Online Appendix
Hypothesis Testing with Error Correction Models

April 6, 2020

1 Approaches to Testing for Cointegration

1.1 Engle-Granger Method

Borrowing notation from Sjö (2011), let $x_{j,t}$ be a series which is stationary after differencing d times and is integrated of order d, expressed as $x_{j,t} \sim I(d)$. A vector, $\vec{x}_t = (x_{1,t}, \ldots, x_{j,t}, \ldots, x_{J,t})'$, has cointegrated components of order d, b, represented as $\vec{x}_t \sim CI(d, b)$, if $\vec{x}_t$ is integrated of order d *and* there is a nonzero vector where $\vec{\beta}'\vec{x}_t \sim I(d-b)$ with $d \geq b > 0$. The vector, $\vec{\beta}$, is the cointegrating vector (Engle and Granger 1987).

To test for cointegration among two variables, we can use a residual based approach like the Engle and Granger (1987) two-step procedure. To begin, estimate the following equation:

$$y_t = \alpha + \beta x_{1,t} + \varepsilon_t, \tag{1}$$

where y_t and $x_{1,t}$ are both $I(1)$ and the residuals are $\hat{\varepsilon}_t$. In the presence of cointegration, $\hat{\varepsilon}_t$ will be $I(0)$. Performing an Augmented Dickey Fuller test on $\hat{\varepsilon}_t$ gives

$$\Delta\hat{\varepsilon}_t = \alpha + \pi\hat{\varepsilon}_{t-1} + \sum_{i=1}^{k} \Delta\hat{\varepsilon}_{t-i} + \zeta_t. \tag{2}$$

In this framework, we consider the null hypothesis that $\pi = 0$ against the alternative hypothesis $\pi < 0$. Rejecting the null leads us to conclude the variables are cointegrated and the cointegration parameter is β. The composition of the variable as the product of multiple stochastic processes not only requires the use of nonstandard critical values (e.g., Engle and Granger 1987; Banerjee, Dolado, Galbraith, and Hendry 1993) but also leads the test statistic to change depending on how many variables are in the model.

1.2 The Johansen Approach

Moreover, the Engle and Granger method assumes there is only one cointegrating vector. The addition of an extra unit root variable, $x_{2,t}$, makes our regression equation,

$$y_t = \alpha + \beta x_{1,t} + \beta x_{2,t} + \eta_t. \tag{3}$$

If, as before, y_t and $x_{1,t}$ are cointegrating, the resulting linear combination will be stationary. However, if y_t and $x_{1,t}$ are not cointegrating, it is possible that adding $x_{2,t}$ can create a cointegrating relationship. As a result, when using the Engle and Granger procedure, a researcher should throoughly investigate the various combinations of cointegrating hypotheses.

What if we would like to test whether we have integration among more than two variables? To do so, we can use the VAR representation of a system and apply Johansen's test (Johansen 1988). Consider the following VAR (adapted from Ericsson and MacKinnon 2002):

$$\vec{x}_t = \sum_{i=1}^{\ell} \boldsymbol{\pi_i} \vec{x}_{t-1} + \boldsymbol{\Phi} \vec{D}_t + \vec{\varepsilon}_t, \tag{4}$$

where $\vec{x}_t$ is a vector of k variables at time t; $\boldsymbol{\pi_i}$ is a matrix of coefficients on the ith lag of $\vec{x}_t$, $\vec{\varepsilon}_t \sim IN(0, \Omega)$; $\vec{D}_t$ is a vector including the constant and other deterministic components, $\boldsymbol{\Phi}$ is the matrix of coefficients for $\vec{D}_t$, and t ranges from 1 to T. $\vec{x}_t$ is of order $I(d)$ with $0 \leq d \leq 1$, requiring dth differencing to make stationary.

This equation represented in vector error correction form is

$$\Delta\vec{x}_t = \boldsymbol{\pi}\vec{x}_{t-1} + \sum_{i=1}^{\ell-1}\boldsymbol{\Gamma_i}\Delta\vec{x}_{t-i} + \boldsymbol{\Phi}\vec{D}_t + \vec{\varepsilon}_t, \qquad t = 1,\ldots,T \tag{5}$$

such that

$$\boldsymbol{\pi} = \left(\sum_{i=1}^{\ell}\pi_i\right) - \boldsymbol{I}_k \tag{6}$$

and

$$\boldsymbol{\Gamma}_i = -(\pi_{i+1} + \ldots + \pi_\ell), \tag{7}$$

with I_t being the identity matrix of dimension k.

