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RESEARCH ARTICLE

Supplementary Information: Stability of schooling patterns of a fish pair swimming against a flow

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S1. Expressions for fluid flow model

The flow velocity induced by each of the two fish (point dipoles) in the model is given by

$$\begin{aligned}
\boldsymbol{u}_{f\,1} &= r_0^2 v_0 \frac{(x-x_1+y-y_1)(x-x_1-y+y_1)\cos\theta_1 + 2(x-x_1)(y-y_1)\sin\theta_1}{((x-x_1)^2+(y-y_1)^2)^2}\hat{\boldsymbol{i}} \\
&+ r_0^2 v_0 \frac{2(x-x_1)(y-y_1)\cos\theta_1 - (x-x_1+y-y_1)(x-x_1-y+y_1)\sin\theta_1}{((x-x_1)^2+(y-y_1)^2)^2}\hat{\boldsymbol{j}}, \qquad (1)\\
\boldsymbol{u}_{f\,2} &= r_0^2 v_0 \frac{(x-x_2+y-y_2)(x-x_2-y+y_2)\cos\theta_2 + 2(x-x_2)(y-y_2)\sin\theta_2}{((x-x_2)^2+(y-y_2)^2)^2}\hat{\boldsymbol{i}} \\
&+ r_0^2 v_0 \frac{2(x-x_2)(y-y_2)\cos\theta_2 - (x-x_2+y-y_2)(x-x_2-y+y_2)\sin\theta_2}{((x-x_2)^2+(y-y_2)^2)^2}\hat{\boldsymbol{j}} \qquad (2)
\end{aligned}$$

The flow velocity due to the presence of the walls

$$\begin{aligned}
\boldsymbol{u}_w &= r_0^2 v_0 \Bigg[\left(\frac{\cos\theta_1}{(x-x_1)^2+(y-y_1)^2} - \frac{1}{4} f_w(x_1,y_1,\theta_1) - \frac{2(x-x_1)((x-x_1)\cos\theta_1+(y-y_1)\sin\theta_1)}{((x-x_1)^2+(y-y_1)^2)^2}\right) \\
&+ \left(\frac{\cos\theta_2}{(x-x_2)^2+(y-y_2)^2} - \frac{1}{4} f_w(x_2,y_2,\theta_2) - \frac{2(x-x_2)((x-x_2)\cos\theta_2+(y-y_2)\sin\theta_2)}{((x-x_2)^2+(y-y_2)^2)^2}\right)\Bigg]\hat{\boldsymbol{i}} \\
&+ r_0^2 v_0 \Bigg[\left(\frac{\sin\theta_1}{(x-x_1)^2+(y-y_1)^2} - \frac{1}{4} f_{\tilde{w}}(x_1,y_1,\theta_1) - \frac{2(y-y_1)((x-x_1)\cos\theta_1+(y-y_1)\sin\theta_1)}{((x-x_1)^2+(y-y_1)^2)^2}\right) \\
&+ \left(\frac{\sin\theta_2}{(x-x_2)^2+(y-y_2)^2} - \frac{1}{4} f_{\tilde{w}}(x_2,y_2,\theta_2) - \frac{2(y-y_2)((x-x_2)\cos\theta_2+(y-y_2)\sin\theta_2)}{((x-x_2)^2+(y-y_2)^2)^2}\right)\Bigg]\hat{\boldsymbol{j}}, \qquad (3)
\end{aligned}$$

where

$$\begin{aligned}
f_w(x_k,y_k,\theta_k) &= -\frac{\pi^2 e^{-\mathrm{i}\theta_k}}{2h^2}\left(e^{2\mathrm{i}\theta_k}\operatorname{csch}^2\left(\frac{\pi(x-x_k+\mathrm{i}(y-y_k))}{2h}\right) + \operatorname{csch}^2\left(\frac{\pi(x-x_k-\mathrm{i}(y-y_k))}{2h}\right)\right. \\
&\left.+ e^{2\mathrm{i}\theta_k}\operatorname{csch}^2\left(\frac{\pi(x-x_k-\mathrm{i}(y+y_k))}{2h}\right) + \operatorname{csch}^2\left(\frac{\pi(x-x_k+\mathrm{i}(y+y_k))}{2h}\right)\right), \qquad (4a)\\
f_{\tilde{w}}(x_k,y_k,\theta_k) &= \frac{\pi^2 \mathrm{i} e^{-\mathrm{i}\theta_k}}{2h^2}\left(-e^{2\mathrm{i}\theta_k}\operatorname{csch}^2\left(\frac{\pi(x-x_k+\mathrm{i}(y-y_k))}{2h}\right) + \operatorname{csch}^2\left(\frac{\pi(x-x_k-\mathrm{i}(y-y_k))}{2h}\right)\right. \\
&\left.+ e^{2\mathrm{i}\theta_k}\operatorname{csch}^2\left(\frac{\pi(x-x_k-\mathrm{i}(y+y_k))}{2h}\right) - \operatorname{csch}^2\left(\frac{\pi(x-x_k+\mathrm{i}(y+y_k))}{2h}\right)\right). \qquad (4b)
\end{aligned}$$

