Hybrid axisymmetric model for forced heave of a shallowly submerged cylindrical wave energy converter: Supplementary Material

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A Linear frequency-domain matched model

As an aside to the development of the time-domain hybrid model, it is useful to consider a fully linear model that incorporates the matching method described in section 2 (main text). That is, a model in which the exterior and core regions are modelled using linear potential flow theory, but the core is matched to the exterior using only the surface elevation and piston velocity at the boundary. Such a model does not take proper account of the potential flow around the corner, as in the full linear potential flow model, but serves to test the matching method and gives an indication of the discrepancy introduced by this choice. This model requires a potential flow solution for the core region with an oscillating piston boundary. This will be described below and then matched to the exterior solution in the frequency domain.

A.1 Core solution

In the core region a velocity potential ϕ satisfying the free surface condition ($-\omega^2\phi + g\partial_z\phi = 0\big|_{z=0}$, where the subscript denotes the partial differentiation variable), piston condition ($\partial_r\phi = 1|_{r=a,-s\le z\le 0}$) and the boundary condition on top of the cylinder ($\partial_z\phi = 0|_{0\le r\le a, z=-s}$) is:

$$\phi^{TC}(r) = \frac{J_0(\kappa r)}{-\kappa J_1(\kappa a)}, \tag{1}$$

where κ is the solution to the shallow water dispersion equation $\kappa = \omega/\sqrt{gs}$ and J_n denotes Bessel functions of the first kind in the standard notation. This core solution is for a fixed cylinder; the heave rigid body mode is accounted for separately in the next section.

A.2 Matching method

In order to solve the decomposed heave radiation problem in the frequency domain, we match the surface elevation at the boundary between the core and exterior domains. We adopt notation where $\hat{}$ denotes a complex amplitude, such that for example the heave motion ζ is given as $\zeta = \mathbf{Re}\{\hat{\zeta}e^{i\omega t}\}$. In the exterior region, the surface elevation at the boundary is the sum of the surface-piercing cylinder value proportional to the heave velocity $\hat{u}_h = i\omega\hat{\zeta}$, and the exterior piston value proportional to the piston velocity $\hat{u}_r$, such that

$$\hat{\eta}_{\mathrm{E}} = \hat{\eta}_{\mathrm{SP}}\hat{u}_h + \hat{\eta}_{\mathrm{PE}}\hat{u}_r. \tag{2}$$

In the core region the surface elevation at the boundary is the sum of the surface elevation due to the rigid body mode $\hat{\eta}_{\mathrm{H}} = \hat{\zeta}$ and that due to the motion of the piston on the interior:

$$\hat{\eta}_{\mathrm{I}} = \hat{\zeta} + \hat{\eta}_{\mathrm{PI}}\hat{u}_r. \tag{3}$$

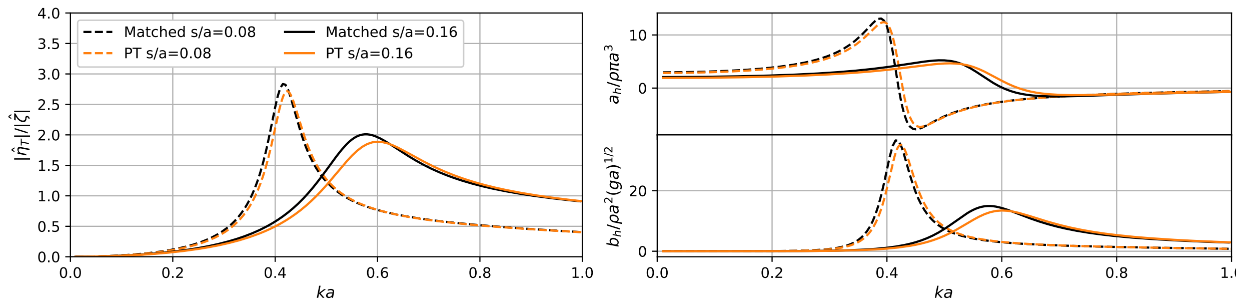

Figure 1: Comparison of the frequency-domain matched component solution ('matched') and full linear potential flow solution (PT). Left: Surface elevation at the boundary ($r = a$). Right: Added mass (top row) and damping (bottom row). $T/a = 0.4$ and $h/a = 2.4$.

