**Appendix 2: Further mathematical derivations**

**The distribution function for *B*–*V* when *B* and *V* are uncorrelated; *α* = 1**

The mathematical derivations in this appendix are included mainly to provide a basis for the simulations and simulation results presented in Figures 1-11 in the main text, and Figures 12-14 in this appendix. This appendix also yields additional insights to the analysis in the paper, in particular by rewriting earlier formulations in terms of conditional probabilities.

To derive the distribution for *B* – *V* = *Z*, call the cdf of the random variable *B*, *F*(*B*), and denote its pdf by *f*(*B*). Considering the cdf for realized positive values of *Z* (cases where the forest is saved), conditional on *H*, this is found as the convolution

 (A1) .

Substituting *B* = *Z*+*V* in (11) and taking its derivative with respect to *Z* (= *B-V*), we find the conditional pdf for *Z* for given *H*, as follows:

 (A2) .

Figure 12 shows a simulation of the pdf given that *B* and *V* are both independent and normal, and given *EB* = 1 = *H*, *EV* = 0.5, *σB* = 1, *σV* =1, for *α* = 1.



**Figure 12: The pdf for *Z = B-V*; *α* = 1, zero correlation**

The mean (first moment) of this distribution is given by

 (A3) 

which can be simplified as follows (assuming that *f*, *g* are continuous and integrable):

 

This leads to

 (A3a) .

Note that

 (A4) 

is simply the expected value of *V*, conditional on *H*. We then also have from (A3a):

 (A3b) .

 is here an increasing function of *H*. Intuitively, when the payment *H* from buyer to seller increases, additional forest–saving projects are induced with gradually higher *V* levels (constrained only by *V*  *H*), thus increasing , but bounded above by its unconditional expectation, *EV*. Thus  must be a decreasing function of *H*, and must always have value less than the unconditional expectation of *B, EB*; and value higher than *EB – EV* (both unconditional).

By a similar procedure we can derive the conditional variance of *B*–*V* for given *H*, with result:

 (A5) .

Figures 13-14 now provide simulations of *EZ* and the standard deviation on *Z* as functions of *H*, respectively.



**Figure 13: *EZ* (= *E*(*B*–*V*)) under parametric variation of *H*; *α* = 1, zero correlation**



**Figure 14: Standard Deviation of *Z* = *B*–*V* under parametric variation in *H*; *α* = 1, zero correlation**

**Positive correlation between *B* and *V*; *α* = 1**

We will now study cases where *B* and *V* can be correlated, for *α* = 1, focusing on joint normal distributions for *B* and *V*. The buyer then solves the following optimization problem, setting *h*(*B*,*V*) to be the joint multivariate normal distribution

(1a) 

This equation now replaces (1). If *B* and *V* are independent, then *h*(*B*,*V*) = *f*(*B*)*g*(*V*) and the equation (1a) above simplifies to (1) in our paper. When *B* and *V* are correlated, we need to apply (1a). It is here easiest to integrate over *B* first (we can switch the order of the integral since we assume integrable, continuous functions)

(A6) 

(A7) 

Now let *ρ* represent the correlation between *B* and *V.*

(A8) 

Now maximizing with respect to *H*:

(A9) 

We find, solving for *H*:

 (A10) 

Note that when EB = EV, *H* does not change with changes in correlation, and is equal to EB:

(A11) 

We can take the second order condition for (A9) to obtain a second order condition

(A12) 

When *EB* = *EV* = *H*, a necessary (and sufficient) condition for the second order condition to hold is .

The intuition is straightforward: when *α* = 1. Having correlated *B* and *V* does not allow any room for increasing expected total payoff by strategically changing *H*. Setting *H* = *EB* = *EV* allows the optimal welfare solution.

However, payoff under incomplete information decreases linearly in *ρ* when *EB* = *EV* for :

(A13) 

Intuitively, *H* stays the same for all values of correlation. Further, this payoff is written as the composition of a difference of random variables. An increase in correlation leads to a decrease in the variance of a difference in bivariate, correlated, normal random variables. Note that the rate of this change is dependent on .

We now consider the positively correlated case for the payoff under complete information. Note that payoff under complete information is given by:

A(14) 

The distribution *m*(*B* – *V*) has mean (*EB* – *EV*) and standard deviation . Thus higher correlation reduces the uncertainty for this distribution. This will make the payoff under full information decrease with increasing correlation, because the integral is over values where *B* – *V* > 0. Higher uncertainty in this difference is the same as lower correlation between the random variables, and means more opportunities for trade to happen.

A direct result is that when *EB* = *EV*, and when , both payoffs under full and incomplete information go to zero.

**Positive correlation between *B* and *V*; *α* = 0**

We will now study cases where *B* and *V* can be correlated, given *α* = 0, still focusing on joint normal distributions for *B* and *V*. The buyer then solves the following optimization problem, setting h(*B*,*V*) to be the joint multivariate normal distribution

(A15) 

(A15) now replaces (1). When *B* and *V* are independent, *h*(*B*,*V*) = *f*(*B*)*g*(*V*) and(A15) simplifies to (1) for *α* = 0. When *B* and *V* are instead correlated, further analysis must be based on (A15). It is here easiest to integrate over *B* first (we can switch the order of the integral since we assume integrable, continuous functions)

(A16) 

(A17) 

Now let *ρ* represent the correlation between *B* and *V.*

(A18) 

Maximizing with respect to *H* yields:

(A19) 

We then get the relationship:

 (A20) 

Note that when *EB = EV*, the relationship can be rewritten as:

(A21) 

We can from (A19) obtain a second order condition for this problem, which takes the same form as (A12).

When *EB* = *EV* = *H*, necessary and sufficient conditions for the second order condition to hold are *g’*(*H*) < 0 and .

We now look at the positively correlated case for the payoff under full information. Note that payoff under full information is given by:



The distribution **  has a mean of and standard deviation . Thus, increasing correlation reduces the uncertainty for this distribution.