S2. Expressions for fish dynamics model

The advection velocities of the two fish are given by the following expressions:

$$
\begin{aligned}
\boldsymbol{U}_1 = & \Bigg[U_0\left(1-\epsilon+\frac{4\epsilon y_1}{h}\left(1-\frac{y_1}{h}\right)\right)-\frac{\pi^2 r_0^2 v_0}{24h^2}\Bigg\{e^{-\mathrm{i}\theta_1}+e^{\mathrm{i}\theta_1}+3e^{-\mathrm{i}\theta_1}\csc^2\left(\frac{\pi y_1}{h}\right)+3e^{\mathrm{i}\theta_1}\csc^2\left(\frac{\pi y_1}{h}\right) \\
& -3e^{\mathrm{i}\theta_2}\operatorname{csch}^2\left(\frac{\pi(x_1-x_2+\mathrm{i}(y_1-y_2))}{2h}\right)-3e^{-\mathrm{i}\theta_2}\operatorname{csch}^2\left(\frac{\pi(x_1-x_2-\mathrm{i}(y_1-y_2))}{2h}\right) \\
& -3e^{\mathrm{i}\theta_2}\operatorname{csch}^2\left(\frac{\pi(x_1-x_2-\mathrm{i}(y_1+y_2))}{2h}\right)-3e^{-\mathrm{i}\theta_2}\operatorname{csch}^2\left(\frac{\pi(x_1-x_2+\mathrm{i}(y_1+y_2))}{2h}\right)\Bigg\}\Bigg]\hat{i} \\
& +\Bigg[\frac{\pi^2 r_0^2 v_0}{24h^2}e^{-\mathrm{i}(\theta_1+\theta_2)}\Bigg\{e^{\mathrm{i}\theta_2}-e^{\mathrm{i}(2\theta_1+\theta_2)}-3e^{\mathrm{i}\theta_2}\csc^2\left(\frac{\pi y_1}{h}\right)+3e^{\mathrm{i}(2\theta_1+\theta_2)}\csc^2\left(\frac{\pi y_1}{h}\right) \\
& +3e^{\mathrm{i}(\theta_1+2\theta_2)}\operatorname{csch}^2\left(\frac{\pi(x_1-x_2+\mathrm{i}(y_1-y_2))}{2h}\right)-3e^{\mathrm{i}\theta_1}\operatorname{csch}^2\left(\frac{\pi(x_1-x_2-\mathrm{i}(y_1-y_2))}{2h}\right) \\
& -3e^{\mathrm{i}(\theta_1+2\theta_2)}\operatorname{csch}^2\left(\frac{\pi(x_1-x_2-\mathrm{i}(y_1+y_2))}{2h}\right)+3e^{\mathrm{i}\theta_1}\operatorname{csch}^2\left(\frac{\pi(x_1-x_2+\mathrm{i}(y_1+y_2))}{2h}\right)\Bigg\}\Bigg]\hat{j},
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
\boldsymbol{U}_2 = & \Bigg[U_0\left(1-\epsilon+\frac{4\epsilon y_2}{h}\left(1-\frac{y_2}{h}\right)\right)-\frac{\pi^2 r_0^2 v_0}{24h^2}\Bigg\{e^{-\mathrm{i}\theta_2}+e^{\mathrm{i}\theta_2}+3e^{-\mathrm{i}\theta_2}\csc^2\left(\frac{\pi y_2}{h}\right)+3e^{\mathrm{i}\theta_2}\csc^2\left(\frac{\pi y_2}{h}\right) \\
& -3e^{\mathrm{i}\theta_1}\operatorname{csch}^2\left(\frac{\pi(x_2-x_1+\mathrm{i}(y_2-y_1))}{2h}\right)-3e^{-\mathrm{i}\theta_1}\operatorname{csch}^2\left(\frac{\pi(x_2-x_1-\mathrm{i}(y_2-y_1))}{2h}\right) \\
& -3e^{\mathrm{i}\theta_1}\operatorname{csch}^2\left(\frac{\pi(x_2-x_1-\mathrm{i}(y_1+y_2))}{2h}\right)-3e^{-\mathrm{i}\theta_1}\operatorname{csch}^2\left(\frac{\pi(x_2-x_1+\mathrm{i}(y_1+y_2))}{2h}\right)\Bigg\}\Bigg]\hat{i} \\
& +\Bigg[\frac{\pi^2 r_0^2 v_0}{24h^2}e^{-\mathrm{i}(\theta_1+\theta_2)}\Bigg\{e^{\mathrm{i}\theta_1}-e^{\mathrm{i}(\theta_1+2\theta_2)}-3e^{\mathrm{i}\theta_1}\csc^2\left(\frac{\pi y_2}{h}\right)+3e^{\mathrm{i}(\theta_1+2\theta_2)}\csc^2\left(\frac{\pi y_2}{h}\right) \\
& +3e^{\mathrm{i}(2\theta_1+\theta_2)}\operatorname{csch}^2\left(\frac{\pi(x_2-x_1+\mathrm{i}(y_2-y_1))}{2h}\right)-3e^{\mathrm{i}\theta_2}\operatorname{csch}^2\left(\frac{\pi(x_2-x_1-\mathrm{i}(y_2-y_1))}{2h}\right) \\
& -3e^{\mathrm{i}(2\theta_1+\theta_2)}\operatorname{csch}^2\left(\frac{\pi(x_2-x_1-\mathrm{i}(y_1+y_2))}{2h}\right)+3e^{\mathrm{i}\theta_2}\operatorname{csch}^2\left(\frac{\pi(x_2-x_1+\mathrm{i}(y_1+y_2))}{2h}\right)\Bigg\}\Bigg]\hat{j}.
\end{aligned}
\tag{6}
$$