Then if we equate the surface elevation in the interior and exterior, $\eta_{\mathrm{E}} = \eta_{\mathrm{I}} \equiv \eta_{\mathrm{T}}$, we can solve for the radial velocity of the piston ($\hat{u}_r$) and surface elevation at the boundary ($\hat{\eta}_{\mathrm{T}}$) for given input heave velocity ($\hat{u}_h$).

$$\hat{u}_r = \frac{1/(\mathrm{i}\omega) - \hat{\eta}_{\mathrm{SP}}}{\hat{\eta}_{\mathrm{PE}} - \hat{\eta}_{\mathrm{PI}}} \hat{u}_h. \tag{4}$$

The modulus of the complex amplitude of the surface elevation at the matching boundary $\hat{\eta}_{\mathrm{T}}$ (normalised by $\hat{\zeta}$) is shown in figure 1 (left side), plotted against exterior propagating wavenumber for moderate and small submergence. The full linear potential flow theory solution as described in McCauley *et al.* (2018) is also shown. The peak in surface elevation corresponds to the first axisymmetric resonance which occurs in the shallow water region. As noted by McIver & Evans (1984), this resonance occurs around the zeros of $J_0(\kappa a)$ for very shallow submergence. Looking at equations (4) and (1), and noting that $\hat{\eta}_{\mathrm{PE}}$ is shown in figure 3 (main text) while $\hat{\eta}_{\mathrm{SP}}$ is rather flat with frequency, it is apparent that the limiting behaviour will be the same, though for the parameter range considered here the coupling to the external flow shifts the resonant frequency significantly. The resonance is amplified and shifted to lower frequency for the smaller submergence case. We see that the linear matched model closely matches the full solution, with the largest discrepancy at the resonant peak. The discrepancy increases with increasing submergence, as is expected from the underlying assumption of the matched model - that the flow in the core region is shallow, with no vertical variation in velocity at the edge of the cylinder. The heave added mass and damping is shown for the same cases in figure 1. Equation 4 indicates that the piston contribution to the interior free surface keeps the radial velocity in phase with the heave velocity below resonance, such that the added mass is large and the damping small. At resonance, the damping increases, while above resonance the radial velocity and heave velocity are close to 180° out of phase and the added mass is negative. These results indicate that the decomposition matching method described in the main text introduces some discrepancy for the linear frequency-domain matched case but that the error is small for shallow submergence. The comparison between the linear potential flow model and frequency-domain matched model may be considered analogous to the comparison between the solution of Newman *et al.* (1984) (which accounts for flow at the corner) to the linear analytical solution of Grue (1992) for flow over a shallowly submerged step (which allows only horizontal velocity at the matching boundary). Substituting Grue's analytical solution (see Grue, 1992, p. 468) into the expression for heave force in Newman *et al.* (1984) indicates a similar shift in the damping peak to lower frequency and higher amplitude. The matching method of Newman *et al.* (1984) and Grue (1992) is applicable for our parameter range since ($ks \ll 1$), however it will not pursued further here.

B Shallow submergence case

Comparison between the hybrid model and experiments for the small submergence case $s/a = 0.08$ are shown in figure 2. These cases are for relatively small amplitude $A/s = 0.14 - 0.17$. For this smaller submergence the non-linear hybrid model performs significantly better than the linear models in predicting the forces, as would be expected when the flow on top of the cylinder becomes very shallow (here $426 < U < 695$).