Box-Steffensmeier, Freeman, Hitt, and Pevehouse (2014) note the rank of $\boldsymbol{\pi}$, r, is the number of nonzero characteristic roots and can range from 0 to k, the number of equations in the system. If $\boldsymbol{\pi}$ is of rank 0, there is no cointegration because all of the variables are unit roots. No linear combination of them is stationary. If $\boldsymbol{\pi}$ is of full rank, or $r = k$, all variables are stationary. For cointegration to be present, $0 < r < k$ with r indicating the number of cointegrating vectors. We can rewrite $\boldsymbol{\pi}$ as $\boldsymbol{\alpha\beta}'$, making Equation 5,

$$\Delta\vec{x}_t = \boldsymbol{\alpha\beta}'\vec{x}_{t-1} + \sum_{i=1}^{\ell-1}\boldsymbol{\Gamma_i}\Delta\vec{x}_{t-i} + \boldsymbol{\Phi}\vec{D}_t + \vec{\varepsilon}_t. \tag{8}$$

$\boldsymbol{\alpha}$ is a matrix of adjustment parameters for the cointegrating matrix, $\boldsymbol{\beta}$.

Several statistics have been proposed to determine the rank of $\boldsymbol{\pi}$ including $TRACE$ and MAX statistics (Johansen 1995; Box-Steffensmeier et al. 2014). The λ_{TRACE} statistic is

$$\lambda_{TRACE}(r) = -T\sum_{i=r+1}^{k} ln(1-\hat{\lambda}_i), \tag{9}$$

and the λ_{MAX} statistic is

$$\lambda_{MAX}(r, r-1) = -T(ln(1-\hat{\lambda}_{r+1})). \tag{10}$$

For each equation, $\hat{\lambda}$ are eigenvalues, or estimates of characteristic roots, in $\boldsymbol{\pi}$. The number of observations is denoted as T. An eigenvalue is computed for each variable in the system (λ_1 to λ_k), resulting in k values of each statistic. The λ_{TRACE} and λ_{MAX} statistics get higher the larger the value of the characteristic root. That is, the farther the root is away from a pure unit root, or zero, the greater the value of the $TRACE$ and MAX statistics.

The null hypothesis for λ_{TRACE} is that the number of cointegrating vectors is less than r, while λ_{MAX} tests whether the number of cointegrating vectors is equal to r. Used in combination, the tests can determine the presence of cointegrating vectors. Note that the tests use nonstandard critical values (Johansen 1995). Box-Steffensmeier et al. (2014, 164) list several advantages of the Johansen approach such as determining r rather than imposing a restriction on its value. Also, it does not pose any exogeneity restrictions on the variables. However, in so doing it requires a thorough specification of the full system. Note also that the test is sensitive to lag length.

1.3 Returning to the Single Equation Error Correction Model

Most of the work in political science on error correction models (ECMs) focuses not on a system of equations but on a single equation. If the marginal processes for Equation 8 are weakly exogenous for β, cointegration can be determined using the single, conditional model (e.g., Ericsson and MacKinnon 2002).

By way of explanation, let's briefly return to a system of equations with variables of interest y_t and x_t that are both $I(1)$:

$$\Delta y_t = \boldsymbol{\pi}_{(11)} y_{t-1} + \boldsymbol{\pi}_{(12)} x_{t-1} + \varepsilon_{1t} \tag{11}$$

$$\Delta x_t = \boldsymbol{\pi}_{(21)} y_{t-1} + \boldsymbol{\pi}_{(22)} x_{t-1} + \varepsilon_{2t}. \tag{12}$$

Recall that we can represent $\boldsymbol{\pi}$ as $\boldsymbol{\alpha\beta'}$ (see Equation 8), making each equation,

$$\Delta y_t = \boldsymbol{\alpha}_1 \boldsymbol{\beta}' z_{t-1} + \varepsilon_{1t} \tag{13}$$

$$\Delta x_t = \boldsymbol{\alpha}_2 \boldsymbol{\beta}' z_{t-1} + \varepsilon_{2t}. \tag{14}$$

After partitioning the error term, $\varepsilon_{1t} = \nu_{1t} + \gamma_0' \varepsilon_{2t}$, Equation 13 can be expressed as,

$$\Delta y_t = \gamma_0' \Delta x_t + \gamma_1 \boldsymbol{\beta}' z_{t-1} + \nu_{1t} \tag{15}$$

$$\Delta x_t = \varepsilon_{2t}. \tag{16}$$

x_t is weakly exogenous if α_2 in Equation 14 is equal to zero, and we can assess cointegration using Equation 15 alone.