The flow-induced angular velocity of each of the two fish are given by:

$$
\begin{aligned}
\Omega_1 = {} & \frac{1}{16h^3} e^{-\mathrm{i}(2\theta_1+\theta_2)} \Bigg[32 U_0 y_1 h \epsilon e^{\mathrm{i}\theta_2} \left(1 + e^{2\mathrm{i}\theta_1}\right)^2 - 64 U_0 h^2 \epsilon e^{\mathrm{i}(2\theta_1+\theta_2)} \cos^2(\theta_1) \\
& + \pi^3 {r_0}^2 v_0 \Bigg\{ e^{\mathrm{i}(5\theta_1+\theta_2)} \cot\left(\frac{\pi y_1}{h}\right) \csc^2\left(\frac{\pi y_1}{h}\right) - e^{\mathrm{i}(\theta_1+\theta_2)} \cot\left(\frac{\pi y_1}{h}\right) \csc^2\left(\frac{\pi y_1}{h}\right) \\
& - 2e^{\mathrm{i}(\theta_1+\theta_2)} \cos^2(\theta_1) \cot\left(\frac{\pi y_1}{h}\right) \csc^2\left(\frac{\pi y_1}{h}\right) - 2e^{\mathrm{i}(3\theta_1+\theta_2)} \cos^2(\theta_1) \cot\left(\frac{\pi y_1}{h}\right) \csc^2\left(\frac{\pi y_1}{h}\right) \\
& - 2\mathrm{i} \coth\left(\frac{\pi(x_1 - x_2 - \mathrm{i}(y_1 - y_2))}{2h}\right) \operatorname{csch}^2\left(\frac{\pi(x_1 - x_2 - \mathrm{i}(y_1 - y_2))}{2h}\right) \\
& + 2\mathrm{i} e^{2\mathrm{i}(2\theta_1+\theta_2)} \coth\left(\frac{\pi(x_1 - x_2 + \mathrm{i}(y_1 - y_2))}{2h}\right) \operatorname{csch}^2\left(\frac{\pi(x_1 - x_2 + \mathrm{i}(y_1 - y_2))}{2h}\right) \\
& - 2\mathrm{i} e^{2\mathrm{i}(\theta_1+\theta_2)} \cos^2(\theta_1) \coth\left(\frac{\pi(x_1 - x_2 - \mathrm{i}(y_1 + y_2))}{2h}\right) \operatorname{csch}^2\left(\frac{\pi(x_1 - x_2 - \mathrm{i}(y_1 + y_2))}{2h}\right) \\
& + \mathrm{i}\left(-1 + e^{4\mathrm{i}\theta_1}\right) \coth\left(\frac{\pi(x_1 - x_2 + \mathrm{i}(y_1 + y_2))}{2h}\right) \operatorname{csch}^2\left(\frac{\pi(x_1 - x_2 + \mathrm{i}(y_1 + y_2))}{2h}\right) \\
& + 2\mathrm{i} e^{2\mathrm{i}\theta_1} \cos^2(\theta_1) \coth\left(\frac{\pi(x_1 - x_2 + \mathrm{i}(y_1 + y_2))}{2h}\right) \operatorname{csch}^2\left(\frac{\pi(x_1 - x_2 + \mathrm{i}(y_1 + y_2))}{2h}\right) \\
& - 4\mathrm{i} e^{\mathrm{i}(\theta_1+\theta_2)} \cos(\theta_1) \cot\left(\frac{\pi y_1}{h}\right) \csc^2\left(\frac{\pi y_1}{h}\right) \sin(\theta_1) \\
& - 4e^{2\mathrm{i}(\theta_1+\theta_2)} \cos(\theta_1) \coth\left(\frac{\pi(x_1 - x_2 - \mathrm{i}(y_1 + y_2))}{2h}\right) \operatorname{csch}^2\left(\frac{\pi(x_1 - x_2 - \mathrm{i}(y_1 + y_2))}{2h}\right) \sin(\theta_1) \\
& + 2e^{\mathrm{i}(\theta_1+\theta_2)} \cot\left(\frac{\pi y_1}{h}\right) \csc^2\left(\frac{\pi y_1}{h}\right) \sin^2(\theta_1) + 2e^{\mathrm{i}(3\theta_1+\theta_2)} \cot\left(\frac{\pi y_1}{h}\right) \csc^2\left(\frac{\pi y_1}{h}\right) \sin^2(\theta_1) \\
& + 2\mathrm{i} e^{2\mathrm{i}(\theta_1+\theta_2)} \coth\left(\frac{\pi(x_1 - x_2 - \mathrm{i}(y_1 + y_2))}{2h}\right) \operatorname{csch}^2\left(\frac{\pi(x_1 - x_2 - \mathrm{i}(y_1 + y_2))}{2h}\right) \sin^2(\theta_1) \\
& - 2\mathrm{i} e^{2i\theta_1} \coth\left(\frac{\pi(x_1 - x_2 + \mathrm{i}(y_1 + y_2))}{2h}\right) \operatorname{csch}^2\left(\frac{\pi(x_1 - x_2 + \mathrm{i}(y_1 + y_2))}{2h}\right) \sin^2(\theta_1) \Bigg\} \Bigg],
