hearing a case if she will be in the dispositional minority, as she would incur cost $c \geq 0$ to get an inferior final ruling. ■

Proof of Lemma 2. Set $\check{x}_0 = -\infty$ and fix $i \in \{0, 1, \ldots, \frac{n-1}{2}\}$. Consider $x_{sq} \in (\check{x}_i, \check{x}_{i+1}]$. If $x_{sq} \in [x^*_i, \check{x}_{i+1})$, then the case is heard iff $U_{P_R}(x_{sq}) - c \leq u_{P_R}(x_{sq})$. Otherwise, the case is heard iff $U_{P_R}(x^*_i) - c \geq u_{P_R}(x_{sq})$, which is equivalent to $x_{sq} \leq \tilde{x}_i \equiv \hat{x}_{P_R} - \sqrt{c + (x^*_i - \hat{x}_{P_R})^2}$ because $\hat{x}_{P_R} \geq \check{x}_{i+1}$ by Lemma B.1. Note that $\tilde{x}_i < x^*_i$ if $x^*_i \leq \check{x}_{i+1}$. Define $\underline{x}_i = \max\{\check{x}_i, \min\{\check{x}_{i+1}, \tilde{x}_i\}\}$.

Next, set $\hat{x}_{n+1} = \infty$. For $i \in \{\frac{n+1}{2}, \ldots, n+1\}$, we can symmetrically define $\tilde{x}_i = \hat{x}_{P_L} + \sqrt{c + (x^*_i - \hat{x}_{P_L})^2}$ and $\overline{x}_i = \min\{\check{x}_i, \max\{\check{x}_{i-1}, \tilde{x}_i\}\}$. Also symmetric to the preceding case, $\tilde{x}_i > x^*_i$ if $x^*_i \geq \check{x}_i$.

Setting $\underline{x} = \max\{\underline{x}_i | \underline{x}_i > \check{x}_i\}$ and $\overline{x} = \min\{\overline{x}_i | \overline{x}_i < \check{x}_i\}$, the case is not heard if $x_{sq} \in [\underline{x}, \overline{x}]$. ■

Proposition 1. *If $\hat{x}_{P_R}$ increases to $\hat{x}'_{P_R}$, then $\underline{x} \leq \underline{x}'$. Similarly, if $\hat{x}_{P_L}$ decreases to $\hat{x}'_{P_L}$, then $\overline{x}' \leq \overline{x}$.*

Proof. By Lemma 2, we have $\underline{x} \in \{\hat{x}_{P_R} - \sqrt{c + (x_i^* - \hat{x}_{P_R})^2}, \check{x}_{i+1}\}$ for some $i < \frac{n+1}{2}$. We show $\frac{\partial \underline{x}}{\partial \hat{x}_{P_R}} \geq 0$. Analogous arguments imply $\overline{x}$ decreases as $\hat{x}_{P_L}$ shifts leftward.

Case 1. Suppose $\underline{x} = \hat{x}_{P_R} - \sqrt{c + (x_i^* - \hat{x}_{P_R})^2}$. There are two possible subcases: $x_i^* \leq \hat{x}_{P_R}$ and $x_i^* > \hat{x}_{P_R}$.

First, if $x_i^* \leq \hat{x}_{P_R}$, then

$$\begin{aligned}
\frac{\partial \underline{x}}{\partial \hat{x}_{P_R}} &= 1 + \frac{x_i^* - \hat{x}_{P_R}}{\sqrt{c + (x_i^* - \hat{x}_{P_R})^2}} \left(1 - \frac{\partial x_i^*}{\partial \hat{x}_{P_R}}\right) \\
&= 1 + \frac{\hat{x}_{P_R} - x_i^*}{\sqrt{c + (x_i^* - \hat{x}_{P_R})^2}} \left(\frac{\partial x_i^*}{\partial \hat{x}_{P_R}} - 1\right) \\
&\geq 1 - \frac{\hat{x}_{P_R} - x_i^*}{\sqrt{c + (x_i^* - \hat{x}_{P_R})^2}} \qquad (3) \\
&\geq 0, \qquad (4)
\end{aligned}$$

where (3) follows from Property 1 and $x_i^* \leq \hat{x}_{P_R}$, and (4) because $c \geq 0$ implies $\sqrt{c + (x_i^* - \hat{x}_{P_R})^2} \geq \hat{x}_{P_R} - x_i^*$.