There are different ways of assessing the rank of the cointegrating vector. Harbo, Johansen, Neilsen, and Rahbek (1998) test the null that the rank of the cointegrating vector is zero. Rejection of the null does not necessarily imply that y and x are cointegrated. In a model with multiple independent variables – multiple x's – rejecting the null merely indicates the rank of the cointegrating vector is not zero, indicating the presence of some sort of cointegration; it does not indicate whether it includes y or is present only among the x's.

We can also test for cointegration in the single-equation framework motivated by consideration of the re-parameterized autoregressive distributed lag model: the GECM (e.g., Hendry 1984; Banerjee, Dolado, and Mestre 1998). Consider the following model:

$$\Delta y_t = \alpha_0 + \alpha_1^* y_{t-1} + \beta_0 \Delta x_t + \beta_1 x_{t-1} + \varepsilon_t. \tag{17}$$

Cointegration can be assessed by testing the significance of α_1^* on the lagged dependent variable. The null hypothesis is that y and x are not cointegrated. This is a special case of the Johansen procedure described above where the cointegrating vectors appear only in the equation of interest rather than also appearing in the other equations in the system.

Just as with the Engle-Granger and Johansen approaches, there are several assumptions worth highlighting here. First, weak exogeneity is assumed. If weak exogeneity does not hold, the critical values to determine the significance of α_1^* are affected (Hendry 1995). The conditional ECM also usually imposes $r \leq 1$ (Ericsson and MacKinnon 2002).

2 Examining α_1^* across Simulation Scenarios

Figure 1 displays average α_1^* values for the simulation scenario described in the paper. Additional x's move α_1^* further from zero which might lead a researcher to erroneously describe a faster error correction rate.

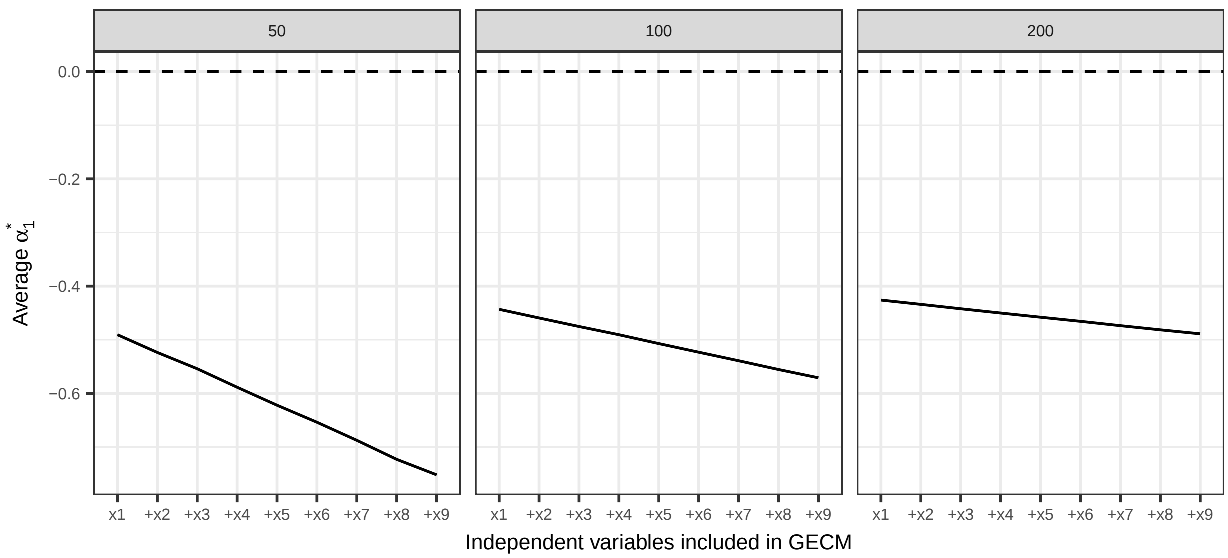

Figure 1: The consequences of GECMs with unbalanced equations. Adding unrelated I(1) regressors moves α_1^* further from zero.