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
\Omega_2 =\ & \frac{1}{16h^3} e^{-\mathrm{i}(2\theta_2+\theta_1)} \Bigg[32U_0 y_2 h \epsilon e^{\mathrm{i}\theta_1} \left(1+e^{2i\theta_2}\right)^2 - 64U_0 h^2 \epsilon e^{\mathrm{i}(2\theta_2+\theta_1)} \cos^2(\theta_2) \\
& + \pi^3 {r_0}^2 v_0 \Bigg\{ e^{\mathrm{i}(5\theta_2+\theta_1)} \cot\left(\frac{\pi y_2}{h}\right) \csc^2\left(\frac{\pi y_2}{h}\right) - e^{\mathrm{i}(\theta_2+\theta_1)} \cot\left(\frac{\pi y_2}{h}\right) \csc^2\left(\frac{\pi y_2}{h}\right) \\
& - 2e^{\mathrm{i}(\theta_2+\theta_1)} \cos^2(\theta_2) \cot\left(\frac{\pi y_2}{h}\right) \csc^2\left(\frac{\pi y_2}{h}\right) - 2e^{\mathrm{i}(3\theta_2+\theta_1)} \cos^2(\theta_2) \cot\left(\frac{\pi y_2}{h}\right) \csc^2\left(\frac{\pi y_2}{h}\right) \\
& - 2\mathrm{i} \coth\left(\frac{\pi(x_2-x_1-\mathrm{i}(y_2-y_1))}{2h}\right) \mathrm{csch}^2\left(\frac{\pi(x_2-x_1-\mathrm{i}(y_2-y_1))}{2h}\right) \\
& + 2\mathrm{i}e^{2\mathrm{i}(2\theta_2+\theta_1)} \coth\left(\frac{\pi(x_2-x_1+\mathrm{i}(y_2-y_1))}{2h}\right) \mathrm{csch}^2\left(\frac{\pi(x_2-x_1+\mathrm{i}(y_2-y_1))}{2h}\right) \\
& - 2\mathrm{i}e^{2\mathrm{i}(\theta_2+\theta_1)} \cos^2(\theta_2) \coth\left(\frac{\pi(x_2-x_1-\mathrm{i}(y_2+y_1))}{2h}\right) \mathrm{csch}^2\left(\frac{\pi(x_2-x_1-\mathrm{i}(y_2+y_1))}{2h}\right) \\
& + \mathrm{i}\left(-1+e^{4\mathrm{i}\theta_2}\right) \coth\left(\frac{\pi(x_2-x_1+\mathrm{i}(y_2+y_1))}{2h}\right) \mathrm{csch}^2\left(\frac{\pi(x_2-x_1+\mathrm{i}(y_2+y_1))}{2h}\right) \\
& + 2\mathrm{i}e^{2\mathrm{i}\theta_2} \cos^2(\theta_2) \coth\left(\frac{\pi(x_2-x_1+\mathrm{i}(y_2+y_1))}{2h}\right) \mathrm{csch}^2\left(\frac{\pi(x_2-x_1+\mathrm{i}(y_2+y_1))}{2h}\right) \\
& - 4\mathrm{i}e^{\mathrm{i}(\theta_2+\theta_1)} \cos(\theta_2) \cot\left(\frac{\pi y_2}{h}\right) \csc^2\left(\frac{\pi y_2}{h}\right) \sin(\theta_2) \\
& - 4e^{2\mathrm{i}(\theta_2+\theta_1)} \cos(\theta_2) \coth\left(\frac{\pi(x_2-x_1-\mathrm{i}(y_2+y_1))}{2h}\right) \mathrm{csch}^2\left(\frac{\pi(x_2-x_1-\mathrm{i}(y_2+y_1))}{2h}\right) \sin(\theta_2) \\
& + 2e^{\mathrm{i}(\theta_2+\theta_1)} \cot\left(\frac{\pi y_2}{h}\right) \csc^2\left(\frac{\pi y_2}{h}\right) \sin^2(\theta_2) + 2e^{\mathrm{i}(3\theta_2+\theta_1)} \cot\left(\frac{\pi y_2}{h}\right) \csc^2\left(\frac{\pi y_2}{h}\right) \sin^2(\theta_2) \\
& + 2\mathrm{i}e^{2\mathrm{i}(\theta_2+\theta_1)} \coth\left(\frac{\pi(x_2-x_1-\mathrm{i}(y_2+y_1))}{2h}\right) \mathrm{csch}^2\left(\frac{\pi(x_2-x_1-\mathrm{i}(y_2+y_1))}{2h}\right) \sin^2(\theta_2) \\
& - 2\mathrm{i}e^{2\mathrm{i}\theta_2} \coth\left(\frac{\pi(x_2-x_1+\mathrm{i}(y_2+y_1))}{2h}\right) \mathrm{csch}^2\left(\frac{\pi(x_2-x_1+\mathrm{i}(y_2+y_1))}{2h}\right) \sin^2(\theta_2) \Bigg\} \Bigg]. \qquad (8)
\end{aligned}
$$