For $x_i^* > \hat{x}_{P_R}$,

$$\begin{aligned}
\frac{\partial \underline{x}}{\partial \hat{x}_R} &= 1 + \frac{x_i^* - \hat{x}_{P_R}}{\sqrt{c + (x_i^* - \hat{x}_{P_R})^2}} \left(1 - \frac{\partial x_i^*}{\partial \hat{x}_{P_R}}\right) \\
&\geq \frac{x_i^* - \hat{x}_{P_R}}{\sqrt{c + (x_i^* - \hat{x}_{P_R})^2}} \left(1 - \frac{\partial x_i^*}{\partial \hat{x}_{P_R}}\right) \qquad (5) \\
&\geq 0, \qquad (6)
\end{aligned}$$

where (6) follows from (5) by Property 1.

Case 2. Suppose $\underline{x} = \check{x}_{i+1}$. Then $\underline{x} \geq \hat{x}_{i+1}$ and satisfies $u_{i+1}(x_{i+1}^*) = u_{i+1}(x_i^*) + l(\underline{x} - \hat{x}_{i+1})$.

Applying the implicit function theorem,

$$\frac{\partial \underline{x}}{\partial \hat{x}_{P_R}} = \left(\frac{\partial u_{i+1}(x^*_{i+1})}{\partial x^*_{i+1}} \frac{\partial x^*_{i+1}}{\partial \hat{x}_{P_R}} - \frac{\partial u_{i+1}(x^*_i)}{\partial x^*_i} \frac{\partial x^*_i}{\partial \hat{x}_{P_R}}\right)\left(\frac{\partial l(\underline{x}-\hat{x}_{i+1})}{\partial \underline{x}}\right)^{-1} \tag{7}$$

$$\propto \frac{\partial u_{i+1}(x^*_i)}{\partial x^*_i} \frac{\partial x^*_i}{\partial \hat{x}_{P_R}} - \frac{\partial u_i(x^*_{i+1})}{\partial x^*_{i+1}} \frac{\partial x^*_{i+1}}{\partial \hat{x}_{P_R}}. \tag{8}$$

Thus, $\frac{\partial \underline{x}}{\partial \hat{x}_{P_R}} \geq 0$ iff

$$\frac{\partial u_{i+1}(x^*_i)}{\partial x^*_i} \frac{\partial x^*_i}{\partial \hat{x}_{P_R}} \geq \frac{\partial u_i(x^*_{i+1})}{\partial x^*_{i+1}} \frac{\partial x^*_{i+1}}{\partial \hat{x}_{P_R}}. \tag{9}$$

If $\frac{\partial x^*_i}{\partial \hat{x}_{P_R}} = 0$, then (9) holds because $\hat{x}_{i+1} < x^*_{i+1}$ by Property 3, so $\frac{\partial u_{i+1}(x^*_{i+1})}{\partial x^*_{i+1}} < 0$. If $\frac{\partial x^*_i}{\partial \hat{x}_{P_R}} > 0$, then (9) is equivalent to

$$\frac{\frac{\partial u_{i+1}(x^*_i)}{\partial x^*_i}}{\frac{\partial u_{i+1}(x^*_{i+1})}{\partial x^*_{i+1}}} \leq \frac{\frac{\partial x^*_{i+1}}{\partial \hat{x}_{P_R}}}{\frac{\partial x^*_i}{\partial \hat{x}_{P_R}}}. \tag{10}$$

The LHS of (10) is in $[0,1]$ because $\hat{x}_{i+1} \leq x^*_i \leq x^*_{i+1}$ with at least one strict inequality, and the RHS is greater than or equal to 1 by Properties 1 and 4. Thus, (10) holds. ■

The interval $[\underline{x}, \overline{x}]$ studied in Proposition 1 does not always fully characterize the set of heard cases. We now fully characterize this set, extend Proposition 1, and prove Proposition 3.