3 Replicating Simulation Results with Alternative DGP

In order to check the robustness of the Monte Carlo analysis, we repeat the simulation with an alternative DGP. More specifically, we directly simulate a GECM process with cointegration between y and x_1 while all the remaining x's are independent unit roots. As before, we start by simulating 1000 datasets each containing 9 independent unit root variables ($k = 9$) with $T \in \{50, 100, 200\}$:

$$x_{j,t} = x_{j,t-1} + \varepsilon_{j,t}, \quad \varepsilon_{j,t} \sim \mathrm{N}(0,1), \qquad j = 1, \ldots, k; \quad t = 1, \ldots, T. \tag{18}$$

Then, we draw an initial $y_{t=1} \sim N(0,1)$ and simulate all subsequent y_t's according to the following process:

$$y_t = y_{t-1} + \Delta y_t, \tag{19}$$

$$\Delta y_t = 1 - 0.4 y_{t-1} + 0.5 \Delta x_{1,t} + 0.5 x_{1,t-1} + \zeta_t, \qquad \zeta_t \sim \mathrm{N}(0,1) \tag{20}$$

This DGP results in a cointegrating relationship between y and x_1 while the remaining independent variables $(x_2, \ldots, x_9)$ are each unrelated unit roots. Compared to the simple DGP in the main text, we observe similar patterns regarding the significance of α_1^* as well as the false positive rates on the unrelated LRMs (see Figure 2 and 3).

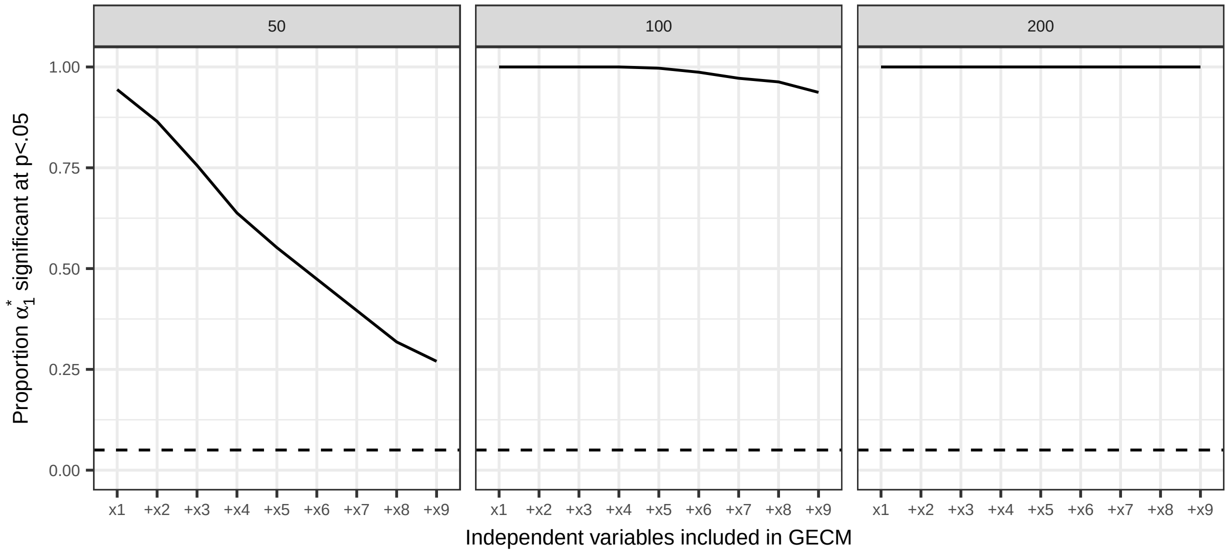

Figure 2: The consequences of GECMs with unbalanced equations (alternative DGP). Adding unrelated I(1) regressors does not sufficiently diminish the statistical significance of α_1^*.