S3. Expressions for dynamical system

The final system of equations of the dynamical system governing the position and orientation of both the fish swimming in an infinite channel is given by Eq. (14). The expressions of all the required functions are given below:

$$
\begin{aligned}
f_\xi(\xi_1, \xi_2, \theta_1, \theta_2, \Lambda) =\ & \frac{\pi^2}{24} \Bigg[-3\mathrm{i}e^{-\mathrm{i}\theta_2} \Big\{ \mathrm{csch}^2(\psi_1) + \mathrm{sech}^2(\psi_2) \Big\} + 3\mathrm{i}e^{\mathrm{i}\theta_2} \Big\{ \mathrm{csch}^2(\psi_3) + \mathrm{sech}^2(\psi_2) \Big\} \\
& + (\cos(2\pi\xi_1) - 5) \sec^2(\pi\xi_1) \sin\theta_1 \Bigg], \qquad (9a)
\end{aligned}
$$

$$f_\theta(\xi_1,\xi_2,\theta_1,\theta_2,\Lambda) = \frac{\pi^3}{8}(\sin(\theta_2)+\mathrm{i}\cos(\theta_2))\Bigg[-\cos(2\theta_1)\coth\left(\psi_2-\mathrm{i}\frac{\pi}{2}\right)\mathrm{csch}^2\left(\psi_2-\mathrm{i}\frac{\pi}{2}\right)$$
$$+\cos(2\theta_1)\cos(\theta_2)^2\coth\left(\psi_4+\mathrm{i}\frac{\pi}{2}\right)\mathrm{csch}^2\left(\psi_4+\mathrm{i}\frac{\pi}{2}\right)$$
$$+\coth(\psi_1)\,\mathrm{csch}^2(\psi_1)\,(\cos(2\theta_1)-\mathrm{i}\sin(2\theta_1))$$
$$-\mathrm{i}\coth\left(\psi_2-\mathrm{i}\frac{\pi}{2}\right)\mathrm{csch}^2\left(\psi_2-\mathrm{i}\frac{\pi}{2}\right)\sin(2\theta_1)$$
$$-\coth\left(\psi_4+\mathrm{i}\frac{\pi}{2}\right)\mathrm{csch}^2\left(\psi_4+\mathrm{i}\frac{\pi}{2}\right)\Big\{\mathrm{i}\cos^2(\theta_2)\sin(2\theta_1)+\cos(2\theta_1)\sin^2(\theta_2)$$
$$-\mathrm{i}\sin(2\theta_1)\sin^2(\theta_2)-\mathrm{i}\cos(2\theta_1)\sin(2\theta_2)-\sin(2\theta_1)\sin(2\theta_2)\Big\}$$
$$-(\cos(2\theta_1+2\theta_2)+\mathrm{i}\sin(2\theta_1+2\theta_2))\coth(\psi_3)\,\mathrm{csch}(\psi_3)$$
$$-2\mathrm{i}\cos(\theta_1)\cos(\theta_2)\sec^2(\pi\xi_1)\tan(\pi\xi_1)+2\cos(\theta_1)\sec^2(\pi\xi_1)\sin(\theta_2)\tan(\pi\xi_1)\Bigg], \tag{9b}$$