The proof of Lemma 2 implies that the non-heard set is

$$\mathscr{G} = \{\cup_{i<\frac{n+1}{2}} [\underline{x}_i, \check{x}_{i+1}]\} \cup \{\cup_{i>\frac{n+1}{2}} [\check{x}_{i-1}, \overline{x}_i]\}. \tag{11}$$

and $[\underline{x}, \overline{x}] \subseteq \mathscr{G}$. In general, $\mathscr{G}$ may also contain intervals of unheard cases disjoint from $[\underline{x}, \overline{x}]$. With sufficiently strong dispositional motivation, we prove a more general version of Proposition 1.

Proof of Proposition 2. Without loss of generality, suppose $\hat{x}_{P_R}$ increases. If $\mathscr{G} = [\underline{x}, \overline{x}]$, then Proposition 1 yields the result. Suppose $\mathscr{G}$ contains at least one interval $[\underline{x}', \overline{x}']$ disjoint from $[\underline{x}, \overline{x}]$, with $\overline{x}' < \underline{x}$.

The characterization for $\mathscr{G}$ in (11) implies $\overline{x}' = \check{x}_i$ for some $i < \frac{n+1}{2}$. Analogous to (7), applying the implicit function theorem yields $\frac{\partial \overline{x}'}{\partial \hat{x}_{P_R}} = \left(\frac{\partial u_i(x^*_{i+1})}{\partial x^*_{i+1}} \frac{\partial x^*_{i+1}}{\partial \hat{x}_{P_R}} - \frac{\partial u_i(x^*_i)}{\partial x^*_i} \frac{\partial x^*_i}{\partial \hat{x}_{P_R}} \right) \left(\frac{\partial l(\underline{x} - \hat{x}_i)}{\partial \underline{x}} \right)^{-1}$. It follows that $\frac{\partial \overline{x}'}{\partial \hat{x}_{P_R}}$ goes to zero as $|\frac{\partial l(\underline{x} - \hat{x}_i)}{\partial \underline{x}}|$ gets large.

Next, we claim $\underline{x}' = \tilde{x}_j$ for some $j \leq i-1$. By Case 1 in the proof of Proposition 1, proving this claim implies that $\underline{x}'$ shifts inward. To show the claim, we proceed by contradiction. Suppose $\underline{x}' = \check{x}_j$ for some $j < i-1$. This requires $\tilde{x}_j \leq \check{x}_j \leq \tilde{x}_{j-1}$. Properties 2 and 3 imply $x^*_{i'}$ increases monotonically over $i' \leq \frac{n+1}{2}$. Thus, $\tilde{x}_j \leq \tilde{x}_{j-1}$ implies $\tilde{x}_{k+1} \leq \tilde{x}_k$ for all $k \in \{j, \ldots, \frac{n-1}{2}\}$. But then we must have a contradiction: $\tilde{x}_i \leq \tilde{x}_j \leq \check{x}_j \leq \check{x}_i < \tilde{x}_i$, where the strict inequality follows because $\check{x}_i = \overline{x}' < \underline{x}$ implies $\check{x}_i < \underline{x}_i$, and thus $\check{x}_i < \tilde{x}_i$.

We have shown that $\underline{x}$ and $\underline{x}'$ shift inward, and the effect on $\overline{x}'$ vanishes as $|\frac{\partial l(\underline{x} - \hat{x}_i)}{\partial \underline{x}}|$ gets large. Because analogous arguments apply to any interval of policies disjoint from $[\underline{x}, \overline{x}]$, sufficiently strong dispositional motivations yield the desired result. ■