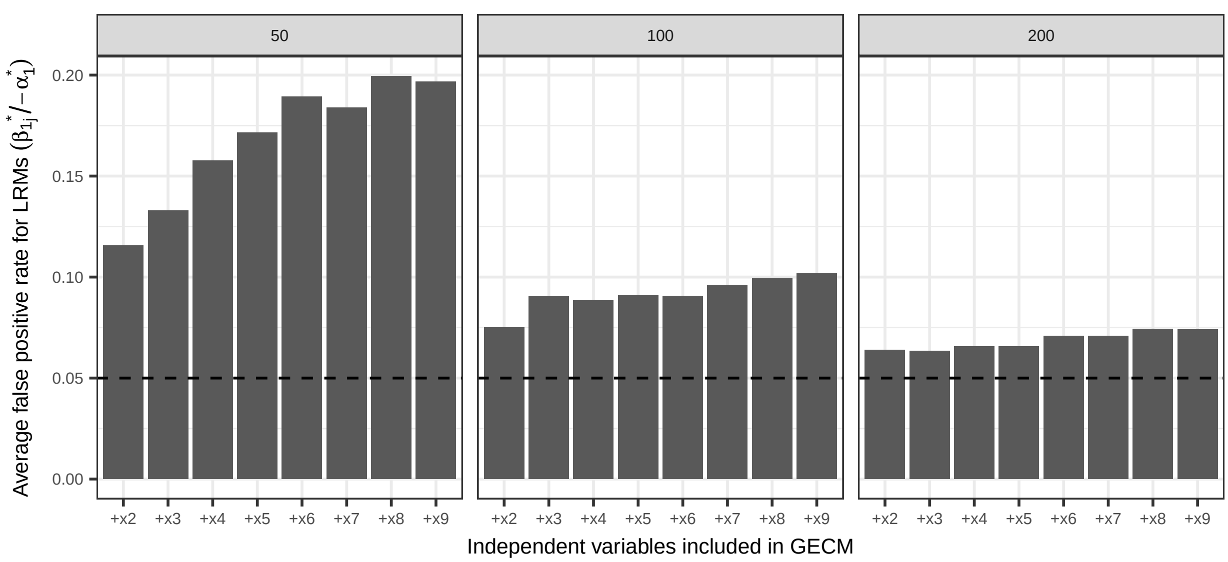

Figure 3: The consequences of GECMs with unbalanced equations (alternative DGP). We observe inflated false positives on long run multipliers for unrelated regressors. The horizontal line indicates an acceptable significance rate of 0.05.

4 Violating the Exogeneity Assumption

We can additionally explore how violations of the exogeneity assumption for unrelated regressors affects false positive rates. We repeat the same DGP as above (based on the GECM with the same parameters) and induce endogeneity by letting $\tilde{x}_j \forall j \neq 1$ be correlated with the GECM error term ζ_t:

$$\tilde{x}_{j,t} = x_{j,t} + \zeta_t + \eta_{j,t}, \qquad \eta_{j,t} \sim \mathrm{N}(0,1), \quad j = 2, \ldots, k \tag{21}$$

Again, we simulated a cointegrating relationship between y and x_1. The remaining independent variables $(x_2, \ldots, x_9)$ are still unit roots, but they are now correlated with the GECM error term. The results for this DGP are displayed in Figure 4 and 5. Adding endogenous regressors that are not cointegrated with y reduces the proportion of times α_1^* surpasses MacKinnon's critical values. However, violating the exogeneity assumption substantially increases false positive rates on unrelated LRMs.