$$g_\theta(\xi_1,\theta_1) = 8\xi_1\cos^2\theta_1, \tag{9c}$$

$$f_\Lambda(\xi_1,\xi_2,\theta_1,\theta_2,\Lambda) = \frac{\pi^2}{24}\Bigg[\cos\theta_1(7+\cos(2\pi\xi_1))\sec^2(\pi\xi_1)-\cos\theta_2(7+\cos(2\pi\xi_2))\sec^2(\pi\xi_2)$$
$$+3(\cos\theta_1-\mathrm{i}\sin\theta_1)\Big\{\mathrm{csch}^2(\psi_1)-\cos(2\theta_1)\mathrm{sech}^2(\psi_2)-\mathrm{sech}^2(\psi_4)$$
$$+\mathrm{csch}^2(\psi_1)(\cos(2\theta_1)+\mathrm{i}\sin(2\theta_1))-\mathrm{i}\,\mathrm{sech}^2(\psi_2)\sin(2\theta_1)\Big\}$$
$$-3(\cos\theta_2-\mathrm{i}\sin\theta_2)\Big\{\mathrm{csch}^2(\psi_1)-\mathrm{sech}^2(\psi_2)-\cos(2\theta_2)\mathrm{sech}^2(\psi_4)$$
$$+\mathrm{csch}^2(\psi_3)(\cos(2\theta_2)+\mathrm{i}\sin(2\theta_2))-\mathrm{i}\,\mathrm{sech}^2(\psi_4)\sin(2\theta_2)\Big\}\Bigg], \tag{9d}$$

where

$$\psi_1 = \frac{\pi}{2}(\Lambda+\mathrm{i}(\xi_1-\xi_2)),\quad \psi_2 = \frac{\pi}{2}(\Lambda-\mathrm{i}(\xi_1+\xi_2)),$$
$$\psi_3 = \frac{\pi}{2}(\Lambda-\mathrm{i}(\xi_1-\xi_2)),\quad \psi_4 = \frac{\pi}{2}(\Lambda+\mathrm{i}(\xi_1+\xi_2)). \tag{10}$$

S4. Time trajectories near equilibrium configurations

Sample time traces of the positions and orientations of a fish pair originating from points in the vicinity of the equilibria of the system at different Λ values are shown in Figs. 1-4. These time trajectories, obtained by numerically integrating the complete nonlinear equations (Eq. (14) in main manuscript), illustrate the validity of our findings from linear stability theory. When perturbed from an unstable equilibrium configuration, such as staggered upstream in $\Lambda = 0$ (Fig. 1a), both the fish depart from the equilibrium conditions. Starting near an asymptotically stable configuration, such as wall-perpendicular configuration in $\Lambda = 0$ (Fig. 1c), the fish trajectories converge to the equilibrium configuration. Near a

