

9 Appendix: Technical Details for DCCM

The predicted networks were generated using a Metropolis-Hastings algorithm with target distribution based on equation (25). Use of Metropolis-Hastings algorithm requires evaluation of the acceptance probability, as described in equations (7). Since (24) provides the probability mass function for $P_{\mathcal{G}_t}$, we only need to calculate $f(C_g, C_h)$.

Though our analysis considers mixing based on only political party membership, the equations below are generalized to allow for mixing between individuals based on an arbitrary number of covariate patterns. We present the quantities for the four cases that must be evaluated in order to calculate $f(c_{\eta(g'_t|g_{t-1})}, c_{\eta(gp_t|g_{t-1})})$. Let edge (i, j) be the required edge toggle to move from g'_t to gp_t , and let $S^{l,k}(g) = \{E_{ij} : E_{ij} \in g, m_i = l, \text{ and } m_j = k\}$. The four cases are associated with whether (i, j) exists in g'_t or g_{t-1} or both or neither.

Case 1: $(i, j) \in g'_t$ and $(i, j) \in g_{t-1}$. Therefore,

$$\eta_s^{l,k}(gp_t) * M_{m_i}(g'_t) = \eta_s^{l,k}(g'_t) * M_{m_i}(g'_t) - I_{\{m_i=l, m_j=k\}}, \quad (30)$$

and

$$\eta_d^{l,k}(gp_t|g_{t-1}) * M_{m_i}(g'_t) = \eta_d^{l,k}(g'_t|g_{t-1}) * M_{m_i}(g'_t) - I_{\{m_i=l, m_j=k\}}. \quad (31)$$

Toggleing any edge in $S^{l,k}(g'_t) \cap S^{l,k}(g_{t-1})$ would satisfy equations (30) and (31); since this logic holds for any $g \in c_{\eta(g'_t|g_{t-1})}$ and $|S^{l,k}(g'_t) \cap S^{l,k}(g_{t-1})|$ is constant across $g \in c_{\eta(g'_t|g_{t-1})}$,

$$f(c_{\eta(g'_t|g_{t-1})}, c_{\eta(gp_t|g_{t-1})}) = \eta_d^{m_i, m_j}(g'_t|g_{t-1}) * M_{m_i}(g'_t). \quad (32)$$

Case 2: $(i, j) \in g'_t$ and $(i, j) \notin g_{t-1}$. Therefore,

$$\eta_s^{l,k}(gp_t) * M_{m_i}(g'_t) = \eta_s^{l,k}(g'_t) * M_{m_i}(g'_t) - I_{\{m_i=l, m_j=k\}}, \quad (33)$$

and

$$\eta_d^{l,k}(gp_t|g_{t-1}) * M_{m_i}(g'_t) = \eta_d^{l,k}(g'_t|g_{t-1}) * M_{m_i}(g'_t). \quad (34)$$

Any edge from $S^{l,k}(g'_t)/S^{l,k}(g_{t-1})$ can be toggled to satisfy equations (33) and (34). Again, because this reasoning holds for any $g \in c_{\eta(g'_t|g_{t-1})}$ and because $|S^{l,k}(g'_t)/S^{l,k}(g_{t-1})|$ is constant across $g \in c_{\eta(g'_t|g_{t-1})}$,

$$f(c_{\eta(g'_t|g_{t-1})}, c_{\eta(gp_t|g_{t-1})}) = \eta_s^{m_i, m_j}(g'_t) * M_{m_i}(g'_t) - \eta_d^{m_i, m_j}(g'_t|g_{t-1}) * M_{m_i}(g'_t). \quad (35)$$

Case 3: $(i, j) \notin g'_t$ and $(i, j) \in g_{t-1}$. Therefore,

$$\eta_s^{l,k}(gp_t) * M_{m_i}(g'_t) = \eta_s^{l,k}(g'_t) * M_{m_i}(g'_t) + I_{\{m_i=l, m_j=k\}}, \quad (36)$$

and

$$\eta_d^{l,k}(gp_t|g_{t-1}) * M_{m_i}(g'_t) = \eta_d^{l,k}(g'_t|g_{t-1}) * M_{m_i}(g'_t) + I_{\{m_i=l, m_j=k\}}. \quad (37)$$

An edge from $S^{l,k}(g_{t-1})/S^{l,k}(g'_t)$ can be toggled to satisfy equations (36) and (37). Therefore,

$$f(c_{\eta(g'_t|g_{t-1})}, c_{\eta(gp_t|g_{t-1})}) = \eta_s^{m_i, m_j}(g_{t-1}) * M_{m_i}(g'_t) - \eta_d^{m_i, m_j}(g'_t|g_{t-1}) * M_{m_i}(g'_t) \quad (38)$$

for similar reasons as the previous cases.

Case 4: $(i, j) \notin g'_t$ and $(i, j) \notin g_{t-1}$. Therefore,

$$\eta_s^{l,k}(gp_t) * M_{m_i}(g'_t) = \eta_s^{l,k}(g'_t) * M_{m_i}(g'_t) + I_{\{m_i=l, m_j=k\}}, \quad (39)$$

and

$$\eta_d^{l,k}(gp_t|g_{t-1}) * M_{m_i}(g'_t) = \eta_d^{l,k}(g'_t|g_{t-1}) * M_{m_i}(g'_t). \quad (40)$$

An edge from all possible edges connecting an m_i node to an m_j node that is not in $S^{l,k}(g'_t) \cup S^{l,k}(g_{t-1})$ can be toggled to satisfy equations (39) and (40). Therefore,

$$f(c_{\eta(g'_t|g_{t-1})}, c_{\eta(gp_t|g_{t-1})}) = M_{m_i, m_j}(g'_t) * M_{m_i}(g'_t) - [\eta_s^{m_i, m_j}(g_{t-1}) * M_{m_i}(g'_t) - \eta_d^{m_i, m_j}(g'_t|g_{t-1}) * M_{m_i}(g'_t)], \quad (41)$$

where

$$M_{m_i, m_j}(g'_t) = \begin{cases} [(M_{m_i}(g'_t) * M_{m_j}(g'_t)) - \eta_s^{m_i, m_j}(g'_t) * M_{m_i}(g'_t)] & \text{if } m_i \neq m_j \\ (M_{m_i}(g'_t)) - \eta_s^{m_i, m_j}(g'_t) * M_{m_i}(g'_t) & \text{if } m_i = m_j, \end{cases} \quad (42)$$

for similar reasons as the previous cases. The calculations for $f(c_{\eta(gp_t|g_{t-1})}, c_{\eta(g'_t|g_{t-1})})$ are similar to $f(c_{\eta(g'_t|g_{t-1})}, c_{\eta(gp_t|g_{t-1})})$.