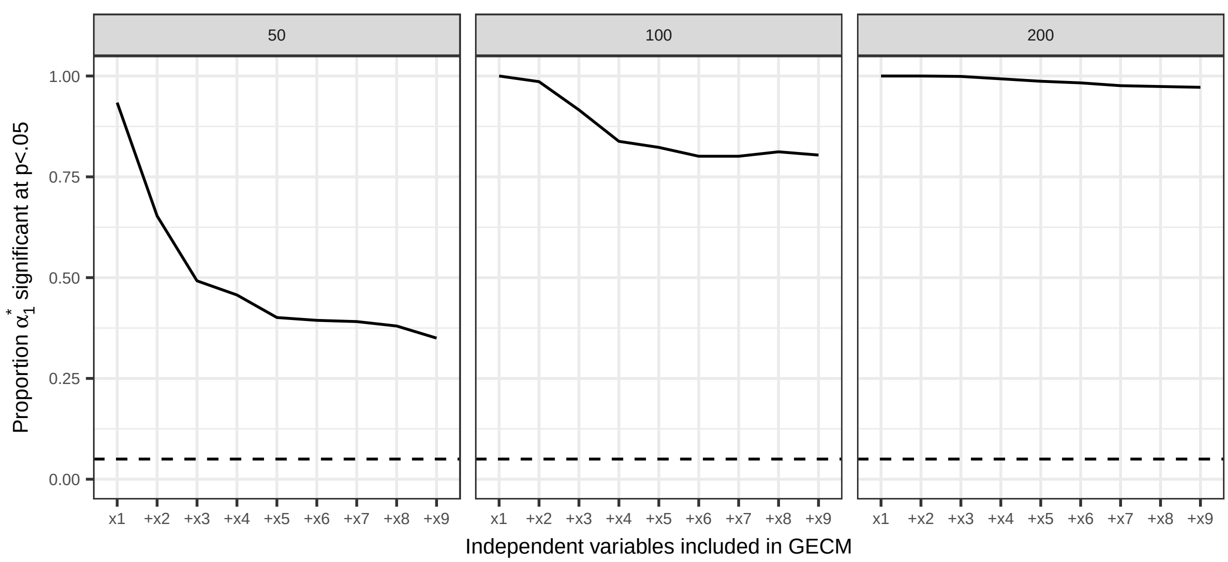

Figure 4: The consequences of GECMs with unbalanced equations (violating exogeneity). Adding unrelated I(1) regressors does not sufficiently diminish the statistical significance of α_1^*.

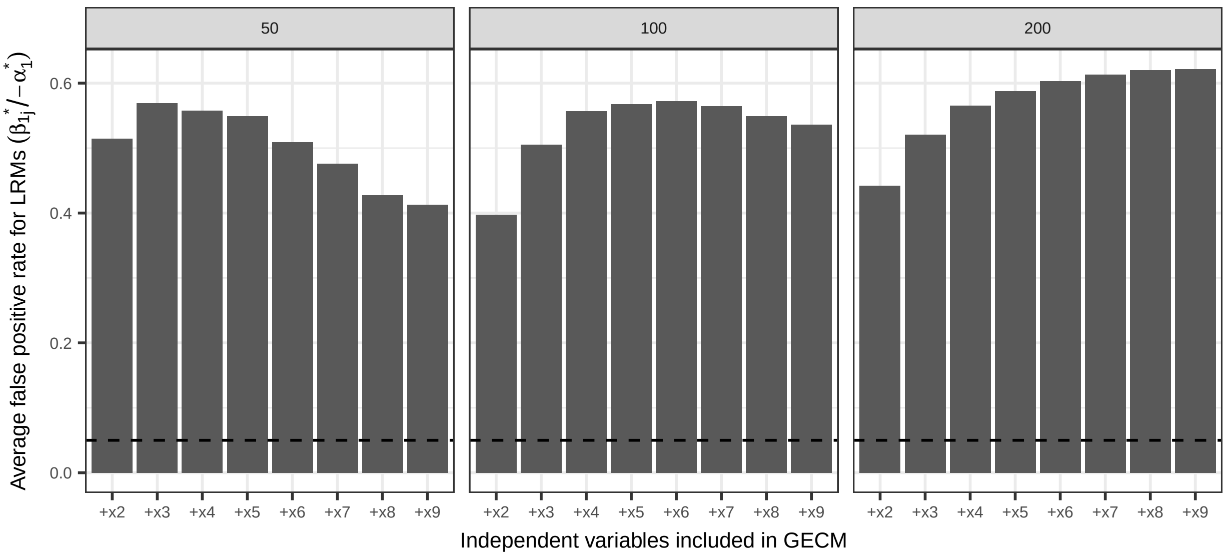

Figure 5: The consequences of GECMs with unbalanced equations (violating exogeneity). We observe inflated false positives on long run multipliers for unrelated regressors. The horizontal line indicates an acceptable significance rate of 0.05.

5 Alternative specification for Volscho and Kelly (2012)

Table 1: Alternative specification for Volscho and Kelly (2012)

	(1)
	Δ Top 1%$_{t}$
Top 1%$_{t\text{-}1}$	-0.759***
	(0.123)
% Cong Dems$_{t\text{-}1}$	-0.0500**
	(0.0162)
% Divided Govt$_{t\text{-}1}$	-0.308
	(0.177)
% Union Membership$_{t\text{-}1}$	-0.362***
	(0.0825)
% Top Marginal Tax$_{t\text{-}1}$	-0.0273*
	(0.0110)
Capital Gains Tax Rate$_{t\text{-}1}$	-0.0720***
	(0.0176)
Δ Unemplyment$_{t}$	-0.0703
	(0.121)
Unemplyment$_{t\text{-}1}$	-0.140
	(0.118)
Trade Openness$_{t}$	0.151
	(0.104)
Log RGDP$_{t\text{-}1}$	-5.930***
	(1.315)
Δ Real S&P 500 Index$_{t}$	0.0635***
	(0.00755)
Real S&P 500 Index$_{t\text{-}1}$	0.0352***
	(0.00705)
Shiller HPI$_{t\text{-}1}$	0.350***
	(0.0820)
Constant	69.56***
	(14.06)
N	60
R^2	0.811

References

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