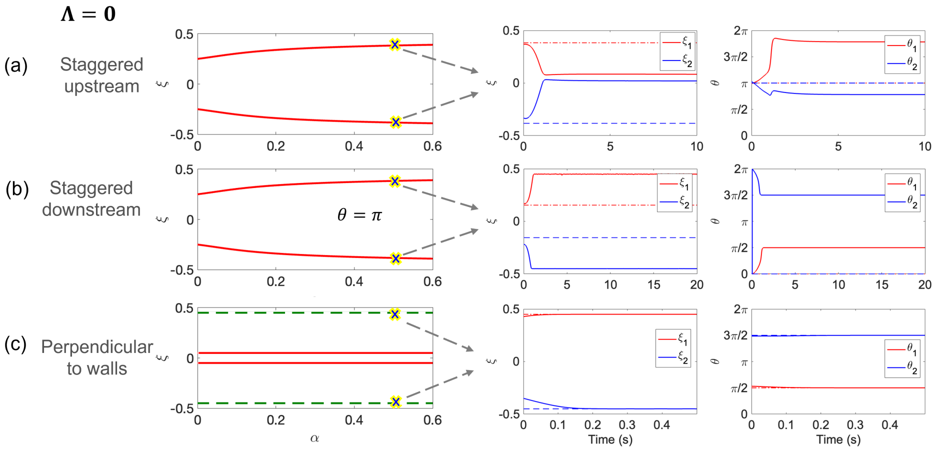

Figure 1: Sample trajectories in cross-stream coordinate (ξ_k) and orientation angle (θ_k) of the two fish ($k = 1, 2$) beginning from an initial configuration close to (a) staggered upstream, (b) staggered downstream, and (c) wall-perpendicular, at $\Lambda = 0$. In the ξ-α plots (first column), red lines represent unstable equilibria and green lines represent stable equilibria (light green solid: marginally stable, dark green dashed: asymptotically stable). In the time trajectories (second and third columns), the equilibrium conditions are marked by dashed lines.

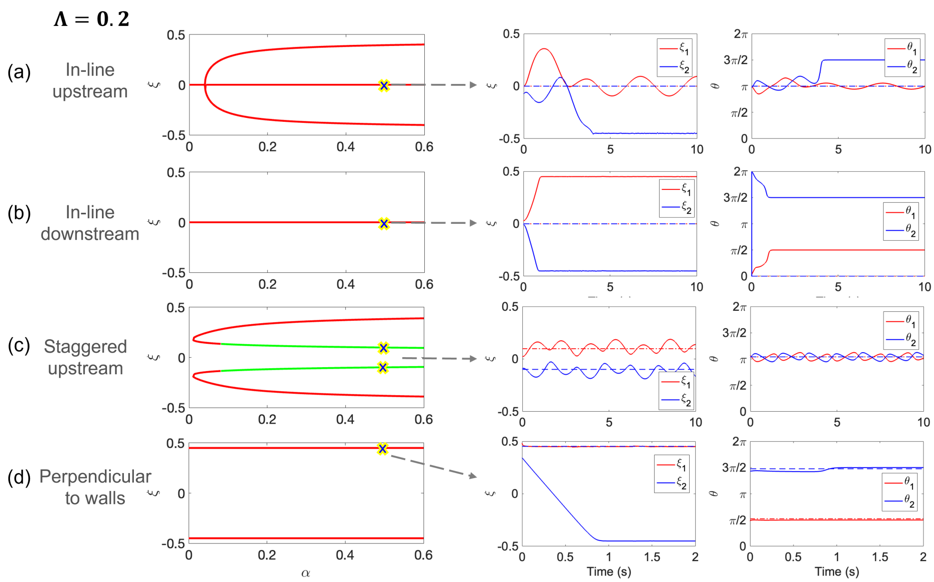

Figure 2: Sample trajectories in cross-stream coordinate (ξ_k) and orientation angle (θ_k) of the two fish ($k = 1, 2$) beginning from an initial configuration close to (a) in-line upstream, (b) in-line downstream, (c) staggered upstream, and (d) wall-perpendicular, at $\Lambda = 0.2$. In the ξ-α plots (first column), red lines represent unstable equilibria and green lines represent stable equilibria (light green solid: marginally stable, dark green dashed: asymptotically stable). In the time trajectories (second and third columns), the equilibrium conditions are marked by dashed lines.