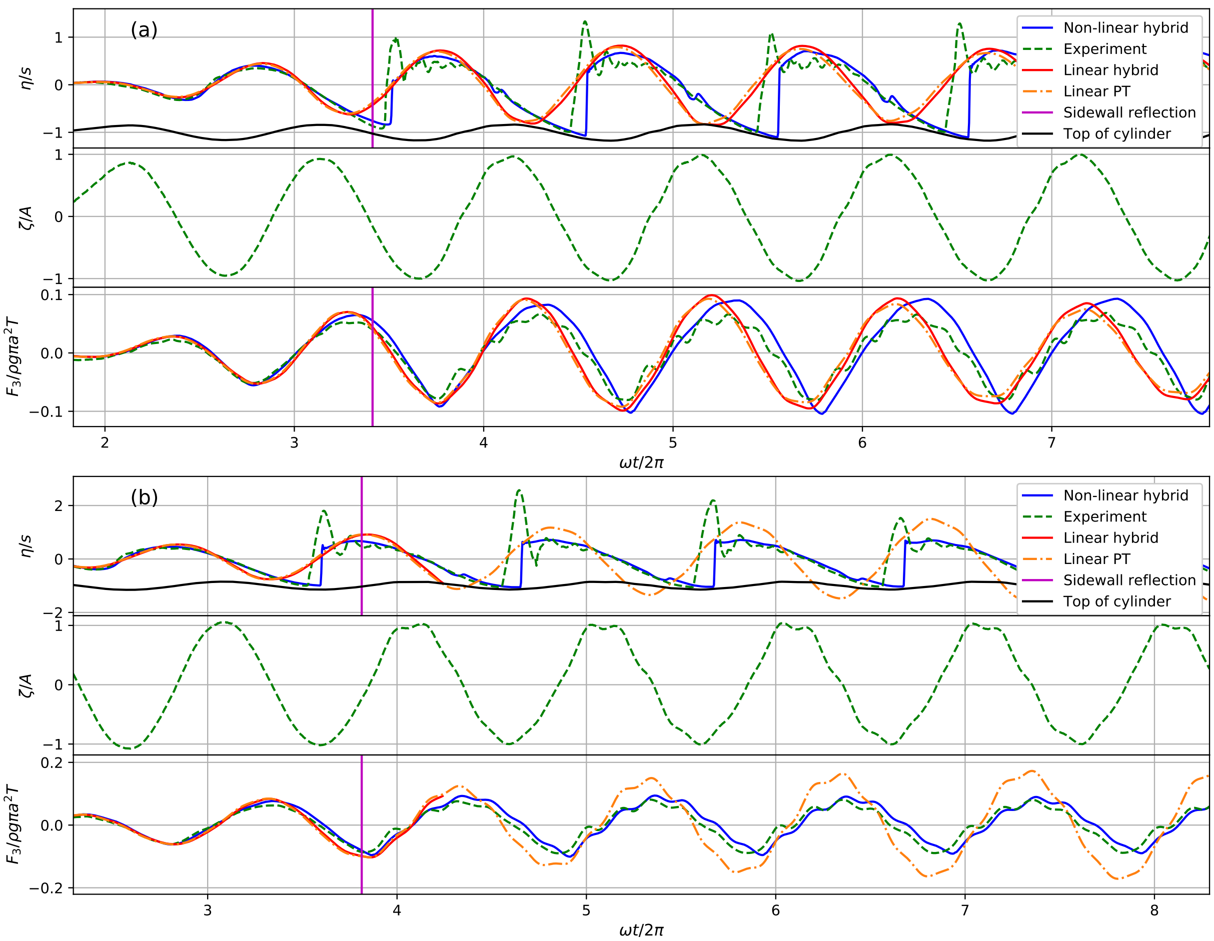

Figure 2: Surface elevation at the cylinder centre (top), heave displacement (middle) and heave hydrodynamic force (bottom). (a) $ka = 0.35$, $A/s = 0.17$, (b) $ka = 0.42$, $A/s = 0.14$. All plots: $s/a = 0.08$, $h/a = 2.4$, $T/a = 0.4$.

References

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