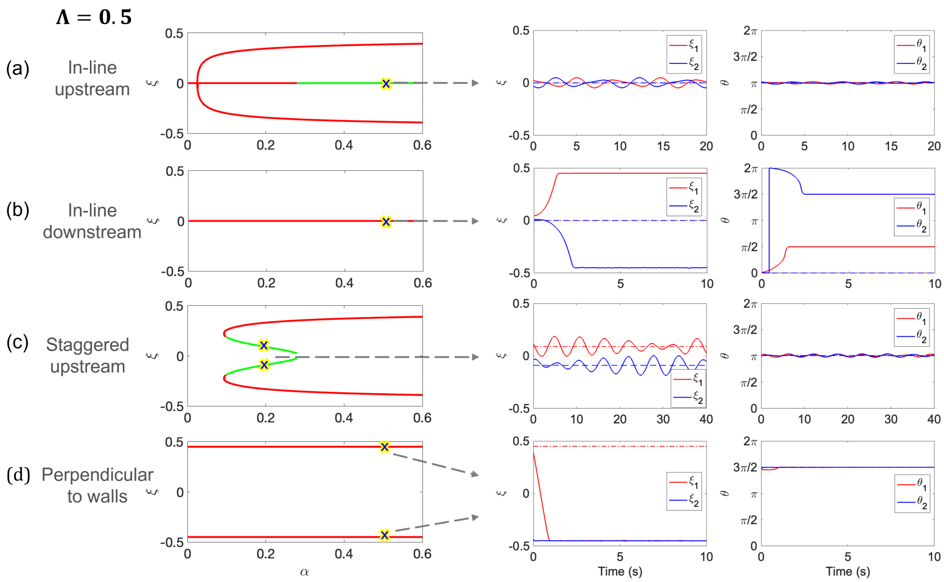

Figure 3: Sample trajectories in cross-stream coordinate (ξ_k) and orientation angle (θ_k) of the two fish ($k = 1, 2$) beginning from an initial configuration close to (a) in-line upstream, (b) in-line downstream, (c) staggered upstream, and (d) wall-perpendicular, at $\Lambda = 0.5$. In the ξ-α plots (first column), red lines represent unstable equilibria and green lines represent stable equilibria (light green solid: marginally stable, dark green dashed: asymptotically stable). In the time trajectories (second and third columns), the equilibrium conditions are marked by dashed lines.

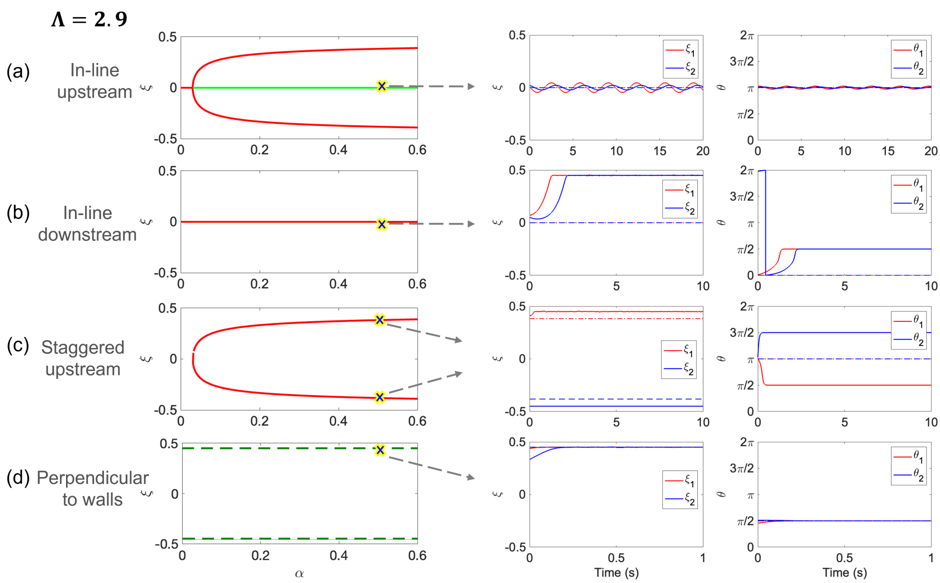

Figure 4: Sample trajectories in cross-stream coordinate (ξ_k) and orientation angle (θ_k) of the two fish ($k = 1, 2$) beginning from an initial configuration close to (a) in-line upstream, (b) in-line downstream, (c) staggered upstream, and (d) wall-perpendicular, at $\Lambda = 2.9$. In the ξ-α plots (first column), red lines represent unstable equilibria and green lines represent stable equilibria (light green solid: marginally stable, dark green dashed: asymptotically stable). In the time trajectories (second and third columns), the equilibrium conditions are marked by dashed lines.

neutrally stable equilibrium point, trajectories oscillate about the equilibrium (for example, in $\Lambda = 0.5$ case in Fig. 1a). Similar behavior is observed at other Λ values.

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